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CONTROL OF SOUND NOISE RADIATED FROM A PLATE USING DYNAMIC ABSORBERS UNDER THE OPTIMIZATION BY NEURAL NETWORK

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This paper reports a system for reducing sound radiated from a plate using electromagnetic vibration absorbers. Conventional design techniques for vibration absorbers are not appropriate for sound reduction absorbers, because higher-mode vibrations exert strong effects on noise. Hence, an analytical expression for noise level including higher modes has been first derived, then a method for obtaining optimal parameters for noise reduction absorbers is presented. In this method, an integrated value of the sound pressures in a frequency domain is taken as a cost function, and the parameters are decided by using a neural network procedure. An algorithm of the neural network for obtaining the parameters is given. Numerical calculations and experimental tests are carried out for some important cases.

1. INTRODUCTION

Vibration absorbers have been utilized for reducing vibrations of machines and structures, and a number of reports have been published concerning methods of tuning of absorbers [1, 2]. The tuning of the absorbers as just mentioned are based on the usual concept of minimizing vibrations. However, the optimal values of absorbers will be different from those when minimizing sound power radiation. From this situation, Fuller and Silcox [3] presented a method using active forces in conjunction with a radiated sound cost function to minimize radiated sound (the technique is called ASAC). For the vibration absorbers, an interesting method has been presented by Fuller *et al*. [4] for tuning absorbers based on ASAC which was applied to a cylindrical shell, but there was no experimental check. In the paper, the mass and locations of the absorbers were not included in the tuning parameters, so only the frequency ratio was tuned for a target frequency. Since the problem is complex, these parameters have been decided by considering mode shapes as given in reference [4], but the parameters do not always give correct optimal values. Structures are excited, in general, by mechanical vibrations involving various frequency components, so it is important to design the absorber by considering higher modes. But in previous studies the number of absorbers have been the same as that of the modes under consideration, so many absorbers would be required to control a number of higher modes. Hence a tuning method which sets all tuning parameters, including consideration of higher modes is desirable.

The present article discusses a method of tuning of absorbers without the disadvantages. Since the mass and locations of the absorbers have not been included in the tuning parameters in previous studies, the designed absorbers were not optimum. In the present article, all physical values of absorbers such as mass, spring constant, damping coefficient

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and location are taken to be the tuning parameters. When peak values involving higher modes are reduced, vibration amplitude decreases even if the exciting loads have high frequency components. This is also applicable to reducing vibrations for impulse response [1]. By considering this phenomena in the present article, the integration of the sound pressures over the considered frequency region is taken to be a cost function. This implies that the higher modes can be controlled by a few absorbers. But, when using the present method, the equation for predicting tuning parameters becomes non-linear with respect to the tuning parameters, and a number of parameters must be obtained from the non-linear equation. To solve the equation, a neural network procedure is applied.

Numerical calculations are carried out for a rectangular plate with opposite side clamped and the others free. To validate the method and the analysis, experimental tests have been carried out.

2. ANALYSIS OF SOUND RADIATED FROM A PLATE

Consider a rectangular plate with opposite sides built-in and the other ends free under a point sinusoidal excitation force $f = f_0 \sin(\omega t)$ as shown in Figure 1. Dynamic absorbers consisting of magnetic dampers and masses are connected to a surface of the plate as shown in Figure 1. Let the displacement of the mass of the absorber be u, displacement of the plate at the ith point be δ_1. The equation of motion of the absorber is

$$m\, \mathrm{d}^2u/\mathrm{d}t^2 + c(\mathrm{d}u/\mathrm{d}t + \mathrm{d}\delta_i/\mathrm{d}t) + k(u - \delta_i) = 0, \tag{1}$$

where m is the mass of the absorber, k the spring constant and c the damping coefficient. Substituting $\delta_i = \delta_{i0}\, \mathrm{e}^{\mathrm{j}\omega t}$ and $u = A_0\, \mathrm{e}^{\mathrm{j}\omega t}$ into equation (1), one obtains the amplitude of the displacement u:

