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LETTERS TO THE EDITOR

AN APPROXIMATE METHOD FOR THE FREE VIBRATION ANALYSIS OF PARTIALLY FILLED AND SUBMERGED, HORIZONTAL CYLINDRICAL SHELLS

A. Ergin

Faculty of Naval Architecture and Ocean Engineering, Istanbul Technical University, Maslak, 80626, Istanbul, Turkey

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1. INTRODUCTION

The free vibration analysis of partially filled, vertical cylindrical shells has been dealt with by various authors (see, for example, references [1, 2]). However, only a few studies have been found regarding the vibration of partially filled and submerged horizontal cylindrical shells [3–5]. In their studies, Amabili and Dalpiaz [3] and Amabili [4] investigated the natural frequencies and mode shapes of a partially filled cylindrical shell experimentally, and both experimentally and analytically. Amabili [5], on the other hand, studied the free vibration characteristics of the shell when partially filled and submerged.

This letter presents an approximate solution method for the free vibration analysis of horizontal cylindrical shells partially filled and/or submerged in fluid. For the vibration of the thin cylindrical shell Love's shell theory is adopted. In the investigation, it is assumed that the fluid is ideal, i.e., inviscid, incompressible and its motion is irrotational. Therefore, the fluid forces are associated with the inertial effect of the fluid: i.e., the fluid pressure on the wetted surface of the structure is in phase with the structural acceleration. The fluid velocity potential function adopted does not impose an appropriate boundary condition on the fluid's free surface. However, it satisfies the kinematic boundary condition on the wetted surface of the shell. The fluid–structure interaction problem is solved by using the Rayleigh–Ritz method, and the natural frequencies and associated mode shapes are calculated to assess the influence of the internal and external fluids on the dynamic behaviour of the shell. To assess the validity of the method, comparison of the predictions is made with the experimental result found in the literature. A very good agreement between the predictions and experiments is obtained.

2. THEORETICAL FORMULATION

2.1. *Motion of shell*

Figure 1 shows the cylindrical shell partially filled and submerged with a horizontal axis. The thin cylindrical shell is of length L, mean radius R, shell thickness h and is simply supported at both ends. The shell material is assumed to be homogeneous and isotropic. Let the quantities x, θ and r denote the axial, circumferential and radial co-ordinates, respectively, while u, v and w represent their corresponding displacement components of a point in the middle surface of the shell. The displacement components for simply supported end conditions can be expressed in the form [6]

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$$u = \sum_m \sum_n U_{mn} \cos\left(\frac{m\pi x}{L}\right) \Psi_{un}\, e^{i\omega t}; \qquad v = \sum_m \sum_n V_{mn} \sin\left(\frac{m\pi x}{L}\right) \Psi_{vn}\, e^{i\omega t};$$

$$w = \sum \sum W_{mn} \sin\left(\frac{m\pi x}{L}\right) \Psi_{wn}\, e^{i\omega t}, \quad (1)$$

where U_{mn}, V_{mn} and W_{mn} are amplitudes of the component vibrations, m and n define the nodal arrangements along the cylindrical shell and around the circumference, respectively, and ω and t respectively denote the circular frequency and time. It should be noted, for the shell partially filled and submerged, that the vibration of the shell involves symmetric and antisymmetric modes. The symmetric (antisymmetric) modes refer to those having normal displacements (w) that are symmetric (antisymmetric) with respect to the plane through the center of the shell and perpendicular to the free surface of the fluid. Therefore, the circumferential functions that are used for the shell may be expressed as; for symmetric modes,

$$\Psi_{un} = \cos(n\theta), \qquad \Psi_{vn} = \sin(n\theta), \qquad \Psi_{wn} = \cos(n\theta), \quad (2)$$

and, for antisymmetric modes,

$$\Psi_{un} = \sin(n\theta), \qquad \Psi_{vn} = -\cos(n\theta), \qquad \Psi_{wn} = \sin(n\theta). \quad (3)$$

The strain and kinetic energies of the shell, therefore, can be easily calculated by using equation (1) as in reference [7].

