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A MODELLING OF THE NOISE FROM SIMPLE COAXIAL JETS, PART II: WITH HEATED PRIMARY FLOW

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This paper reports the second part of a continuing study of the noise of coaxial jets, and describes modifications to the model developed previously to allow for the effects of a heated flow from the primary nozzle. The essential feature of the model described previously for the prediction of the noise from isothermal coaxial jets was the identification of three flow regions, within the coaxial jet flow, the noise production of which could be estimated from single-jet prediction methods. In particular, it was shown that noise from the principal interaction zone could be calculated by using single-jet prediction methods as long as account was taken of the fact that measured turbulence levels in this region were lower than those observed in a single isolated jet at the same centerline velocity. For isothermal flows, for which only quadrupole sources exist, allowance for this reduced turbulence level was entirely straightforward. However, for heated flows both dipole and quadrupole sources exist, and these have different dependencies on the turbulence level. Hence to predict the noise one needs to know the relative contributions of the dipole and quadrupole sources. In the present work, use has been made of previously published results for these relative contributions, as a function of jet velocity and temperature, for single jets. This then permits prediction of the noise from the interaction zone, which is subsequently combined with that from the secondary jet shear layer and fully mixed flow region, as before. Comparison between data and prediction over a range of jet velocity, temperature and angle of observation again show very acceptable agreement.

1. INTRODUCTION

Although bypass engines have dominated the civil engine market for over 20 years, the prediction of the jet noise from their coaxial exhausts remains a difficult problem. There are several reasons for this difficulty. First, there is the continuing difficulty of predicting, from first principles, the noise production of single stream jets; see reference [1] for details. Hence, even in this case, heavy reliance is still placed on systematic databases, together with the empirical approaches based thereon [2–5]. However, the extension of this principal to coaxial flows represents a formidable task in view of the large number of both aerodynamic and geometric variables involved, many of which interact in a complex manner.

In the light of these problems, a fundamental research programme was initiated with the objective of first identifying the major noise producing regions of a coaxial jet and

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subsequently exploring the feasibility of predicting the noise of these regions on the basis of our, albeit empirical, knowledge of the noise of single-stream jets.

The first results to emerge from this approach have been previously presented in references [1] and [6]. That work was restricted, in the interests of initial simplicity, to isothermal flows. In this paper, the considerations necessary to extend the approach to include the very practical situation of a heated primary flow are introduced.

To this end, therefore, section 2 provides a brief synopsis of the isothermal flow model, in which a key feature was the observation from turbulence measurements [7], that in the interaction zone the turbulence level was only 10% of the centreline velocity, as opposed to 15% for an isolated jet. The allowance for this reduced turbulence level on the noise production is straightforward for isothermal flows in which only quadrupole sources exist. However, for a heated primary flow this problem becomes a little more challenging due to the presence of both dipole and quadrupole sources. The solution of this problem is described in section 3 and the final acoustic prediction model is assembled in section 4. The paper concludes with some typical comparisons between the predicted and measured noise fields for primary jet temperatures of 600 K and 800 K respectively.

2. THE ISOTHERMAL MODEL

The major noise producing regions of coaxial jet as identified in reference [1] are shown in Figure 1. They comprise the following.

(a) The shear layer separating the primary and secondary flows. For the velocity ratios of interest here ($\lambda \equiv V_s/V_p > 0{\cdot}5$), the low turbulence levels in this region [7] and the limited volume results in this shear layer being a region of negligible noise production, which is not considered further.

(b) The secondary/ambient shear layer, which has the initial flow characteristics [7] of a jet of diameter equal to that of the secondary jet D_s and velocity V_s. This shear layer, however, terminates as the two shear layers interact. Hence the noise from this region is predicted as that of a single jet of velocity V_s and diameter D_s, but with a low frequency spectral cut-off to reflect the termination of this shear layer, as detailed in references [1, 6].

