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INVESTIGATION ON SOUND FIELD MODEL OF PROPELLER AIRCRAFT—THE EFFECT OF RIGID FUSELAGE BOUNDARY*

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An improved sound field model with multiple propeller noise sources and finite fuselage boundary has been developed for the prediction of propeller aircraft noise by using the acoustic analogy method. It involves the effects of fuselage boundary with arbitrary shape and coupling of multiple propeller sources. It is also applicable to solving the interaction between any known boundary and harmonic sound source. The model has been used to calculate the sound field of propeller aircraft Y12 with rigid fuselage boundary and the sound field of rigid sphere in planar harmonic sound wave. The latter has an analytical solution which could be used to check the present method. The calculation results show that the model is reasonable and valuable.

1. INTRODUCTION

The acoustic analogy theory pioneered by Lighthill [1] has become a very important tool in the prediction of propeller noise. It has been successfully used for the calculation of subsonic propeller discrete noise propagating in a free sound field by using the formula developed by Farassat from the FW-H equation [2–4]. However, the existence of fuselage causes the sound field to change greatly. The data of flight and wind-tunnel tests show that, while $M \geqslant 0{\cdot}5$, the effects of layer refraction on the fuselage surface play a dominant role, and the pressure on the surface is lower than that in a free field [5]; while $M \leqslant 0{\cdot}3$, however, the scattering effects play a dominant role, and the pressure on the surface is higher than that in a free field [6].

The sound pressure distribution on the fuselage surface is the basis for noise prediction inside the cabin. In ANOPP, one of the leading prediction tools of propeller aircraft noise, an infinite cylinder model, was developed to rectify the scattering and refraction effects [7]. In 1991, Magliozzi [8] put forth an improved model which can be used to predict sound pressure at any point on the fuselage surface as well as the effects of flight speed, frequency, etc. These models, however, are only for infinite cylinders and not for real fuselage shapes.

Some studies of the interior noise of propeller aircraft have provided evidence that the precision of the sound load on the fuselage surface may be a principal factor that affects the accuracy of prediction [9].

This paper presents a sound field model of the propeller aircraft by which the effects of rigid fuselage boundary in low flying speed can be taken into account on the basis of acoustic analogy given by Ffowcs Williams and Hawkins [10] and the united aerodynamics and aeroacoustics approach given by Farassat and Long [11, 12]. It has been successfully

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used in calculating the blocked sound pressure on the fuselage of Y12 propeller aircraft made in China by this developed model. Generally the model can also be applied to calculate the scattering sound field of any harmonic sound source outside of a body moving evenly and rectilinearly with the sound source, in addition to the sound field of a propeller aircraft. The calculation results of plane wave scattered by a rigid sphere are in satisfactory agreement with an analytical solution.

2. DESCRIPTION OF THE IMPROVED MODEL

The governing equation is still the FW-H equation. It is hypothesized as follows. (1) The aerodynamic load distribution on a propeller surface and the discrete sound field radiated from this propeller is independent of the existence of fuselage, airfoils and sound field radiated from other propellers (for sound is weak pressure perturbation and does not affect the strong fluid field), so that it can be calculated separately. (2) The forward velocity of the fuselage is constant, and the Mach number is less than 0·3, so that the refraction effects of the boundary layer are small and negligible. (3) The fuselage boundary is rigid and inviscid. There are no layer effects on it.

Under these conditions the sound field out of the fuselage could be described by the FW-H equation without quadruple.

Based on the acoustic analogy originally suggested by Lighthill, Ffowcs Williams and Hawkings converted the moving boundary condition to the sources on the right side of the inhomogeneous wave equation [10]. In the calculation of single propeller noise in free field only the sources on the single propeller surface have been taken into account [2–4]. But in the present model the sources on the fuselage surfacfe and that on the other propeller surface must also be taken into account. The equation is now as follows:

$$\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} - \nabla^2 p = \frac{\partial}{\partial t}[\rho V_n|\nabla f_1|\delta(f_1)] - \frac{\partial}{\partial x_j}[l_j|\nabla f_1|\delta(f_1)] + \frac{\partial}{\partial t}[\rho V_n|\nabla f_2|\delta(f_2)] - \frac{\partial}{\partial x_j}[l_j|\nabla f_2|\delta(f_2)] + \frac{\partial}{\partial t}[\rho V_n|\nabla f_3|\delta(f_3)] - \frac{\partial}{\partial x_j}[l_j|\nabla f_3|\delta(f_3)] \quad (1)$$

where $f_1 = 0$, $f_2 = 0$ represent the equation describing the two propeller's surfaces; $f_3 = 0$ represents the equation describing the fuselage's surface.