$$A_0 = \frac{p^2 + 2\mu(\mathrm{j}\omega)}{[p^2 - \omega^2 + 2\mu(\mathrm{j}\omega)]}\, \delta_{i0}, \tag{2}$$

where, $2\mu = c/m$, $p^2 = k/m$ and $\mathrm{j} = \sqrt{-1}$. The force due to the absorber which acts at the point j in the plate is written by

$$Q = c\left(\frac{\mathrm{d}u}{\mathrm{d}t} - \frac{\mathrm{d}\delta_i}{\mathrm{d}t}\right) + k(u - \delta_i) = \frac{p^2 + 2\mu(\mathrm{j}\omega)}{p^2 - \omega^2 + 2\mu(\mathrm{j}\omega)}\, m\omega^2\delta_{i0} = B(\omega) m\omega^2\delta_{i0}. \tag{3}$$

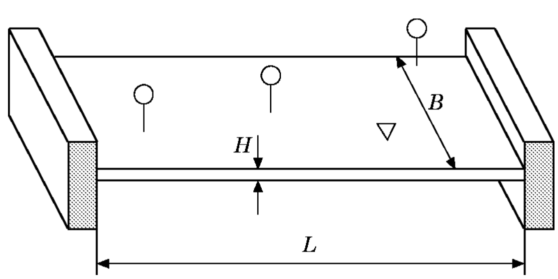

Figure 1. Geometry of a plate with vibration absorbers. $E = 2{\cdot}165$ N/cm^2, p = 7·8 N/cm^3. ○, Absorber setting point; ▽, input point.

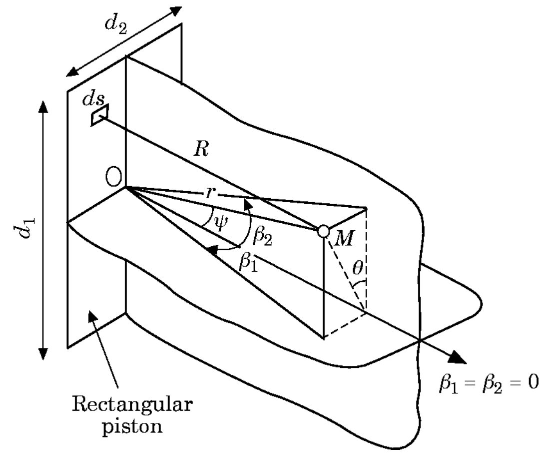

Figure 2. Co-ordinates of a rectangular piston model.

The plate is divided into n rectangular sections, and the finite element method is applied. By combining the force resulting from the absorber with the equation of motion of the plate, one can write in matrix form

$$\left\{-\omega^2\left([\mathbf{M}] - \sum_{k=1}^{n_1} [B(\omega)m\ \delta_k\ (z_{ii})]\right) + \mathrm{j}\omega[\mathbf{C}] + [\mathbf{K}]\right\}\{\boldsymbol{\delta}\} = \{\boldsymbol{f}\}, \tag{5}$$

where $\{\boldsymbol{f}\}$ is the force vector, $[\mathbf{M}]$ the mass matrix, $[\mathbf{C}]$ the damping matrix, $[\mathbf{K}]$ the stiffness matrix (such matrices are used in finite element text books), $\{\boldsymbol{\delta}\}$ the displacement vector, n_1 the number of dynamic absorbers, and $\delta_k\ (z_{ii})$ the Dirac delta function.

When the plate vibrates, an acoustical field is generated in front of the plate. The sound pressure is obtained by integrating small rectangular elements with length d_1 and width d_2 over the area, as given by references [5–8]:

$$p_m = j\rho f \int_S \dot{\delta}(x, y) \frac{\mathrm{e}^{-\mathrm{j}\omega t}}{R} \prod_{q=1}^{2} \frac{\sin \alpha_q}{\alpha_q} \, \mathrm{d}s, \tag{6}$$

where

$$\alpha_q = \pi d_q / \lambda \sin \beta_q$$

and where p_m is the sound pressure at a point M in the space (see Figure 2), f the vibration frequency, ρ the density of air, $\dot{\delta}(x, y)$ the vibration velocity of the plate ($\dot{\delta} = \mathrm{j}\omega\delta$ for the sinusoidal force), λ the wave length, β_q the angle as shown in Figure 2 and d_q the length of the small segment, subscript q the number of small segments and r the length measured from the center of segment to point M.

