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LETTERS TO THE EDITOR

A NOTE ON TRANSMISSION OF SOUND IN A WIDE PIPE WITH MEAN FLOW AND VISCOTHERMAL ATTENUATION

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G. Kirchhoff demonstrated in 1868 that the propagation of sound waves in a pipe is dispersive because of viscothermal losses at the pipe walls. His solution for a wide pipe shows that the inviscid wavenumber, $k = \omega/c_0$, where ω is the radian frequency and c_0 is the speed of sound, is modified to kK_0, where the propagation constant K_0 is given by [1]

$$K_0 = 1 + ((1 + \mathrm{i})/s)(1 + (\gamma - 1)/\sigma)/\sqrt{2}. \tag{1}$$

Here γ is the ratio of specific heat coefficients, $\sigma^2 = \mu c_p/\kappa$ is the Prandtl number, $s = a\sqrt{\rho_0\omega/\mu}$ is the shear wavenumber, μ is the shear viscosity coefficient, κ is the thermal conductivity, c_p is the specific heat coefficient at constant pressure, ρ_0 is the ambient density, a is the pipe radius and i denotes the unit imaginary number, $\exp(-\mathrm{i}\omega t)$ time dependence being assumed for all the fluctuating quantities.

Kirchhoff's wide pipe solution is valid for a homogeneous medium in the absence of mean flow. A one-dimensional *ad hoc* extension of the Kirchhoff solution to the viscothermal acoustic wave motion in a pipe carrying a uniform mean flow has been described by Davies [2]. This indicates that, in the presence of a superimposed uniform mean flow, the propagation constant for the wave motion in the direction of the mean flow, K^+ say, is different from the propagation constant for the wave motion in the opposite direction, K^- say, and these are given by, respectively,

$$K^+ = K_0/(1 + M), \qquad K^- = -K_0/(1 - M). \tag{2a, b}$$

Here M denotes the Mach number of the mean flow.

An alternative solution to the problem of viscothermal acoustic wave motion in a homogeneous wide pipe carrying a superimposed uniform mean flow has been indicated in reference [3]. This is obtained by asymptotic expansion, for large shear wavenumbers and small mean flow Mach numbers, of the solution of convective axisymmetric acoustic equations simplified in the manner of the Zwikker and Kosten theory, which is known to represent the full Kirchhoff solution accurately for large shear wavenumbers [1]. In this asymptotic theory, the propagation constants are given by

$$K^+ = K_0/(1 + MK_0), \qquad K^- = -K_0/(1 - MK_0), \tag{3a, b}$$

which differ from the expressions predicted by the one dimensional theory, equations (2a) and (2b), by the factor K_0 in the denominator. The general eigenequation for these wavenumbers is [3]

$$(K/(1 - KM))^2 = -(\mathrm{J}_0(\beta a)/\mathrm{J}_2(\beta a))[\gamma + (\gamma - 1)\mathrm{J}_2(\sigma\beta a)/\mathrm{J}_0(\sigma\beta a)], \tag{4}$$

where $\beta a = s\sqrt{\mathrm{i}(1 - KM)}$ and $\mathrm{J}_n(\cdot)$ denotes a Bessel function of order n. The asymptotic expansion of equation (4) is effected by $\mathrm{J}_2(z)/\mathrm{J}_0(z) \approx -1 + \mathrm{i}2/z$, which is valid if $\|z\| \gg 4$. But, for large s and small M, say, $M < 0{\cdot}3$, $K \approx \mp 1$ and $\beta a \approx (1 + \mathrm{i})s/\sqrt{2}$ and the condition $\|z\| \gg 4$ can be expressed as $\sigma s \gg 4$ or, say $s > 40$. Thus, for large s, equation (4) yields equations (3a) and (3b) where K_0 is given by,

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$$K_0 = \sqrt{1 + ((1 + \mathrm{i})/s)(1 + (\gamma - 1)/\sigma)\sqrt{2}}, \tag{5}$$

which is accurately represented by equation (1) for $s > 40$.

In the context of this asymptotic theory, the acoustic pressure distribution along the pipe is given by

$$p(x) = p^+(x) + p^-(x), \tag{6}$$

where x is the pipe axis and

$$p^+(x) = p^+(0)\exp(\mathrm{i}kK^+x), \qquad p^-(x) = p^-(0)\exp(\mathrm{i}kK^-x) \tag{7a, b}$$

in which K^+ and K^- are given by equations (3a) and (3b) and the superscripts "+" and "−" refer, as usual, to the waves travelling in $+x$ and $-x$ directions, respectively. Equation (6) also holds in the one dimensional theory of reference [2] with K^+ and K^- given by equations (2a) and (2b). Although the pressure distribution along the pipe is one-dimensional in the present asymptotic theory, the axial component of acoustic velocity, v, and the acoustic density, ρ, are functions of axial and radial co-ordinates. It has been shown in reference [3] that v and ρ can be expressed as