3. THE SOUND PRESSURE ON THE SURFACE OF THE RIGID BOUNDARY—THE BLOCKED PRESSURE

From the FW-H equation a calculation formulation for the discrete noise of subsonic propeler in the free field is derived by Farassat. It is an integral equation containing derivative [7].

$$4\pi p(x, t) = \frac{1}{c}\int_{f=0}\left[\frac{\dot{l}_j\hat{r}_j}{r(1-M_r)^2}\right]_{ret} \mathrm{d}s + \int_{f=0}\left[\frac{l_r - l_jM_j}{r^2(1-M^r)^2}\right]_{ret} \mathrm{d}s + \int_{f=0}\left[\frac{l_r(r\dot{M}_j\hat{r}_j + cM_r - cM^2}{r^2(1-M_r)^3}\right]_{ret} \mathrm{d}s + \int_{f=0}\left[\frac{\rho V_n(r_j\dot{M}_j + cM_r - cM^2)}{r^2(1-M_r)^3}\right]_{ret} \mathrm{d}s \quad (2)$$

where l_j and V_n represent the load distribution and normal velocity component on the boundary; $r_j = x_j - y_j$, x_j is the coordinate of observer's position; y_j is the coordinate of source point, $r = |\mathbf{x} - \mathbf{y}|$, $\hat{r}_j = r_j/r$; the subscript *ret* denotes that the expression is evaluated at retarded time; and $f = 0$ is the equation describing the blade's surface.

In the Ffowcs Williams–Hawkings acoustic analogy model, it is required to calcuate l_j. In the calculation of propeller noise in free field, l_j could be obtained from some aerodynamic program. In the present model, by hypothesis (1) the load distribution on the blade can also be obtained independently from an aerodynamic program even for the case of existing multiple propellers and fuselage boundary. From formulation (2) the sound pressure of the two propellers in a free field could be caculated independently with p_1, p_2 as expressed hereunder. It is a cyclic signal with its own passing frequency for an observer moving together with the fuselage. The first four source terms on the propeller surface on the right side of (1) could be integrated. The remaining source terms on the fuselage's surface can also be integrated by (2), if the load distribution l_j on the fuselage surface could be known. By hypothesis (3), the viscosity is ignored, and the load on the fuselage surface only involves the normal pressure, written by $l_j = p\mathbf{n}_j = p_n$; $\mathbf{n}_j$ representing the unit normal vector on the fuselage surface.

When the observer's point moves onto the fuselage surface, it can be shown that the near field integral (that with integrand of $O(1/r^2)$ is divergent. This means that the principal value of this integral must be taken into account. By using Long's approach the divergent integral is regulated. The procedure to find the principal value is described in detail in reference [11].

Briefly, the regularization procedure involves: selecting a small square hole at the observer's point on the fuselage surface which gives a convergent integral as the size of the hole is reduced while its shape is retained; moving the observer's point to a small distance δ above the cap of the hole and finding the contribution of the cap as $\delta \to 0$ while the cap size is fixed; and letting the hole size shrink to zero. The result is as follows:

$$4\pi\left(1 - \frac{1}{2\beta_n^2}\right) p(x_0, t) = 4\pi p_1 + 4\pi p_2 + \int_{f_3=0} K_R \,\mathrm{d}s$$

$$= + \,⨍_{f_3=0} K_S \,\mathrm{d}s + 2\pi\rho V_n^2(x_0, t)/\beta_n^2 + O(\epsilon) \tag{3}$$

where x_0 represents the observer's point on the fuselage surface, $⨍$ indicates that a specific small square hole has been removed, and ϵ is the size of the small square hole. It means that in the above regularization procedure the ignored amount is $O(\epsilon)$. In numerical calculation the hole size of ϵ is the same order of magnitude as the size of the surface grid, which means that the accuracy of the numerical method could be controlled by the grid size.