(c) The fully mixed flow region, the characteristic velocity V_m and diameter D_m of which are determined from the conservation of mass and momentum of the combined primary and secondary streams. The noise is predicted as that of a jet of velocity V_m and diameter

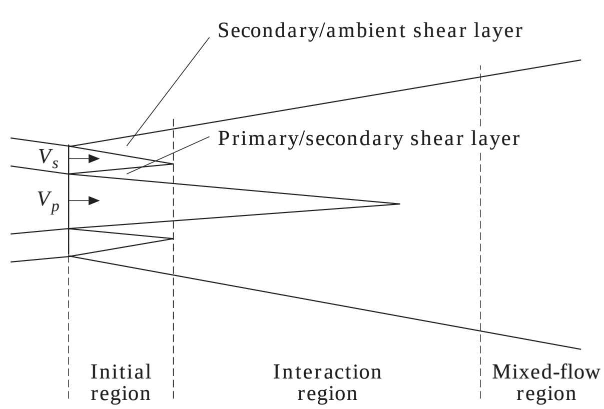

Figure 1. The principal noise producing regions.

D_m, with a high frequency spectral cut-off to reflect the fact that, in reality, only the downstream portion of this jet exists; see references [1, 6] for details.

(d) The effective jet representing the noise produced in the interaction region. This is predicted in reference [1] as the noise produced by an isolated jet of effective diameter D_e, obtained from the mean velocity profiles of [7] as

$$D_e = D_p\,(1 + \lambda^2\beta)^{1/2}, \tag{2.1}$$

where λ is the velocity ratio (V_s/V_p) and β is the geometric area ratio (A_s/A_p), with a characteristic velocity V_p. However, as described in reference [1], it is necessary to attenuate this contribution by 7 dB to allow for the observed 10% turbulence level associated with this flow region as compared to 15% in an isolated jet.

In the next section we consider the modifications necessary to extend this modelling process to a jet in which the primary flow is heated.

3. A MODEL FOR HEATED PRIMARY FLOW

We now consider modifications to the acoustic modelling described above required in the presence of a heated primary flow, for the three major noise producing regions in turn.

3.1. THE SECONDARY SHEAR LAYER

This region is entirely outside the heated flow and can therefore be modelled precisely as for the isothermal case. An identical secondary jet was previously used by Tanna and Morris [8].

3.2. THE FULLY MIXED JET

In the case of isothermal jets, the velocity and diameter of the equivalent fully mixed jet was obtained [1] by attributing to it the total mass flow and momentum of both the primary and secondary jets. In the case of a heated primary flow it is merely necessary to add an energy (enthalpy) equation. Solution of these three equations for the velocity, diameter and temperature of the fully mixed jet then yields

$$\frac{V_m}{V_p} = \frac{(1 + \lambda^2\beta\delta)}{(1 + \lambda\beta\delta)}, \qquad \frac{D_m}{D_p} = \left[\frac{(1 + \lambda\beta)\,(1 + \lambda\beta\delta)}{(1 + \lambda^2\beta\delta)}\right]^{1/2}, \qquad \frac{T_m}{T_p} = \frac{(1 + \lambda\beta)}{(1 + \lambda\beta\delta)}, \tag{3.1–3.3}$$

where λ is the velocity ratio (V_s/V_p), β is the area ratio (A_s/A_p) and δ is the density ratio ($\rho_s/\rho_p = T_p/T_s$).

Hence the noise prediction for the fully mixed region becomes that for a single isolated jet of diameter D_m, velocity V_m and temperature T_m, as given by the above equations, again with a high frequency spectral cut-off as for the isothermal case.

3.3. THE EFFECTIVE JET

While the extension of the acoustic model to a heated primary flow has proved entirely straightforward for the secondary shear layer and fully region, the same is not true for the effective jet or interaction region.

In the absence of any definitive mean flow or turbulence data for a heated primary flow, one might perhaps model this region as jet diameter D_e (as given above), jet velocity V_p (as before), jet temperature T_p and turbulence level 10% (as for the isothermal case).

However, the mixing process is now between fluids of different densities which in the case of isolated jets is known to give rise to a dipole as well as a quadrupole contribution [9, 10], the former arising from the second term of the Lighthill stress tensor

$T_{ij} = \rho v_1 v_j + (p - c_0^2 \rho)\delta_{ij}$. Unfortunately, the commonly available single-jet prediction schemes, on which we have centred the current work, only predict the noise from heated jets with 15% turbulence levels, while the present requirement is to predict their noise at an anticipated 10% turbulence level. Furthermore, as we shall now proceed to demonstrate, the effect of reduced turbulence levels is markedly different for dipole and quadrupole sources so to determine the net reduction also requires some knowledge of their relative contributions. This in turn depends on the velocity and temperature of jet flow.