2.2. *Motion of fluid*

The fluid is assumed ideal, i.e., inviscid, incompressible and its motion is irrotational, and there exists a fluid velocity potential function Φ which satisfies Laplace's equation, $\nabla^2\Phi = 0$, throughout the fluid domain. At the fluid–structure interface, continuity considerations require that the normal velocity of the fluid is equal to that of the structure. Thus,

$$\partial\Phi/\partial r|_{r=R} = \dot{w}, \quad (4)$$

where $\dot{w}$ denotes the local velocity on the wetted surface of the structure.

The solution of Laplace's equation may be assumed as [8]

$$\Phi = \sum_m \sum_n \phi_{mn}(r) \sin\left(\frac{m\pi x}{L}\right) \Psi_{wn}\, e^{i\omega t}. \quad (5)$$

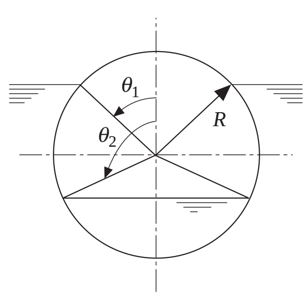

Figure 1. Cylindrical shell partially filled and submerged.

$\phi_{mn}(r)$, for the contained (internal) and surrounding (external) fluids, respectively, may be expressed as

$$\phi^{in}_{mn}(r) = A_{mn}\,\mathrm{I}_n(m\pi r/L), \qquad \phi^{ex}_{mn}(r) = B_{mn}\,\mathrm{K}_n(m\pi r/L), \tag{6, 7}$$

where I_n and K_n represent the modified Bessel functions of order n. A_{mn} and B_{mn} are arbitrary constants and can be obtained by using the kinematic boundary condition (4).

The kinetic energy of the entire fluid, therefore, may be determined by performing the following integration over the wetted surfaces of the shell [9]:

$$T_f = \frac{\rho_f}{2}\left\{\int_0^L\int_{\theta_2}^{2\pi-\theta_2}\left(\Phi^{in}\frac{\partial\Phi^{in}}{\partial r}\right)_{r=R} R\,\mathrm{d}\theta\,\mathrm{d}x - \int_0^L\int_{\theta_1}^{2\pi-\theta_1}\left(\Phi^{ex}\frac{\partial\Phi^{ex}}{\partial r}\right)_{r=R} R\,d\theta\,\mathrm{d}x\right\}, \tag{8}$$

where Φ^{in} and Φ^{ex} denote the velocity potential functions of the internal and external fluids respectively, and the angles θ_1 and θ_2 indicate, respectively, the external and internal dry portions of the half shell.

2.3. *Calculations of frequencies and mode shapes*

By applying the Rayleigh–Ritz method to the Lagrangian function [8], one obtains an eigenvalue problem of the type

$$(-\omega^2\mathbf{M} + \mathbf{K})\mathbf{D} = \mathbf{0}, \tag{9}$$

where **K** and **M** are the stiffness and mass matrices respectively. **D** is the vector of the vibration amplitudes and can be expressed as

$$\mathbf{D} = [U_{mn}, V_{mn}, W_{mn}]^T. \tag{10}$$

In the analysis, it is assumed that the shell structure preserves its modal forms (1) when it is in contact with fluid, and that each mode gives rise to the surface pressure distribution of the shell. However, when the cylindrical shell is partially in contact with internal or (and) external fluid(s), the hydrodynamic forces associated with the inertial effect of the fluid do not have the same spatial distribution as those of the modal forms. Consequently, this produces hydrodynamic coupling between the modes. This coupling effect is introduced into equation (9) through the matrix **M**. By solving the eigenvalue problem, equation (9), the uncoupled modes and associated frequencies of the shell partially in contact with fluid are obtained.

3. NUMERICAL RESULTS AND COMPARISONS

The horizontal cylindrical shell under consideration is partially filled, which is the configuration investigated in references [3–5]. The shell is of length $L = 664$ mm, radius $R = 175$ mm, thickness $h = 1.02$ mm and made of stainless steel. Figure 2 presents the modes and corresponding frequencies of the half filled shell. It must be realized that the mode shapes of the half filled cylindrical shell are either symmetric or antisymmetric about the symmetry plane of the shell–fluid system. The modes in Figure 2 are ordered and numbered simply with resonance frequency increasing; because of that the circumferential mode shapes are not described by the circumferential wave patterns as in equation (1) for the partially filled cylinder. All the modes considered in Figure 2 have a shape with $m = 1$. The predicted frequencies and mode shapes compare very well with the corresponding experimental results, as seen in Figure 2. However, there are differences between the predicted results and experimental measurements. These differences lie in the range between 1·2% and 10%.