$$K_R = \left[\frac{\rho c^2 M_n \dot{M}_r + \dot{p}(1 - M_r)\cos\theta + p\dot{M}_r \cos\theta}{cr(1 - M_r)^3}\right]_{ret} \tag{4}$$

$$K_S = \left\{\frac{\rho c^2 M_n (M_r - M^2) + p[(1 - M^2)\cos\theta - (1 - M_r)M_n]}{r^2(1 - M_r)^3}\right\}_{ret} \tag{5}$$

where $\beta_n = \sqrt{1 - M_n^2}$, $\dot{M}_r = \dot{M}_j \hat{r}_j$ and θ is the angle between $\hat{n}$ and $\hat{r}$.

The p in both the integrand of the integral equation and the left side is the sound pressure on the fuselage surface and is called the blocked sound pressure for rigid fuselage surface [9]. Such sound pressure is not independent, and is to be determined from the governing equation (3).

From hypothesis (2) the flying velocity of aircraft is constant, so that

$$\dot{M}_r = \dot{M}_j \hat{r}_j = 0. \tag{6}$$

For an observer's point and a source point fixed on the fuselage surface the r_j is constant. While the aircraft moves evenly, the M_r and M_n are independent of observer's time t. By removing the steady components, the following equation is obtained:

$$4\pi\left(1 - \frac{1}{2\beta_n^2}\right) p(x_0, t) = 4\pi p_1 + 4\pi p_2 + \frac{1}{c}\int_{f_3=0}\left[\frac{\dot{p}\cos\theta}{cr(1-M_r)^3}\right]_{ret} \mathrm{d}s$$
$$+\int_{f_3=0}\left\{\frac{p[\cos\theta(1-M^2)-(1-M_r)M_n]}{r^2(1-M_r)^3}\right\}_{ret} \mathrm{d}s. \tag{7}$$

The governing equation for the sound pressure on the fuselage surface in the time domain is an integral differential equation with independent variables of time t and position x. In the frame fixed to the aircraft, p_1 and p_2 are cyclic functions with the period of passing frequency, and all the variables in the integrand are independent of t except for p, so that the blocked sound pressure must be a cyclic function with such period as that of p_1, p_2. If the p_1 and p_2 have different periods, the blocked pressure p will comprise of two sets of cyclic functions with the same periods as those of p_1 and p_2, which can be calculated separately. It is more convenient to calculate the blocked sound pressure produced by one propeller.

In equation (7) the p and p_1 are cyclic functions with the same period, and the other variables are independent of t. By the Fourier transformation the variable t could be removed. The governing equation is:

$$4\pi\left(1 - \frac{1}{2\beta_n^2}\right) p_n(x_0) = \int_{f_3=0}\left[\frac{p_n(y)\,\mathrm{e}^{-in\omega r/c} in\omega\cos\theta}{cr(1-M_r)^3}\right]_{ret} \mathrm{d}s$$
$$+\int_{f_3=0}\left\{\frac{p_n(y)\,\mathrm{e}^{-in\omega r/c}[(1-M^2)\cos\theta-(1-M_r)M_n]}{r^2(1-M_r)^3}\right\}_{ret} \mathrm{d}s + 4\pi p_{1n}(x_0) \tag{8}$$

where $p_{1n}(x_0)$ is the nth Fourier component of pressure in free field and $p_n(x)$ is the nth Fourier component of blocked pressure excited by the former. Both $p_{1n}(x)$ and $p_n(x)$ are complex functions of coordinate. The term $\mathrm{e}^{-in\omega r/c}$ in integrand could by interpreted as the phase shift caused by the retarded time. The variables in the parentheses are evaluated at the retarded time, and are dependent on x, y, $n\omega$ and M. Equation (8) is a linear integral equation, independently of t, which is a linking equation between the pressure radiated by the propeller in the free field and blocked pressure on the fuselage surface. The retarded time τ is calculated as follows:

