



ON THE MEASUREMENT OF ANGULAR VIBRATION DISPLACEMENTS:
A LASER TILTMETER

N. A. HALLIWELL, A. HOCKNELL AND S. J. ROTHBERG

*Department of Mechanical Engineering, Loughborough University, Ashby Road,
Loughborough, LE11 3TU, England*

(Received 20 January 1997, and in final form 27 July 1997)

1. INTRODUCTION

A novel modification and use of the optical geometry which is used as the basis for the Laser Torsional Vibrometer [1] is described. This latter instrument measures the torsional oscillation (or local rotational speed fluctuation) of a rotating component of arbitrary shape and in the presence of solid body vibrations of the component itself. The Laser Torsional Vibrometer relies on the mean speed of rotation of the component to produce a time resolved voltage analogue of the angular velocity. There are many cases of engineering interest, however, where a component undergoes an angular oscillation about an axis rather than a simple translation or rotation with a unidirectional angular velocity. In modal analysis, for example, where rotational and translation motions are of equal importance the absence of suitable transducers means rotational mobilities are rarely measured [2]. In what follows, it is shown how a modification to the optical geometry of the Laser Torsional Vibrometer can produce an instrument which is capable of measuring the local angular or tilt velocity in this situation and will therefore solve this problem.

2. THEORETICAL CONSIDERATIONS

A schematic diagram of the optical geometry for the Laser Torsional Vibrometer is shown in Figure 1. A laser beam of wavelength λ is amplitude divided into two equal-intensity parallel beams which are incident on a target rotating with a mean speed of N revolutions per second. Light from each incident laser beam is scattered from the moving target surface and undergoes a Doppler frequency shift. The portion in direct backscatter is collected on to the surface of a photodetector and the difference in frequency shift produces a heterodyne or “beat” frequency in the current output of the detector. It is straightforward to show [1] that the “beat” frequency f_D is given by

$$f_D = \left(\frac{4\pi d \cos \alpha}{\lambda} \right) N, \quad (1)$$

where α is the angle between the normal to the plane of the incident beams and the rotational axis of the component. In this way, when the beat frequency is demodulated and tracked it is possible to produce a time resolved voltage analogue of the rotational speed N . Now consider the optical geometry shown in Figure 2. The laser beam is incident on the surface of a rotating diffraction grating which diffracts the beam into distinct orders. Rotating the diffraction grating at a constant speed produces a frequency difference f_R between the first order of diffraction and the zero order [3]. Alternative means of producing frequency pre-shifted laser beams include Bragg cells [4]. With reference to Figure 2, the first and zero order beams are collimated to produce the same parallel beam geometry as

LETTERS TO THE EDITOR
Photodetector output beat frequency, f_D

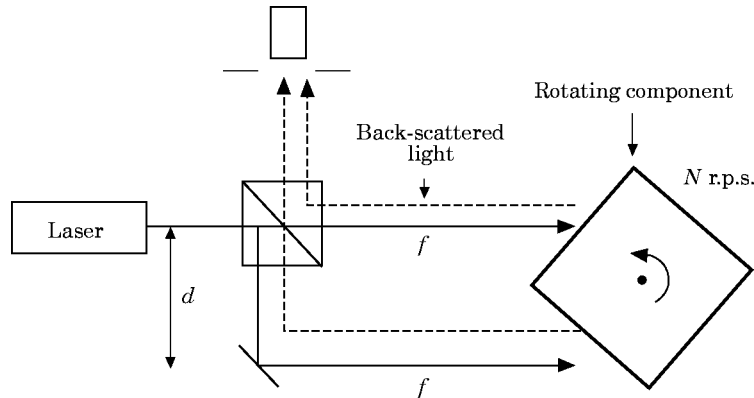


Figure 1. The laser torsional vibrometer. $f_D = (4\pi/\lambda)Nd \cos \alpha$, where α is the angle between one normal to the plane of the incident beam and the rotational axis.

for the Laser Torsional Vibrometer, but now with an inherent frequency difference between the incident beams. The beams are incident on the surface of a component which tilts about an axis rather than rotates with a mean speed. The component shown is a cylinder, but it should be emphasized that the measurement can be taken on a component of arbitrary shape [1].

