Journal of Sound and Vibration (1997) **208**(5), 861–863

LETTERS TO THE EDITOR

COMMENTS ON "DETECTION OF GRAZING ORBITS AND INCIDENT BIFURCATIONS OF A FORCED CONTINUOUS, PIECEWISE-LINEAR OSCILLATOR"

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(*Received* 13 *September* 1996, *and in final form* 3 *July* 1997)

The essential defects of the mathematical model and analytical deductions of reference [1] are shown in this note.

Some problems concerning the grazing phenomena of a forced continuous, piecewise-linear oscillator are discussed in reference [1]. It is interesting to study the dynamics of a piecewise-linear oscillator. However, some essential defects have been found in reference [1]. The details are as follows:

1. EQUATION (1)* IS A GENERAL EQUATION

Some indefinite factors of equation (1) result in indefiniteness of the deduction of reference [1].

A forced continuous, piecewise-linear oscillator is described by equation (1) in [1]. The indefiniteness of equation (1) is shown as follows:

(a) The coefficients of equation (1) may be constant or variable with time.

(b) The functions of the control parameter λ in equation (1) are undetermined. For example, two different functions of control parameter λ are

(i)
$$K_1(x) = \begin{cases} \lambda(x-a) + ka, & x \geqslant a; \\ ka, & x < a. \end{cases}$$

(ii)
$$K_2(x) = \begin{cases} \lambda(x-a) + \lambda ka, & x \geqslant a; \\ \lambda ka, & x < a. \end{cases}$$

We can see that the influences of the λ on the dynamical behaviors are different, especially for the derivatives with respect to λ.

2. DEFECTS OF THE BIFURCATION ANALYSIS IN SECTION 2 OF REFERENCE [1]

(1) The bifurcation point of equation (1) and its type must be studied first in accordance with the non-linear dynamics. However, these analyses are not presented, and cannot be presented because of the indefiniteness of equation (1) in reference [1].

(2) It is invalid to say that the conclusions about $\mathbf{J}(v, \lambda)$ can be applied directly to the problems about $\mathbf{J}^m(v_m, \lambda)$.

(3) Does expression (10) exist? How large may $\operatorname{Tr} \mathbf{J}^m(v_m, \lambda)$ become when $0 < \det \mathbf{J}^m(v_m, \lambda) < 1$? The question cannot be answered because of the indefiniteness of

* For the numbers of the equations and the meaning of the symbols in this note, refer to reference [1].

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equation (1). Therefore, the demonstration of expression (10) in reference [1] is not sufficiently ample.

3. A DEFECT IN SECTION 3 OF REFERENCE [1]

The bifurcation point of equation (1) and its type cannot be determined definitely because of the indefiniteness of equation (1). Under this condition, the test function for predicting the saddle-node bifurcation is used in reference [1]. For this reason, the "failure" in reference [1] has nothing to do with the validity of the test function method for predicting the saddle-node bifurcation.

4. DEFECTS IN SECTION 4 OF REFERENCE [1]

(1) Before using any method for calculating the periodic grazing orbit, the existence of the periodic grazing orbit of equation (1) must be determined first. However, this problem is not dealt with in reference [1]. Indeed, it is not possible for reference [1] to discuss this problem, because of the indefiniteness of equation (1).

(2) If the periodic grazing orbit exists in system (1), according to the characteristics of the periodic grazing orbit, the periodic grazing orbit can be obtained through integrating equation (1) directly from the point ($x = x_s$, $y = 0$). It is not necessary to use the Poincaré map. The fact may be shown in Figure 2 of reference [1].

(3) If the periodic grazing orbit does not exist in system (1), it is not possible to calculate the periodic grazing orbit.

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AUTHOR'S REPLY

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(*Received 19 November 1996*)

The key point of the comments on reference [1] is that the analysis should be made for a specific piecewise-linear oscillator, instead of the general one described by equation (1) in reference [1]. As a matter of fact, the primary aim of the paper was to point out the possible bifurcations incident to periodic grazing orbits of a forced continuous, piecewise-linear oscillator and to locate them numerically. Similarly to other publications for numerical analysis of non-linear dynamical systems, the paper presented the numerical procedure in a general form and enabled one to follow its idea to deal with a specific system; for instance, the oscillator described by equations (2) and (23) in reference [1].

The magnitude of $Tr\, \mathbf{J}^m(v_m, \lambda)$ is dependent on the specific form of equation (1) in reference [1] and can be calculated according to the repeated use of equation (13) in reference [2]. As reference [1] aimed at the numerical analysis, it addressed this problem only qualitatively.

The test function for predicting saddle-node bifurcations was defined in section 3 of reference [1], but used in section 5 for a specific oscillator described by equations (22) and (23). Therefore, there is no indefiniteness in the use of the test function. The failure of the test function in predicting the saddle-node bifurcation did come from the sudden tangential change in the test function in the numerical simulation, even though such a change might, in theory, be traced by using much smaller continuation steps. If one does not know that the grazing phenomenon may change the tangent of the test function suddenly, one never reduces the continuation step when the test function is far from zero.

There is no doubt that the forced piecewise-linear oscillator has at least one periodic grazing orbit, which is the periodic motion with the largest amplitude in the linear range. As for other kinds of periodic grazing orbits, it is extremely hard to study their existence even if the oscillator is very specific. Therefore, it is useful to detect the possible periodic grazing orbits numerically from a practical viewpoint. Similarly to machinery fault diagnosis, one cannot wait only until the problem of existence is fully solved. When there exists a periodic grazing orbit for a forced piecewise-linear oscillator, say, equations (22) and (23), the time at which the periodic orbit grazes the switching plane is not known. That is, the initial condition in the comments should be $x(t_0) = x_s$, $y(t_0) = 0$. Thus, it is necessary to develop a scheme to determine t_0 so that the entire periodic grazing orbit can be numerically integrated.

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