$$t - \tau = |\mathbf{x} - \mathbf{y}|/c. \tag{9}$$

In numerical calculation the governing equation will be reduced to a system of simultaneously algebraic equations, written in matrix form as follows:

$$B(n\omega, M, f_3)\mathbf{P}_n = \mathbf{P}_{1n} \tag{10}$$

where $B(n\omega, M, f_3) = \{B_{ij}(n\omega, M, f_3)\}$ is a coupling matrix, $\mathbf{P}_{1n}$ is a vector of nth harmonic sound pressure distribution on the fuselage surface radiated by one propeller in a free field, and $\mathbf{P}_n$ is a vector of nth harmonic blocked sound pressure on the fuselage surface.

$$B_{ij}(n\omega, M, f_3) = -\left\{\frac{\mathrm{e}^{-in\omega r_{ij}/c}[(1-M^2)\cos\theta_{ij} - (1-M_{rij})M_{nj}]}{r_{ij}^2(1-M_{rij})^3}\right\}_{ret}(1-\delta_{ij})\,\Delta S_j$$

$$-\left[\frac{\mathrm{e}^{-in\omega r_{ij}/c} in\omega\cos\theta_{ij}}{cr_{ij}(1-M_{rij})^3}\right]_{ret}\Delta S_j + 4\pi\left(1-\frac{1}{2\beta_{ni}^2}\right)\delta_{ij}. \tag{11}$$

The subscript i represents the observer's point, the subscript j represents the source's point, and $n\omega$ is the angle frequency of the harmonic component.

4. SCATTERING SOUND PRESSURE IN THE CASE OF EXISTING BOUNDARY

From the blocked pressure on the fuselage suface, the formulation of the scattering sound field in the case of existing boundary in the time domain could be derived from equation (2) based on acoustic analogy. As explained in the calculation of the blocked pressure hereabove, the scattering sound pressure in frequency domain could also be obtained by Fourier transformation.

$$4\pi p_n'(x) = \int_{f_3=0}\left[\frac{p_n(y)\,\mathrm{e}^{-in\omega r/c} in\omega\cos\theta}{cr(1-M_r)^3}\right]_{ret}\mathrm{d}s$$

$$+\int_{f_3=0}\left\{\frac{p_n(y)\,\mathrm{e}^{-in\omega r/c}[(1-M^2)\cos\theta - (1-M_r)M_n]}{r^2(1-M_r)^3}\right\}_{ret}\mathrm{d}s \tag{12}$$

where $p_n(y)$ is the nth component of blocked pressure, and $p_n'(x)$ is the component of scattering pressure at the observer's point.

5. DEMONSTRATING BY BLOCKED AND SCATTERING PRESSURE OF A RIGID SPHERE IN HARMONIC PLANE SOUND FIELD

The blocked pressure on a rigid sphere in harmonic plane sound field has a classical analytical solution [13]. If the planar wave is

$$p_p = A\,\mathrm{e}^{-ik(r\cos\theta - ct)} \tag{13}$$

where $k = \omega/c$ is wave number, r is the distance from the sphere center to the observer's point and θ is the spherical coordinate of observer's point. The total sound pressure on a rigid sphere surface, i.e., the blocked pressure could be calculated by the following analytical formulation:

$$p_b = A\,e^{i\omega t}\left(\frac{1}{ka}\right)^2 \sum_{m=0}^{\infty} \frac{2m+1}{B_m}\, p_m(\cos\theta)\, e^{-i(\delta_m - 1/2m\pi)} \tag{14}$$

where a is sphere radius, p_m is the mth Legendre function and B_m and δ_m are the amplitude and phase related to sound radiation of the spherical wave, determined by ka.

The scattering sound intensity in far field could also be calculated by an analytical formulation. Its directivity of scattering sound intensity is as follows:

$$I_{non} = \frac{I_s k^2 r^2}{I}$$

$$= \sum_{m,n=0}^{\infty} (2m+1)(2n+1)\sin\delta_m \sin\delta_n \cos(\delta_m - \delta_n) p_m(\cos\theta) p_n(\cos\theta). \tag{15}$$

For ka at a given value, the blocked pressure and the scattering sound intensity could be calculated by the above analytical formula and the acoustic analogy approach respectively. The results are compared to check the present model and calculation method.