$$\rho_0 c_0 v(x, r) = h^+(r)p^+(x) + h^-(r)p^-(x), \tag{8}$$

$$c_0^2\rho(x, r) = g^+(r)p^+(x) + g^-(r)p^-(x), \tag{9}$$

where $h^+(r)$, $h^-(r)$, $g^+(r)$ and $g^-(r)$ are functions of the radial coordinate, r. For relatively large shear wavenumbers, the radial distribution of these functions is fairly uniform across the cross-section, except in the very close vicinity of the pipe wall. Therefore, for most practical purposes, equations (8) and (9) can be implemented for a wide pipe in the cross-sectionally averaged forms

$$\rho_0 c_0 v_m(x) = h_m^+ p^+(x) + h_m^- p^-(x), \qquad c_0^2\rho_m(x) = g_m^+ p^+(x) + g_m^- p^-(x), \tag{10, 11}$$

where the subscript m denotes a cross-sectionally averaged value and, from reference [3],

$$h_m^+ = -[K^+/(1 - K^+M)]\mathrm{J}_2(\beta^+a)/\mathrm{J}_0(\beta^+a), \tag{12a}$$

$$h_m^- = -[K^-/(1 - K^-M)]\mathrm{J}_2(\beta^-a)/\mathrm{J}_0(\beta^-a), \tag{12b}$$

$$g_m^+ = 1 + [2(\gamma - 1)/\sigma\beta^+a]\mathrm{J}_1(\sigma\beta^+a)/\mathrm{J}_0(\sigma\beta^+a), \tag{13a}$$

$$g_m^- = 1 + [2(\gamma - 1)/\sigma\beta^-a]\mathrm{J}_1(\sigma\beta^-a)/\mathrm{J}_0(\sigma\beta^-a), \tag{13b}$$

in which

$$\beta^+a = (1 + \mathrm{i})s/\sqrt{2(1 + K_0M)}, \qquad \beta^-a = (1 + \mathrm{i})s/\sqrt{2(1 - K_0M)}. \tag{14a, b}$$

The above described asymptotic expansion can be applied to these expressions by noting that $\mathrm{J}_1(z)/\mathrm{J}_0(z) \approx \mathrm{i} + 1/2z$. Thus, the asymptotic expressions for h_m^+, h_m^-, g_m^+ and g_m^- are obtained as

$$h_m^+ \approx K_0(1 - ((1 + \mathrm{i})/s)\sqrt{2(1 + K_0M)}), \tag{15a}$$

$$h_m^- \approx K_0(-1 + ((1 + \mathrm{i})/s)\sqrt{2(1 - K_0M)}). \tag{15b}$$

$$g_m^+ \approx 1 + ((1 + \mathrm{i})/s)((\gamma - 1)/\sigma)\sqrt{2(1 + K_0M)}, \tag{16a}$$

$$g_m^- \approx 1 + ((1 + \mathrm{i})/s)((\gamma - 1)/\sigma)\sqrt{2(1 - K_0M)}. \tag{16b}$$

Equation (6) with equations (3a) and (3b), and equations (10) and (11) with equations (15a), (15b), (16a) and (16b) constitute a pseudo-plane-wave theory for viscothermal wave propagation in a wide pipe. Other acoustic quantities can be obtained by using the relevant acoustic state equations. For example, the cross-sectionally averaged temperature and entropy fluctuations, T_m and s^m, can be determined from, respectively,

$$T_m/T_0 = p/p_0 - \rho_m/\rho_0, \qquad s_m/c_v = p/p_0 - \gamma\rho_m/\rho_0, \tag{17, 18}$$

where T_0 is the ambient temperature, p_0 is the ambient pressure and c_v is the specific heat coefficient at constant volume.

It may be of interest to compare equations (10) and (11) of the present pseudo-plane-wave theory with the corresponding relations of the one dimensional *ad hoc* theory described in reference [2]. In the latter theory, the isentropic relationship $\rho = p/c_0^2$ and the inviscid momentum equation $\rho_0(-\mathrm{i}\omega v + v_0\,\partial v/\partial x) + \partial p/\partial x = 0$ are assumed to be valid. Under these assumptions, equation (10) holds with equations (15a) and (15b) replaced by

$$h_m^+ = K_0/[1 + M(1 - K_0)], \qquad h_m^- = K_0/[-1 + M(1 - K_0)], \tag{19a, b}$$

respectively, and equation (11) holds with equations (16a) and (16b) replaced by,

$$g_m^+ = g_m^- = 1. \tag{20}$$

The present theory can be used in the prediction and measurement of acoustic wave transmission in wide pipes. In most practical low Mach number wide pipe applications, the shear wavenumber, s, which is also called the Stokes number, will be large enough to satisfy the conditions for the use of this asymptotic theory.

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