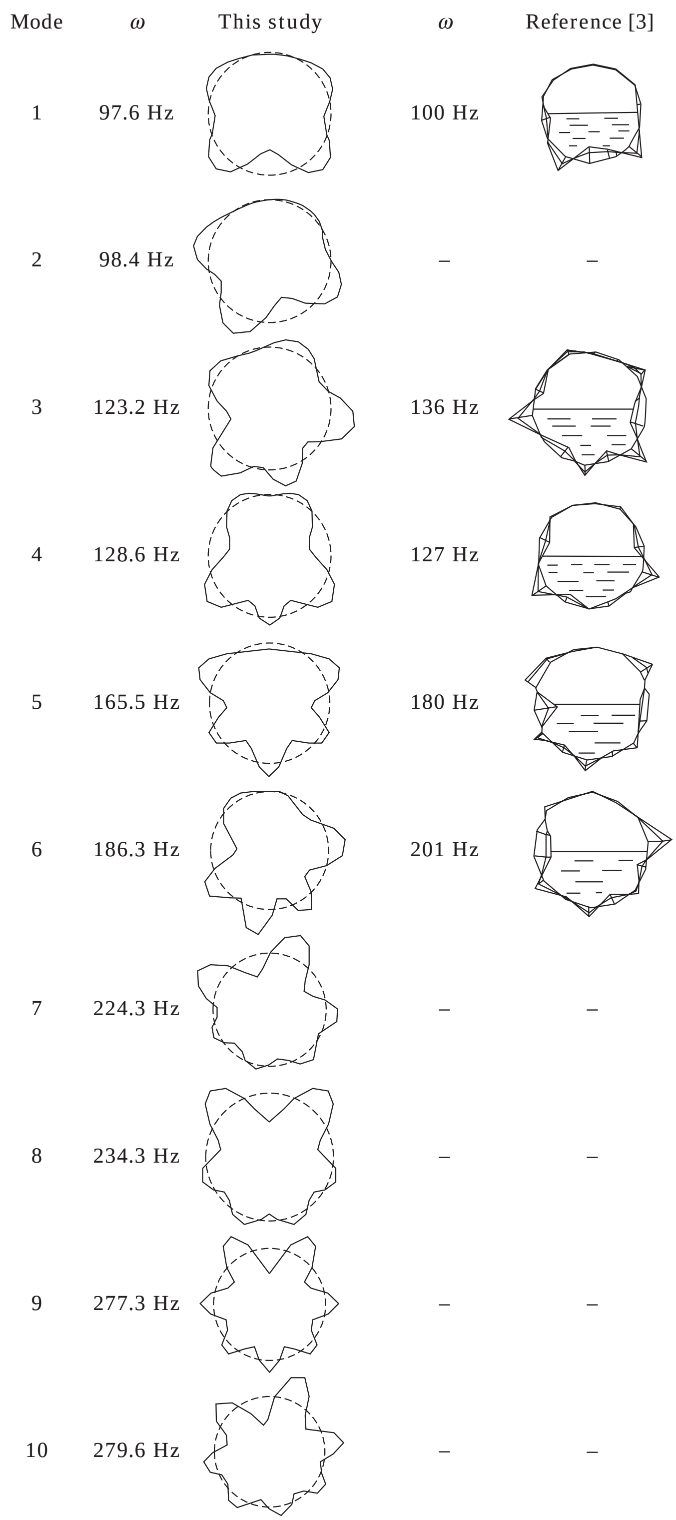

Figure 2. Modes and associated frequencies of half filled cylindrical shell.