3.4. THE DIPOLE AND QUADRUPOLE SOURCES

It was shown in reference [1] that a useful scaling law for quadrupole sources, which contains the influence of turbulence level is

$$\partial(p^2(r_0)) \sim \frac{\rho^2(u')^4\omega^4L^3}{r_0^2c_0^4}\,\mathrm{d}V,$$

where the left side represents the contribution to the mean square pressure made by the local volume $\mathrm{d}V$. We see therefore that the noise output from each local region depends on the fourth power of the root mean square turbulence level, u'. Hence introducing a typical turbulence level $\alpha \equiv u'/U_j$, where U_J is the jet efflux velocity, and making the normal assumptions about the jet flow being Strouhal number dependent, we can obtain a scaling law for the intensity,

$$I_q \sim \alpha^4\rho_s^2\, U_J^8\, D^2/\rho_0\, c_0^5\, r_0^2, \tag{3.4}$$

where following reference [10], ρ_s is the density in the dominant source region.

Conversely, the dipole source strength has been identified by Morfey [11] to be of the form

$$q = -\frac{\partial}{\partial x_i}\left[\frac{\rho - \rho_0}{\rho_0}\frac{\partial p}{\partial x_i}\right],$$

which, again applying standard scaling law procedures, yields a scaling law for acoustic intensity of the form

$$I_d \sim \frac{(\rho-\rho_0)^2U_J^4\,(u')^2D^2}{\rho_0\,c_0^3\,r_0^2} \sim \frac{\alpha^2(\rho-\rho_0)^2U_J^6\,D^2}{\rho_0\,c_0^3\,r_0^2} \tag{3.5}$$

Comparing equations (3.4) and (3.5) we see, therefore, that while the quadrupole contribution depends on the fourth power of the turbulence level, the dipole varies as the square of this level. Hence in reducing the level from 15% (isolated jet) to 10% (effective jet) the quadrupole contribution will reduce by 7 dB, as before, but the dipole contribution is only reduced by 3·5 dB. Hence the attenuation to be applied to the predicted noise of a single hot jet to obtain the noise contribution of the effective jet will lie somewhere between 7 dB (quadrupoles dominant) and 3·5 dB (dipoles dominant). The variation of this attenuation between these two limits, as a function of jet velocity and temperature, is defined in the next section.

3.5. THE ATTENUATION FACTOR

The differing dependencies of the dipole and quadrupole source contributions on turbulence level mean that, for a heated jet, the net noise reduction created by the reduction of turbulence level from 15% in an isolated jet to 10% for the effective jet depends on the relative contribution of dipole and quadrupole noise respectively.

Specifically, the noise reduction is

$$\Delta \text{ dB} = 10 \log_{10}\left[\frac{r^2 I_d + r^4 I_q}{I_d + I_q}\right], \tag{3.6}$$

where r is the ratio of turbulence levels; $r = (10/15)$. Clearly, this expression can only be evaluated if the ratio I_d/I_q is known and, as explained previously, the majority of jet noise prediction methods do not consider this aspect; they merely predict the total.

An exception is the work of reference [12], the background to which is summarized in reference [10]. However, the temperature dependencies used in this work include, unlike the expressions (3.4) and (3.5) above, the fact that the sources radiate into a surrounding flow field. Hence, to use their ratios of a quadrupole to dipole source strength, as a function of Mach number and temperature, it is also essential to use their temperature dependencies.

These are

$$I_d = K_d\left(\frac{T_J - T_0}{T_s}\right)^2\left(\frac{T_0}{T_s}\right)^2 M_J^6, \qquad I_q = K_q\left(\frac{T_0}{T_s}\right)^2 M_J^8, \tag{3.7, 3.8}$$

where K_d and K_q are appropriate constants and $M_J \equiv V_j/a_0$ is the ratio of jet velocity to the ambient speed of sound.