Equation (6) can also be expressed by the velocity vector $\{\dot{\boldsymbol{\delta}}\}$ of the finite element:

$$p_m = j\rho f \int_0^1 \int_0^1 [\mathbf{N}]\{\dot{\boldsymbol{\delta}}\} \frac{\mathrm{e}^{-\mathrm{j}\omega t}(1 + \alpha\xi + \beta\eta)}{r} \prod_{q=1}^{2} \frac{\sin \alpha_q}{\alpha_q} \det [\mathbf{J}] \, \mathrm{d}\xi \, \mathrm{d}\eta, \tag{7}$$

where

$$\alpha = \cos\psi \, \frac{d_2\,\xi}{d_1\sqrt{(d_1\,\eta)^2 + (d_2\,\xi)^2}}, \qquad \beta = \cos\psi \, \frac{d_1\,\eta}{d_2\sqrt{(d_1\,\eta)^2 + (d_2\,\xi)^2}}.$$

where [**N**] is the shape function describing the displacement of the plate:

$$w(x, y) = [\mathbf{N}]\{\boldsymbol{\delta}\}.$$

The function [**N**] is also given in the text book of the finite element method. Equation (7) denotes the sound pressure due to one square element, so the sound pressure p_m due to the plate can be obtained by the addition of the pressure p_m for all elements.

3. DESIGN OF VIBRATION ABSORBERS

The dynamic absorber used in the present article consists of a mass, a plate spring and a magnetic damper as shown in Figure 3. The magnetic damper is constructed from two permanent magnets and a conductor plate made of aluminum or copper. The conductor plate is inserted in the air gap between two magnets one of whose poles is N and the other is S. The spring constant of the absorber is adjusted by varying the length of the plate spring, and the damping is adjusted by varying the air gap.

Since the design of the absorbers in this article is not based on modal analysis [1], it is not necessary to make the number of absorbers coincide with that of the modes. Hence three vibration absorbers are used for controlling up to five vibration modes of the plate. In this case, the number of tuning variables becomes fifteen because for an absorber, the tuning variables are the co-ordinates x, y of the absorber, mass m, spring constant k and damping coefficient c.

Since the sound pressure is a function of the velocity of the plate, the sound pressure and the velocity of the plate are taken to be a cost function as shown in the following non-linear form:

$$J = \int_0^{f_0} \int_0^{L} \int_0^{B} (\varepsilon p_m^2 + g\dot{w}^2)\, \mathrm{d}f\, \mathrm{d}x\, \mathrm{d}y, \tag{8}$$

where p_m is the sound pressure at point M, $\dot{w}$ the velocity at the co-ordinates (x, y), L the length of the plate, B the width, ε and g the weights, and f_0 the upper frequency considered in the design.

A number of papers [9–14] discusses optimization problems. The neural network procedure is straightforward for solving non-linear problems like the present system, because neural networks can be used to approximate simultaneously the optimal function

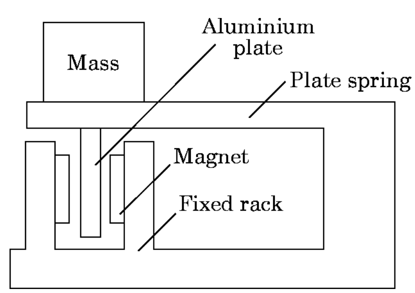

Figure 3. Geometry of the vibration absorber.