In many acoustics books, the scattering of a rigid sphere in the situation of $ka = 1$, $ka = 5$ was used to demonstrate the scattering phenomenon. Here the blocked sound pressure, including the SPL normalized by free field sound pressure and the phase, as well as the scattering intensity level, are calculated by two approaches for the above two situations, as shown in Figures 1–6.

These figures show that the results of acoustic analogy approaches coincide with those of the analytical method. The difference between them could be less than 0·1 dB for SPL, less than 0·15 dB for SIL and less than 0·2° for phase. This is a good check for the validity of the model and the calculation method.

To analyze the effects of motion on the sound field, the sound field of a sphere, moving at $M = 0{\cdot}3$ together with the plane wave in the direction of the propagating direction, was calculated by the acoustic analogy approach. The results are also shown in Figures 1–6 for comparison.

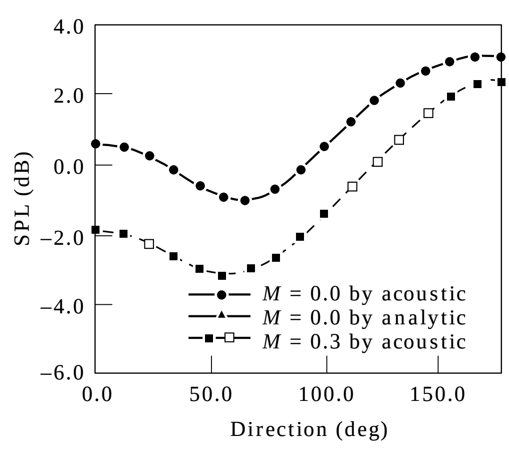

Figure 1. The distribution of block SPL normalized calculated by two approaches for $ka = 1$.

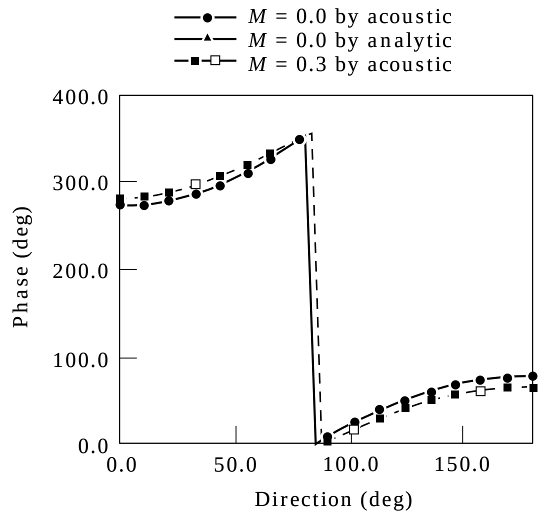

Figure 2. The distribution of phase of block SPL calculated by two approaches for $ka = 1$.

6. EXAMPLE OF Y12 AIRCRAFT, THE CALCULATION OF PRESSURE ON THE FUSELAGE SURFACE

Y12 is a light general purpose propeller airplane developed in China. It is shown in Figure 7. Its maximum T.O. weight is 5000 kg, maximum payload is 1700 kg, and it can be used to accommodate up to 17 passenger seats. It is powered by two PRATT and WHITNEY PT6A-27 engines and Hartzell HC-B3TN-3B/T10173B-3 propellers with 3 blades. The rotation speed is 2200 r.p.m. Its fastest cruising speed is up to 300 km/hr. The calculation was made for cruising flying at an altitude of 300 m. Two propellers have the same rotational speed (2200 r.p.m.) and phase. To reduce calculation complexity, the fuselage shape is simplified as shown in Figure 8. (Note, the middle fuselage is a cylinder part. The under line of the front part of the fuselage is the same as that of the mid-fuselage's under line; while the upper line of the back fuselage is the same as that of the mid-fuselage's upper line. Both front and back fuselage shrink to their respective end, by the way of cylinder-section.)