When the component is stationary the beat frequency in the detector output is constant at f_R . If the component now begins to *tilt* about its axis it follows that the beat frequency f_T is given by

$$f_T = \left| f_R + \left(\frac{4\pi d \cos \alpha}{\lambda} \right) \dot{\theta} \right|, \quad (2)$$

where $\dot{\theta}$ is now the “instantaneous” local angular velocity or tilt velocity. Without the frequency pre-shift f_R , the direction of $\dot{\theta}$ is ambiguous. In the new system tilting of the component frequency modulates the pre-shift and produces the necessary directional output. In the same way as for the Laser Torsional Vibrometer, frequency demodulation of the detector output and “tracking” f_T produces a time resolved voltage analogue of the angular velocity and the measurement can be successfully taken from a component of arbitrary cross-section which is translating or vibrating as a solid body.

Further to this, it should be noted that a simple modification to this geometry will allow combination of scattered light from the target with frequency pre-shifted light directly from the rotating diffraction grating to provide a simultaneous measurement of normal-to-surface velocity.

3. EXPERIMENT

A schematic diagram of the apparatus used to measure the tilt of a small cylinder about its axis of symmetry is included in Figure 2. An accelerometer was used as an alternative means of tilt measurement via a rack and pinion arrangement as shown in the figure. The output was integrated and calibrated to provide an accurate measurement of the local angular velocity to better than one percent over the range of tilt angles tested. A Laser Tiltmeter was constructed by using a rotating diffraction grating to provide the constant frequency pre-shift between the incident beams. The zero and first order beams

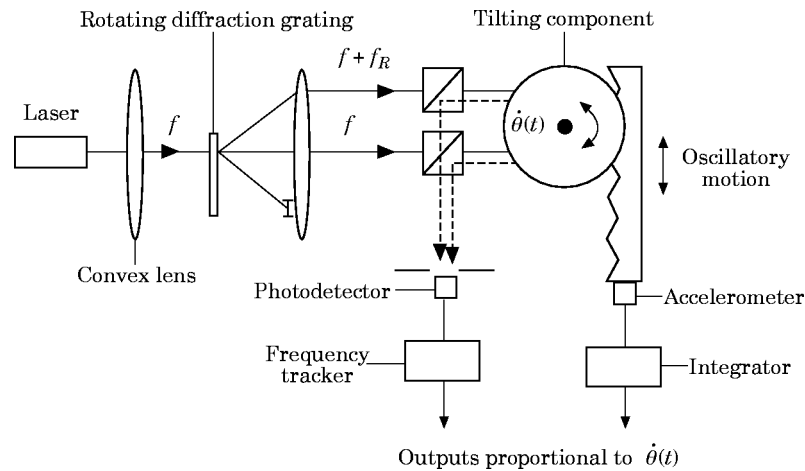


Figure 2. The laser tiltmeter.

from the grating were utilized and the first order beam was pre-shifted by 0.8 MHz. A commercial laser-Doppler frequency tracker was used to provide the output from the laser tiltmeter. A Brüel and Kjaer real time analyzer compared the angular displacement spectra from the two outputs at a nominal tilt frequency of 20 Hz and this result is shown in Figure 3 over a bandwidth of 400 Hz with a resolution of 0.5 Hz. Agreement is excellent until the level of angular displacement approaches the noise floor of the Laser Tiltmeter at *circa* 60 dB, which corresponds to one microradian. The latter level is dictated by laser speckle dynamics on the surface of the photodetector [5].

The maximum level of angular velocity which can be measured is dictated by the demodulation range of the frequency tracker and the magnitude of the frequency pre-shift which is used. It can be anticipated that a dynamic range in excess of 80 dB is readily achievable. With reference to equation (2) it should be noted that for a particular choice