Hence the required ratio for the isolated jet is

$$\frac{I_d}{I_q} = K\left[\frac{T_J - T_0}{T_s}\right]^2\left(\frac{T_s}{T_0}\right) M_J^{-2}, \tag{3.9}$$

where $K \equiv K_d/K_q$ and T_s, the temperature in the principal source region, is taken as $T_s = T_0 + 0{\cdot}65(T_J - T_0)$ or $T_s/T_0 = 1 + 0{\cdot}65(\tau - 1)$, where $\tau \equiv T_J/T_0$. Inserting these definitions into equations (3.9) we find that

$$I_d/I_q = K_y, \tag{3.10}$$

where

$$y \equiv \frac{(\tau - 1)^2}{1 + 0{\cdot}65(\tau - 1)} M_J^{-2}. \tag{3.11}$$

The constant K in equation (3.10) may be evaluated from the "master spectra" given in reference [12] and yields a value of about 7.

Hence the decibel reduction, equation (3.6), reduces to

$$\Delta \text{ dB} = 10 \log_{10}\left[\frac{(1 + 16y)}{5(1 + 7y)}\right], \tag{3.12}$$

with

$$y \equiv \frac{(\tau - 1)^2 M_J^{-2}}{1 + 0{\cdot}65(\tau - 1)}.$$

Note that while the value of K in equation (3.10) is slightly frequently dependent, the observed variation, when inserted into equation (3.12), introduces variations of less than 1 dB and have therefore been ignored.

The variation of this overall attenuation factor with temperature ratio τ at four jet Mach numbers is shown in Figure 2. We see, as expected, that as the temperature ratio tends

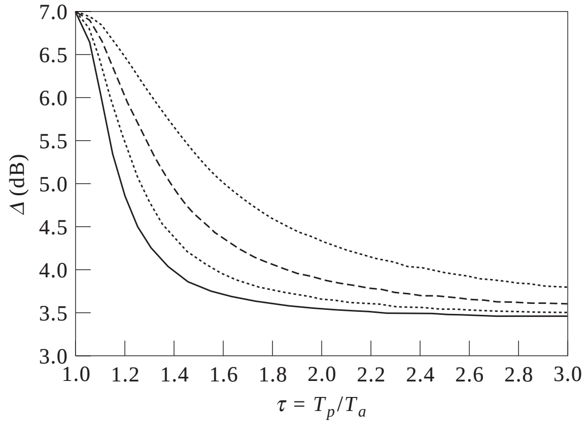

Figure 2. The attenuation due to reduced turbulence level as a function of primary temperature ratio. M_j values: —, 0·5; -----, 0·7; ‒‒‒, 1·0; -----, 1·5.

to unity the attenuation factor asymptotes towards 7 dB as was previously seen for the corresponding isothermal case [1]. Conversely, at high temperature ratio it asymptotes to 3·5 dB, indicating the complete dominance of the dipole sources. However, the transition region depends on jet Mach number, reflecting the differing dependencies of the dipole and quadrupole sources respectively. It can be shown, by using equation (3.12), that the attenuation factor will reach 4·0 dB, i.e., within 0·5 dB of its asymptotic value of 3·5 dB, whenever the parameter y exceeds 0·5. For a jet velocity corresponding to $M_J = 1$, this occurs when the temperature ratio exceeds 1·8 (i.e., $T_J \approx 540$ K), while for $M = 1{\cdot}5$ a temperature ratio of 2·4 ($T_J = 720$ K) is required. Hence in the majority of aero-engine applications it does appear that the dipole contribution will dominate, requiring a 3·5 dB reduction in noise level to account for the lower turbulence level.

4. THE PREDICTION METHOD

We are now in a position to assemble the spectral contributions for the prediction of the coaxial jet as follows.

(i) *The fully mixed jet*. This contribution may be written as

$$SPL_m(\theta, f) = SPL(V_m, T_m, D_m, \theta, f) + 10 \log_{10} F_D(f_1, f), \tag{4.1}$$

where $f_1 D_m / V_m = 1$. This equation gives the sound pressure level contribution of the mixed jet at angle θ and frequency f as the sound pressure level predicted for an isolated jet of diameter D_m, velocity V_m and temperature T_m at the same angle and frequency, cut-off at frequencies above f_1, as described in references [1, 6]. The values of V_m, D_m and T_m are calculated from equations (3.1), (3.2) and (3.3).