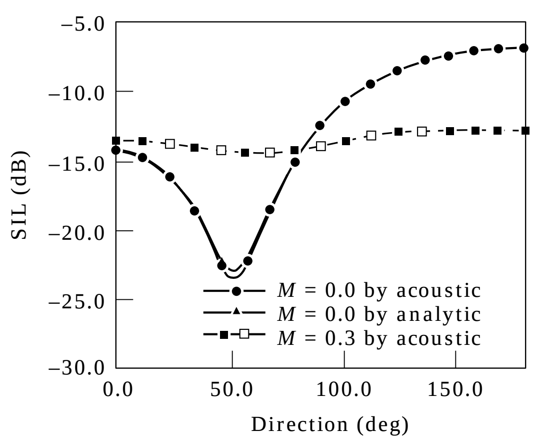

Figure 3. The directivity of scattering SIL calculated by two approaches for $ka = 1$.

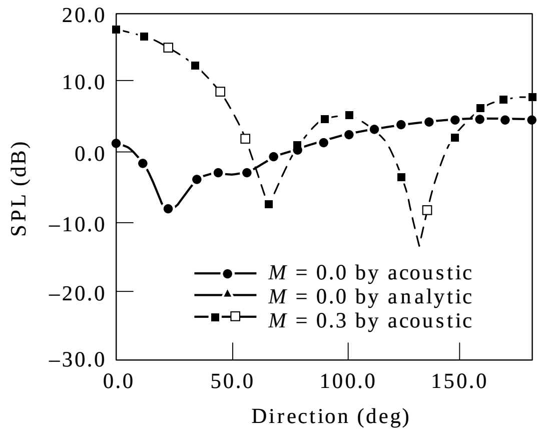

Figure 4. The distribution of block SPL normalized calulated by approaches for $ka = 5$.

In numerical calculation, the fuselage is divided into many small segments, and each small segment is further divided into many grids. In each grid a 4×4 Gauss-Legendre integration scheme is used in calculation. The control point is located in the center of the grid.

The contour is shown on the developed surface of the fuselage, which is cut through an elemental line of the fuselage at the center of its low side. In order to display the contour on the whole fuselage surface more conveniently, the peripheral size of both front and back part of the fuselage is to be so enlarged as to make it equal to that of the mid-fuselage. The flying direction is opposite to the x direction of the contour. The x-axis represents the axial location of the fuselage while the y-axis represents the developed peripheral location of the fuselage. The unit length is meters. The origin of the contour represents the upper center of the front end. The figure comprises 4 pieces of contours: the left-upper one is the contour of free field pressure level; the right-upper one is the contour of blocked pressure level; the left-lower one is the contour of phase of free field pressure; and the right-lower one is the contour of phase of blocked pressure.

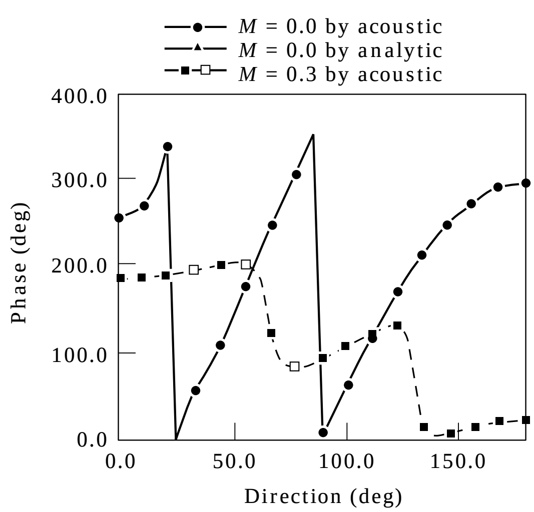

Figure 5. The distribution of phase of block SPL calculated by two approaches for $ka = 5$.

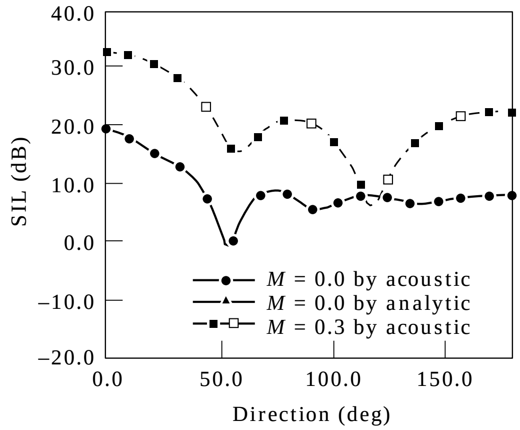

Figure 6. The directivity of scattering SIL calculated by two approaches for $ka = 5$.