and compute the variation in the absorber parameters. Consider a three layered neural network system with an input layer (A), middle layer (B), and output layer (C). The number of units for the input layer is fifteen which correspond to the number of the tuning parameters, that for the middle layer is four, and that for the output layer one, because one cost function is used as has been mentioned. The output function of the neural network is

$$f(x_n, u) = 1/[1 + \exp(-x_n u)].$$

The symbols used in the neural network are O_c, output of the neuron at the output layer C; $O_{b,j}$ the jth output at the middle layer B; $O_{a,i}$, the ith output at the layer A; $W_{bc,j}$, the weight between the jth neuron at the middle layer and the output layer; $W_{ab,ij}$ the weight between the neuron i of the input layer and neuron j of the middle layer; X_i are parameters of the dynamic absorbers; J_n, the cost function at the nth iterations of the neural network calculation; T_n, the output function of the output layer at the nth iteration; E_n, the error function at the nth iteration; $u_{a,i}$, the inclination u of the output function $f(x_n, u)$ at the ith neuron of the input layer; $u_{b,J}$, inclination of the output function at the jth neuron of the middle layer; u_c, the inclination of output function of the output layer and η the coefficient for improving the input values of the neuron.

In the usual neural network procedure, only the weights of connection between neurons are varied for matching the output function of the neural network to the teaching signal. Then the optimal parameters of the absorbers cannot be obtained directly by using the usual neural network. In this article, the following algorithm is combined with the usual neural network. The tuning parameters of the three absorbers are written by the vector:

$$\bar{\mathbf{x}} = (x_1, x_2, \ldots, x_{15})^{\mathrm{T}} = (k_1, c_1, m_1, y_1, k_2, c_2, \ldots, m_3, x_3, y_3)^{\mathrm{T}}. \tag{9}$$

The error function E_n is taken as

$$E_n = (J_n - T_n)^2/2. \tag{10}$$

When one takes the teaching signal in the neural network to be zero, the output function T_n of the neural network becomes zero, so if the error function is reduced to zero, the cost function is also reduced to zero. This can be performed by the following algorithm. The derivation with respect to the input variable is

$$\partial E_n / \partial X_i = (J_n - T_n)\,(\partial J_n / \partial X_i - \partial Tn / \partial X_i). \tag{11}$$

Equation (11) is written by using the three point formula:

$$\frac{\partial E_n}{\partial x_i} = (J_n - T_n)\left\{\frac{3J_n - 4J_{n-1} + J_{n-2}}{2(x_{i,n} - x_{i,n-1})} - u_c\, o_c\,(1 - o_c)\right.$$
$$\left.\times\left[\sum_{j=i}^{4} u_{b,j}\, w_{bc,j}\, o_{b,j}\,(1 - o_{b,j}) u_{a,j}\, w_{ab,ij}\, o_{a,i}\,(1 - o_{a,i})\right]\right\}. \tag{12}$$

The improvement of the input variable is $-\eta(\partial E_n / \partial x_i)$. Hence the input variable at the nth iteration becomes

$$x_{i,n} = x_{i,n-1} - \eta\, \partial E_n / \partial x_i. \tag{13}$$

By using Equation (13), we have the optimal variable X_i which minimizes cost function J. The cost function J can be also minimized by repeating the usual iteration:

$$x_{i,n} = x_{i,n-1} - \eta(\partial J_n / \partial x_i).$$

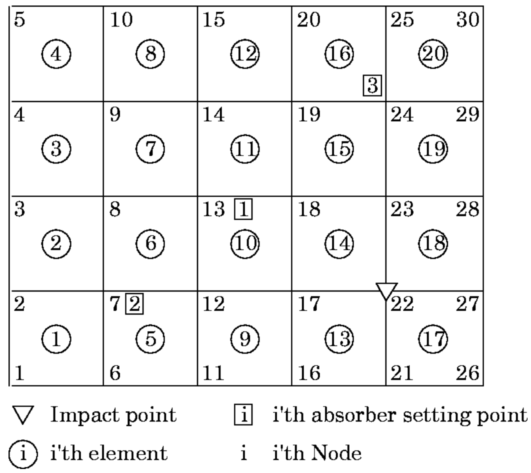

Figure 4. Divisions of the plate. ▽, Impact point; □, *i*th absorber setting point; ○, *i*th element; *i*, *i*th node.