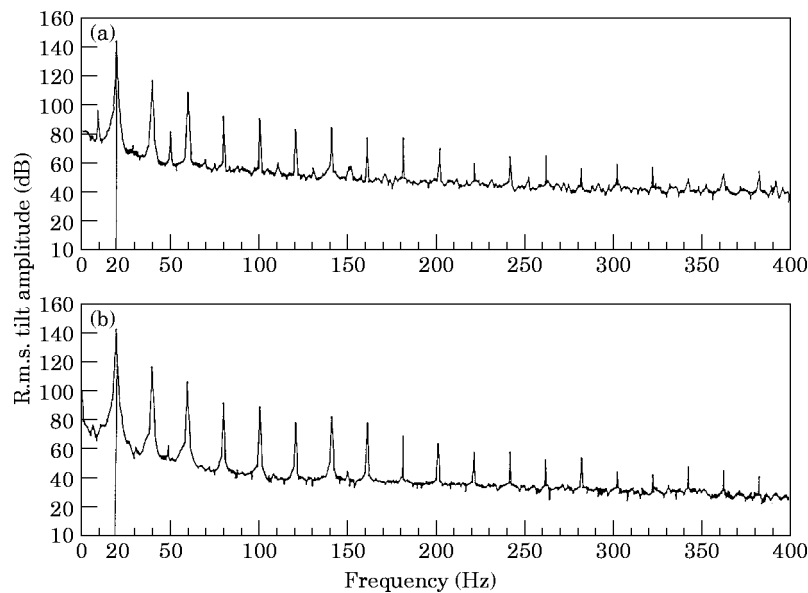


Figure 3. A comparison of tilt amplitude spectra. (a) Accelerometer; (b) laser tiltmeter.

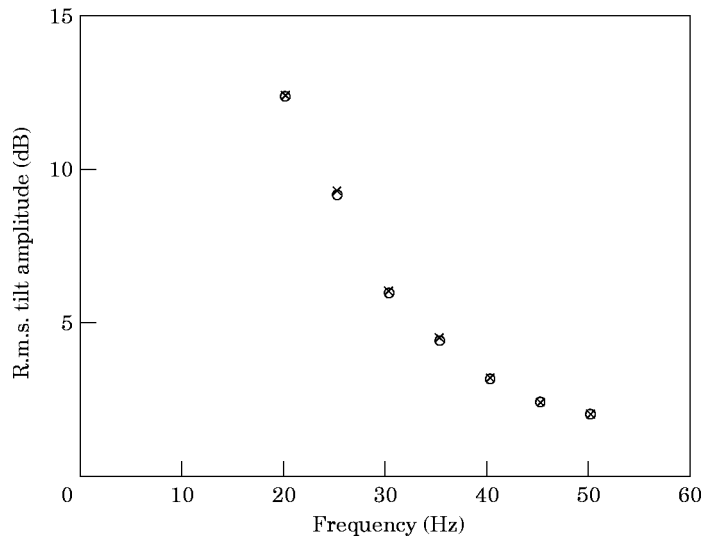


Figure 4. A comparison of tilt amplitudes. ○, Accelerometer; ×, laser tiltmeter.

of frequency pre-shift magnitude f_R the sensitivity of the measurement is flexible through appropriate choice of the beam separation d .

In Figure 4 is shown the result of a series of tilt amplitude measurements taken alternatively by using the accelerometer and laser tiltmeter at 5 Hz intervals over a frequency range 20–50 Hz. Excellent agreement is obtained.

4. CONCLUSIONS

The Laser Torsional Vibrometer [1] solved a particularly difficult measurement problem in providing a time resolved voltage analogue of the torsional vibration velocity (or local angular velocity) of a component which was rotating with a non-zero mean speed. The measurement was independent of the solid body oscillations of the target component or the component shape. Operation of the instrument, however, relied on a non-zero mean rotational speed. In this letter it has been shown how by using a frequency shifting device it is possible to harness the optical geometry of the Laser Torsional Vibrometer to measure the local angular or tilt velocity of a component which has a zero mean speed of rotation. This is achieved while maintaining the advantages of the measurement referred to earlier. Comparison of a Laser Tiltmeter measurement taken from a tilting component showed excellent agreement with results obtained using an accelerometer in a carefully designed and controlled experiment.

REFERENCES

1. N. A. HALLIWELL 1996. *Journal of Sound and Vibration* **190**, 399–418. The laser torsional vibrometer: a step forward in rotating machinery diagnostics.
2. D. EDWINS 1989 *Model Testing: Theory and Practice*. Letchworth: Research Studies Press.
3. J. OLDENGARM, A. H. VON KRIEKEN and H. RATERNIK 1973 *Optics and Laser Technology* **5**, 249–252. Laser Doppler velocimeter with optical frequency shifting.
4. P. BUCHAVE 1975 *Optics and Laser Technology* **7**, 11–16. Laser Doppler velocimeter with variable optical frequency shift.
5. S. J. ROTHBERG, J. R. BAKER and N. A. HALLIWELL 1989 *Journal of Sound and Vibration* **135**, 516–522. Laser vibrometry: pseudo vibrations.