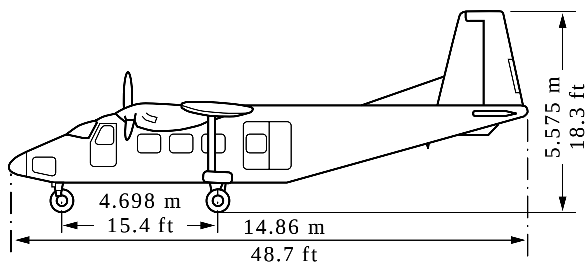

Figure 7. Y12 aircraft scheme.

Figure 9 is the contour of the first harmonic component produced by the right propeller. It shows that the maximum value of both free field sound pressure level and blocked sound pressure level occur in the same place, opposite to the propeller and a little behind. The value of blocked SPL is 5 dB higher than the free field's. It coincides with the empirically fitted curve used in reference [9]. The contours of both free field SPL and blocked SPL show a wave shape, which is expanded from the position of maximum value to outside. But the contour lines of blocked SPL change more than that of the free field's. This implies that the reflecting wave decreases following the increase of the incident angle.

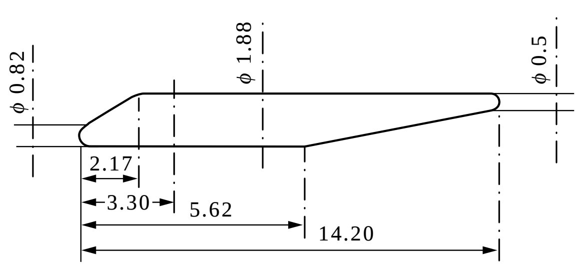

Figure 8. Y12 simplified fuselage model.

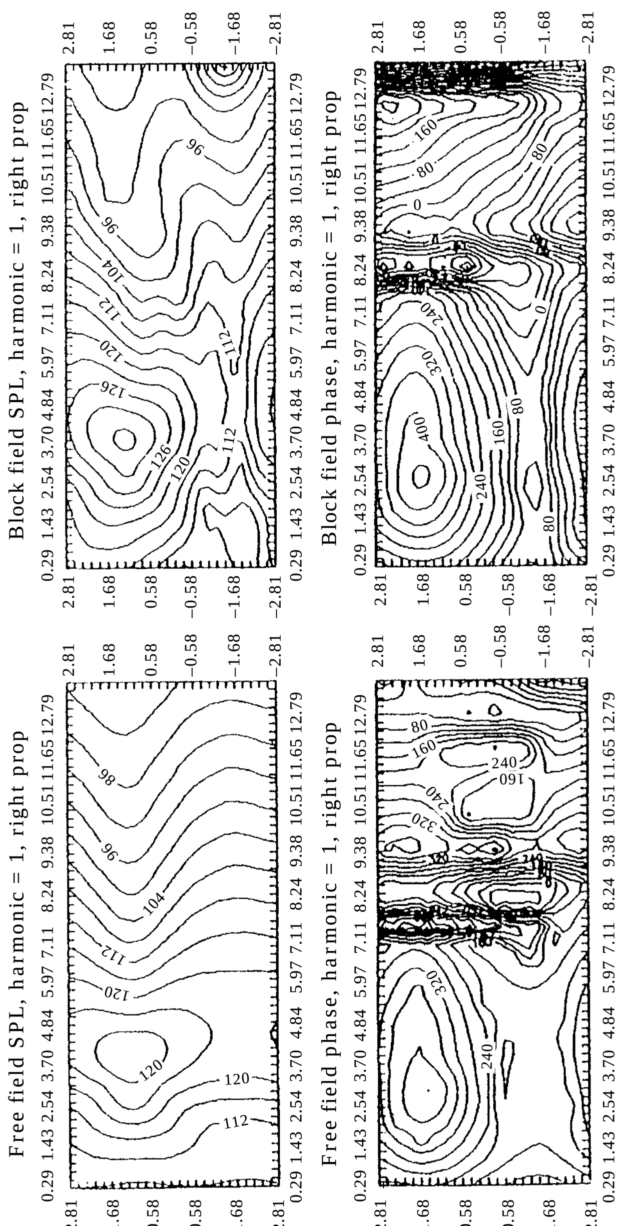

Figure 9. The contour of SPL and phase of free field and blocked field. First harmonic component generated by right propeller.