However the convergence is poor when using the above equation. By using the equation combining the neural network with the proposed algorithm (equation (13)), the convergence becomes very rapid [15].

4. NUMERICAL EXAMPLES

A steel plate with two opposite ends clamped and two other ends free is chosen as a numerical example. The dimensions of the plate are as follows: the length is 400 mm, width is 250 mm, thickness is 1·7 mm. The plate is divided into 20 elements having 30 points. A sinusoidal force acts on point 22 as shown in Figure 4. Since the forcing point is not symmetric about both axes, all modes are induced. Figure 5 depicts the mode shapes of

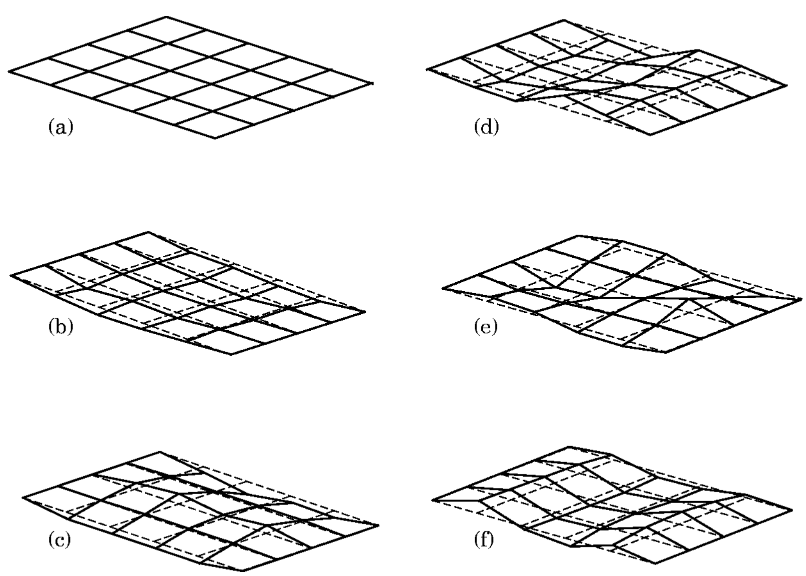

Figure 5. Mode shapes of the plate. (a) Analytical model; (b) First mode, 36·75 Hz; (c) second mode, 77·25 Hz; (d) third mode 117·0 Hz; fourth mode, 206·75 Hz; fifth mode 277·25 Hz.

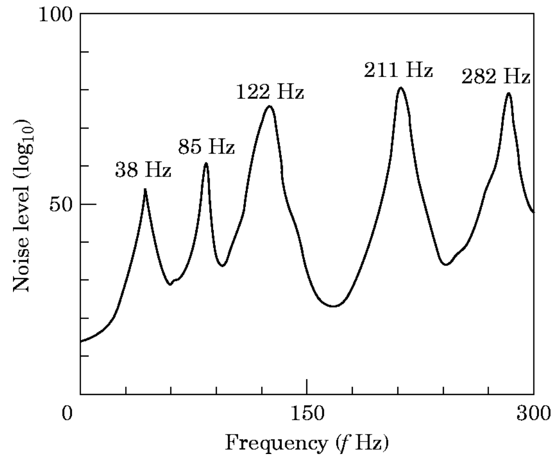

Figure 6. Numerical results of sound presure level for the plate without absorbers.

the plate up to the fifth mode. The amplitudes of vibration of higher modes will be reduced by a small damping. In the present case, the vibration absorbers have dampers, and so the higher modes are reduced by the fluid damper effect. This implies that the sound pressure due to higher modes will be reduced by the dampers. For this reason five modes are controlled in the design of the absorbers. The sound pressure is largest at the center of the plate, and so the calculated or measured point is at the center of the plate with the length from the plate surface being 500 mm in the z direction. Figure 6 depicts the theoretical sound pressure level for the plate without vibration absorbers. There are five resonant peaks in the figure up to 300 Hz. The peak values at high frequencies (fourth and fifth peaks) are larger than those of low frequencies (first and second peaks). This implies that the method of design of usual vibration absorbers based on vibration response is not appropriate, because the absorber is tuned to the low frequency region. The optimal parameters using the cost function based on the sound pressure level are shown in Table 1.