(ii) *The secondary jet*. This is predicted as

$$SPL_s(\theta, f) = SPL(V_s, T_s, D_s, \theta, f) + 10 \log_{10} F_u(f_1, f), \tag{4.2}$$

precisely as used in reference [1].

(iii) *The effective jet*. This contribution is obtained from

$$SPL_e(\theta, f) = SPL(V_P, T_p, D_e, \theta, f) + \Delta \text{ dB}, \tag{4.3}$$

where D_e is given by equation (2.1) and Δ dB is obtained from equation (3.12) and allows for the reduced turbulence level observed for the interaction region. In the present work all the isolated jet predictions are obtained from reference [5].

The total noise prediction is calculated as the incoherent sum of the contributions given in equations (4.1), (4.2) and (4.3) above.

5. THE DATABASE

The coaxial jet noise database used in this work was obtained from a test programme carried out in the large anechoic chamber at DRA, Pyestock. The nozzle configuration, as described in reference [1], was carefully designed to provide parallel, co-planar exit flows. The diameters of the primary and secondary nozzles are 33.2 mm and 58.2 mm, respectively, yielding a geometric area ratio of 2·0.

This configuration was tested at three secondary jet velocities, 135 m/s, 170 m/s and 270 m/s; the secondary air was unheated. At each secondary velocity the primary jet velocity was varied from 170 m/s to 430 m/s in selected logarithmic steps, and for each of these primary velocities three primary jet temperatures were employed; namely, unheated, 600 K and 800 K respectively.

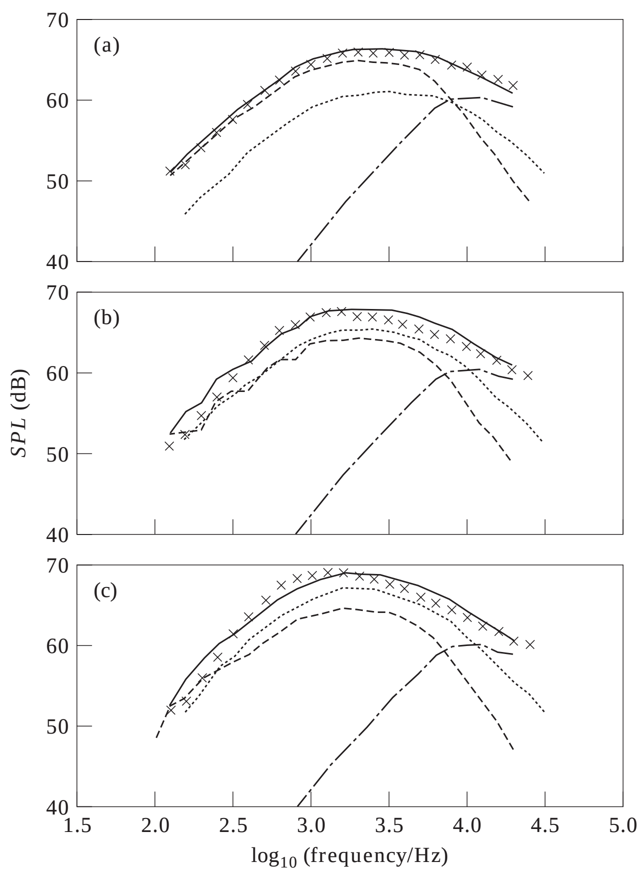

Figure 3. A comparison of data with the prediction at $\theta = 90^{\circ}$ for three jet primary jet temperatures for $V_p = 215$ m/s, $\lambda = 0{\cdot}79$. (a) $T_p =$ ambient; (b) $T_p = 600$ K; (c) $T_p = 800$ K. ·–·–·–, Secondary jet; ----, mixed jet; . . . . . , effective jet; —, prediction; × ×, data points.