In the contour of the blocked SPL, some sound deflection and sound shadow phenomena appear on the other side of the fuselage to the propeller. Right on the opposite side of maximum value of SPL, there exists a very weak SPL with a value 26 dB lower than the maximum value of SPL. It means that a sound shadow occurs. But in the free field, SPL is different from that in the blocked sound pressure. There is not shadow on the reverse side of the fuselage opposite to the propeller. In the contour of the blocked SPL, the SPL is relatively stronger in some places where the sound wave cannot reach according to the sound ray propagating principle. It means that sound deflection has occurred.

The contour of the phase also shows a similar spreading outward wave, but its center falls onto the front and right of the propeller's projection. The contour of the phase is related to the intersection curves of the sound wave front with the boundary at different positions and shows obviously some regularity which has a propagation pattern of the wave front related to a right-turn propeller.

The contour lines of phase of the blocked pressure are closer than those in free field, as the phase of the blocked pressure changes more than that in free field. In the contour of phase there exists some very big phase changes with more then 2π in a narrow distance. But in a Riemann curved surface the phase varies serially.

7. CONCLUSION

The acoustic analogy method has been successfully used to develop a discrete sound field model of propeller aircraft as well as its numerical calculation method which account for the effects of rigid fuselage moving constantly with less than 0·3 Mach number. Generally the model and calculation method could solve the problem of interaction of any known harmonic sound field and rigid boundary.

By using the model, the sound field of a rigid sphere in planar harmonic wave has been calculated. The results are in satisfactory agreement with those obtained by using an analytical method.

The blocked sound pressure of Y12 aircraft has also been calculated by using this approach. From the contour of blocked sound pressure and some phenomena of sound reflection, sound shadow and sound deflection have been discussed.

All of these illustrate the validity of the model and the success of the calculation method.

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APPENDIX: NOMENCLATURE

a	radius of sphere
c	sound velocity
f, f_1, f_2	function describing blade surface
f_3	function describing fuselage surface
I	sound intensity in free field
I_s	scattering sound intensity
I_{non}	non-dimensional sound intensity
$k = \omega/c$	wave number
K_R	regular kernel
K_S	singlular kernel
l_j	load on surface
$\dot{l}_j$	time derivative of load
$M = V/c$	Mach number
$M_n = V_n/c$	Mach number in normal direction
$M_r = V_j r_j/c$	Mach number in propagation direction
$\mathbf{n}$	unit normal vector of surface
$p(x, t)$	sound pressure
$p_{1n}(x)$	Fourier component of SP in free field
$p_n(x)$	Fourier component of blocked SP
$p'_n(x)$	Fourier component of scatter SP
$\mathbf{r} = \mathbf{x}(t) - \mathbf{y}(\tau)$	vector distance between source and observer
$r = \lvert\mathbf{r}\rvert$	
$\hat{r} = \mathbf{r}/r$	
t	observation time
$\mathbf{V}$	velocity of body
V_n	normal velocity of body
$\mathbf{x}$	observer's position
$\mathbf{y}$	source position
$\beta_n = \sqrt{1 - M_n^2}$	
$\delta(f)$	Dirac delta function
ϵ	size of small square hole removed from integral
$\theta = \text{arc cos}\,(\mathbf{n}, \mathbf{r})$	
ρ	density of the undistrubed medium
τ	retarded time (i.e., time of emission)
ω	angular frequency

Subscript

ret	expression is evaluated at retarded time

Superscript

$\circ$	expression of time derivative
$⨍$	integration with a specific hole removed from region of integration.