The numerical results of the sound pressure level are depicted in Figure 7. The peak values in Figure 7 are significantly smaller than those in Figure 6. Therefore the method developed in this paper can be applied to reduce sound noise radiated from a plate. In

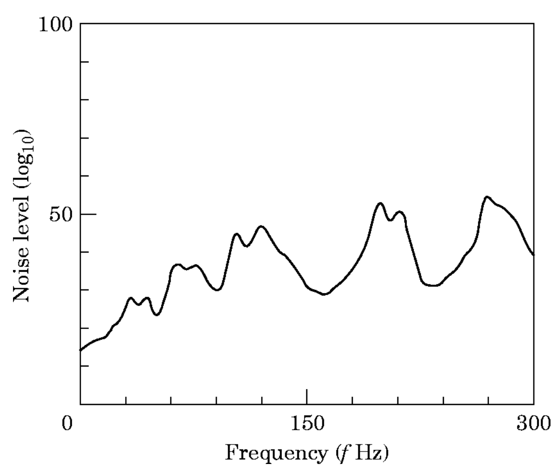

Figure 7. Numerical results of sound pressure level with the present absorbers.

TABLE 1

Optimal parameters obtained by the neural networks method

No.	k (N/mm)	c (N/mm)	m (kg)	x (mm)	y (mm)
1	1·1489	1171·2	0·0793	203·1	117·6
2	3·4890	154·96	0·0513	104·7	62·1
3	21·835	128·01	0·0467	290·4	197·9

particular, in the previous method, the number of absorbers should be equal the number of considered peaks of the response curve which means that the present case, five absorbers are required for controlling five modes. However with the present method, three absorbers can control five modes. This implies that the present method has an advantage over the previous method for designing vibration absorbers.

5. EXPERIMENT

To validate the present method and analysis, experimental tests have been carried out. The plate used in the experiment is the same as that in the numerical calculation. The vibration absorbers were constructed from the magnetic damper and the plate spring. The spring constant and the damping coefficient obtained by the present analysis (Table 1) are tuned by adjusting the length of the plate spring and air gap between the magnets.

First of all the vibration of the plate is investigated. The accelerations at the center of the plate were detected by the acceleration sensor, and the signals were inputted to the FFT analyzer, where the compliance is calculated. The vibration force was applied to the plate by an impulse hammer. Figure 8 shows the compliance versus the frequency of the plate without the absorbers, and Figure 9 the result for the plate with the present tuned absorbers. The resonant peaks are significantly reduced when using the present vibration absorbers in the vibration response.

As the next step, the sound pressure at the center of the plate is discussed. In the experiment concerning noise radiation, the plate was excited by a magnet and the sound pressure is detected by a microphone. The signal was inputted to a FFT analyzer, in which sound pressure level was calculated. Figure 10 shows the experimental result of the sound pressure level radiated from the plate without absorbers, and Figure 11 the result for the plate with the proposed present vibration absorbers. It can be seen that all peak values

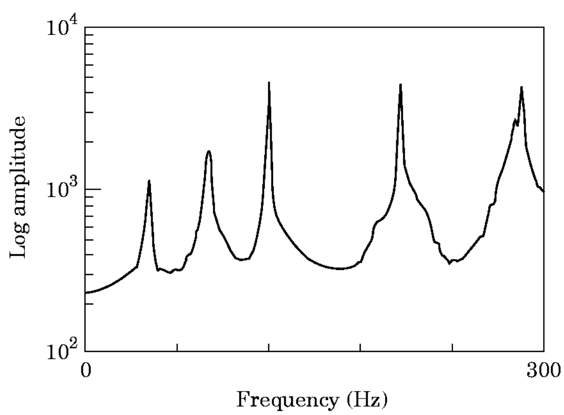

Figure 8. Experimental results of the compliance for the plate without absorbers.