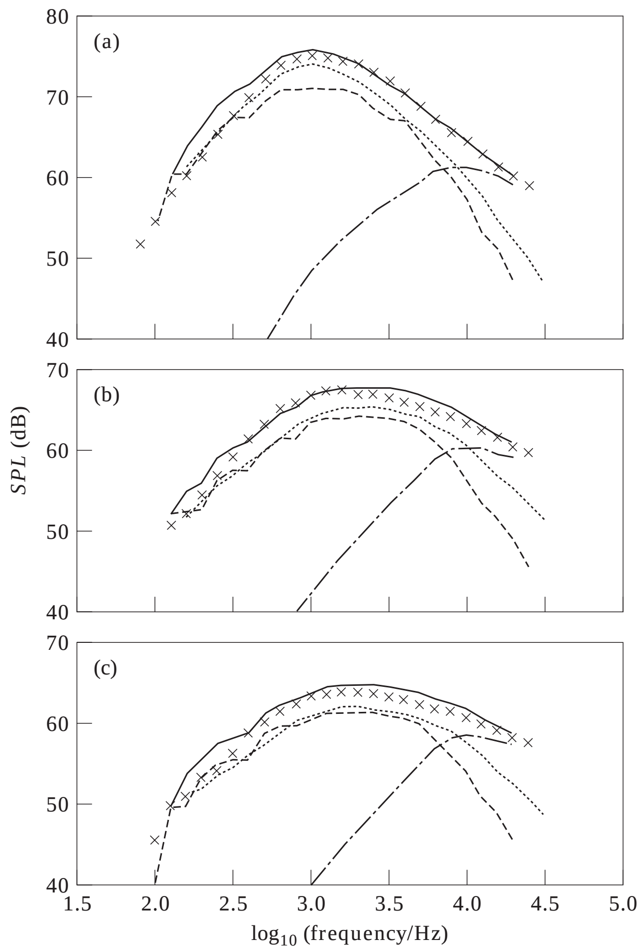

Figure 4. A comparison of data with the prediction at three angles for $V_P = 215$ m/s, $\lambda = 0{\cdot}79$, $T_P = 600$ K. (a) $\theta = 40^\circ$; (b) $\theta = 90^\circ$; (c) $\theta = 120^\circ$. Key as Figure 3.

The noise measurements were made in the far field by using a polar arc of about 12 m radius with microphones at 10° intervals from 30° at 120° to the positive jet axis. The noise levels, corrected for atmospheric absorption to a "lossless" atmosphere, are presented here for a polar distance of 6 m.

6. COMPARISON OF DATA WITH PREDICTION

The comparisons of data with prediction are presented in the form of one-third octave spectra in which the crosses ($\times$) represent the measured values and the full lines (–) are the total prediction. Also shown for information are the spectral contributions made by the full mixed, effective and secondary jets separately.

6.1. RESULTS AT $\theta = 90^\circ$

The general effect of the increasing primary jet temperature is well illustrated in Figures 3(a), 3(b) and 3(c) respectively. This data, for the 90° microphone position, shows the effect of temperature at constant primary and secondary jet velocities for the velocity ratio of 0·79. This velocity ratio is chosen here as the lowest at which the secondary jet makes any real contribution.

Comparison of Figures 3(a), 3(b) and 3(c) shows that the overall effect of increasing the primary temperature from ambient to 800 K is to increase the peak spectral level by about 5 dB, a change which is generally well reflected in the prediction. Examination of the relative contributions to this overall prediction indicates that for the isothermal case the fully mixed jet dominates over the majority of the spectrum, with an important contribution from the secondary jet at high frequencies. Raising the temperature of the primary jet has no effect on the secondary jet contribution and only a small effect on the fully mixed jet, the temperature of which remains relatively modest. However, the effective jet contribution, which is distinctly sub-dominant at isothermal conditions, increases with temperature and dominates at 800 K. This increase is a combination of two effects. First, at these velocities, the predicted level for an isolated jet shows a tendency to increase with increased temperature. Second, while for an isothermal jet the reduced turbulence level of the interaction zone yields a 7 dB reduction (see section 3.5), this reduces to 3·5 dB for a heated, dipole dominated source region. Hence on both counts the contribution of