TABLE 1

Comparison of frequencies (Hz) *for the* 4/5 *filled shell*

m	Mode	Present study	Experiment [4]	Error (%)
1	1	94·2	95·7	−1·6
1	2	95·4	97·1	−1·7
1	3	111·3	113·9	−2·3
1	4	115·0	116·3	−1·1
1	5	139·3	138·7	0·4
1	6	147·4	141·2	4·4
1	7	184·1	180·9	1·7
2	1	214·6	217·2	−1·2
2	2	215·5	219·6	−1·9
2	3	235·8	238·5	−1·1
2	4	240·0	241·3	−0·5
2	5	271·6	270·9	0·3

In a further study, different filling depths of the shell were considered. In Tables 1 and 2, the calculated frequencies obtained by the present method are compared with those of Amabili [4] respectively for the 4/5 filled and 1/5 filled shells. For the shell 4/5 full of water, the values of the present method are in good agreement with the experimental results, and the largest difference obtained is less than 5%. While, for the shell 1/5 full of water, the present results show larger differences in comparison with those of the shell 1/2 and 4/5 full of water. However, the differences lie in the range between 0·2% and 14% for the first eight modes.

For the partially filled shell, convergence studies were carried out to establish the number of modes needed for accurate solutions. A maximum number of 15 modes was adopted in the calculations for the converged results presented in Figures 2 and 3 and Tables 1 and 2.

The analysis was subsequently extended to investigate the effect of the external fluid on the dynamic behaviour of the shell. The calculated resonance frequencies are presented in Figure 3 for the cases of the empty–fully submerged, the empty–half submerged, the half filled–half submerged, the half filled–fully submerged, the completely filled–completely submerged and the empty shell. The frequency values behave as expected. That is to say that the frequencies decrease with increasing area of contact with the fluid. The largest area of contact was in the case of the completely filled–completely submerged shell. Therefore, the lowest frequencies occurred in this case (see Figure 3).

TABLE 2

Comparison of frequencies (Hz) *for the* 1/5 *filled shell*

m	Mode	Present study	Experiment [4]	Error (%)
1	1	102·8	107·9	−4·7
1	2	106·1	113·2	−6·2
1	3	151·7	174·4	−13·0
1	4	157·0	182·4	−13·9
1	5	231·1	237·1	−2·5
1	6	239·2	239·7	−0·2
1	7	239·3	253·2	−5·4
1	8	258·0	258·9	−0·3
2	1	227·4	232·7	−2·3

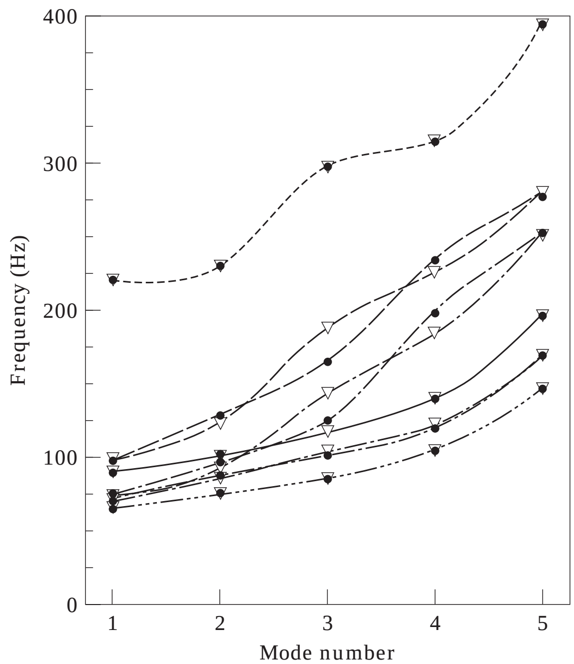

Figure 3. Predicted frequencies of cylindrical shell (●, symmetric mode; ▽, antisymmetric mode). ——, Empty–completely submerged shell; — —, empty–half submerged shell; —-—, half filled–half submerged shell; ·—·—, half filled–completely submerged shell; · · · —· ·—, completely filled–completely submerged shell; -----------, empty shell.

The method presented gives an approximate solution and the fluld velocity potential function adopted does not impose a boundary condition on the fluid's free surface. Thus, a more accurate method may be required. A solution method, imposing appropriate boundary conditions on the fluid's free surface, is under way and willl be reported later.

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