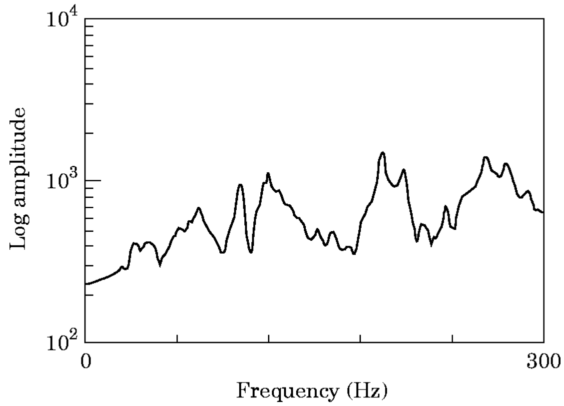

Figure 9. Experimental results of the compliance for the plate with the present absorbers.

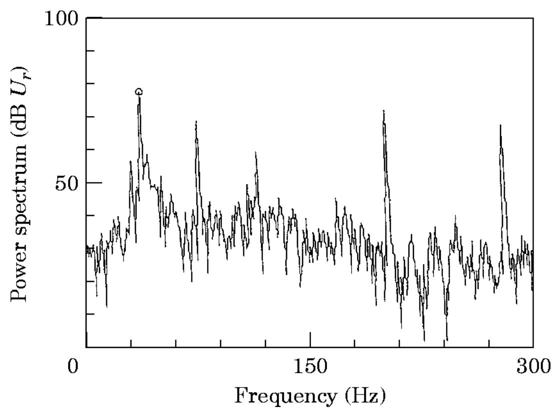

Figure 10. Experimental results of the sound pressure level for the plate without absorbers.

are reduced significantly when the proposed absorbers are used. The value of each peak is reduced to 22·5 dB for the first mode, 18·9 dB for the second mode, 12·6 dB for the third mode, 19·8 dB for the fourth mode, and 18·0 dB for the fifth mode.

The experimental result (Figure 10 and Figure 11) are in good agreement with the theoretical ones (Figure 6 and Figure 7).

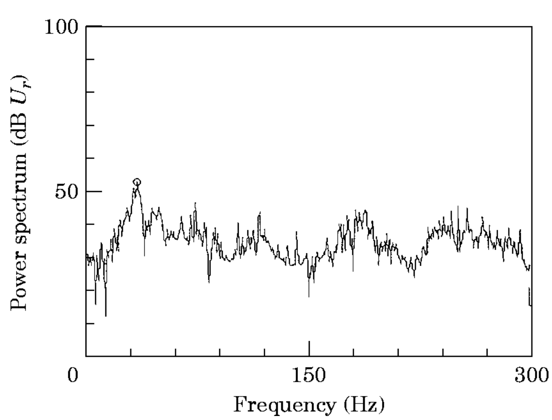

Figure 11. Experimental results of the sound pressure levels for the plate with the present absorbers.

6. CONCLUSION

A method for tuning vibration absorbers for reducing sound radiated from plates is presented, in which sound pressure is taken to be a cost function. The results are summarized as:

(1) A neural network is used to obtain optimal parameters of vibration absorbers by minimizing the cost function in terms of the radiated sound.

(2) Numerical results were obtained for a rectangular plate with two opposite sides clamped and the other free. To validate the method and analysis, experimental tests have been carried out for the same plate treated in the numerical calculation. The sound noise was significantly reduced when using the proposed absorbers.

(3) It is established that when the proposed absorber is used, the higher frequency sound radiation can be reduced by the use of fewer absorbers.

(4) Numerical results are in good agreement with the experimental results. The present analysis and the method are applicable to the design of vibration absorbers for controlling sound radiated from plates.

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