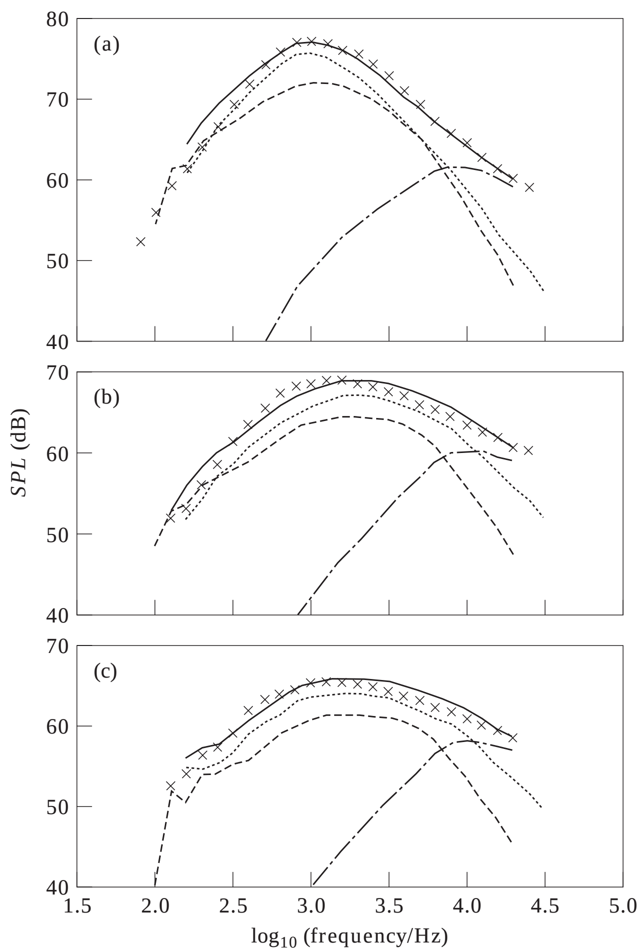

Figure 5. A comparison of data with the prediction at three angles for $V_p = 215$ m/s, $\lambda = 0{\cdot}79$, $T_P = 800$ K. (a) $\theta = 40°$; (b) $\theta = 90°$; (c) $\theta = 120°$. Key as Figure 3.

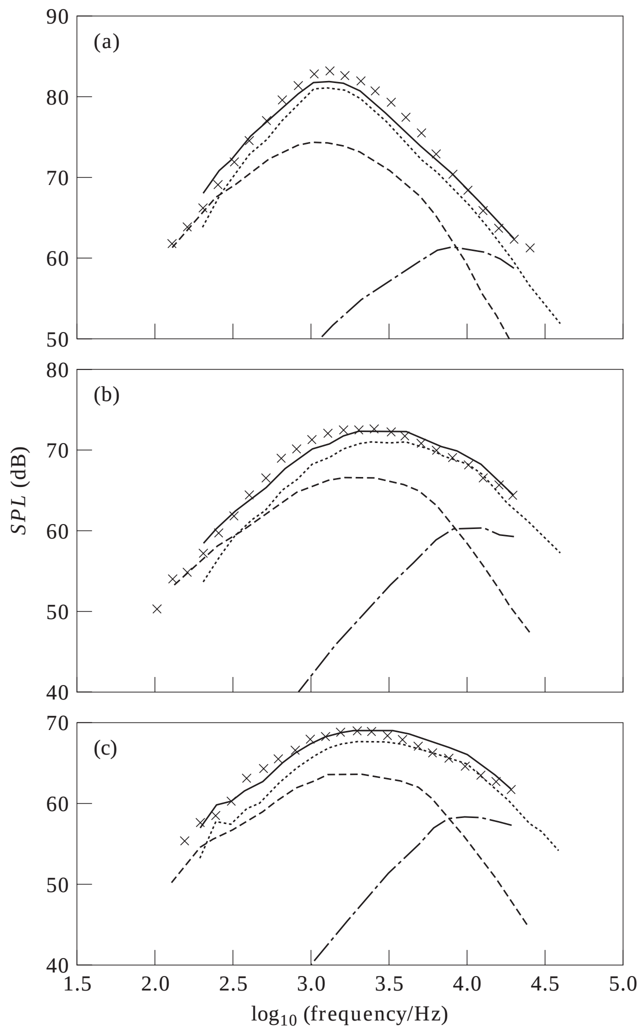

Figure 6. A comparison of data with the prediction at three angles for $V_p = 265$ m/s, $\lambda = 0{\cdot}63$, $T_P = 600$ K. (a) $\theta = 40°$; (b) $\theta = 90°$; (c) $\theta = 120°$. Key as Figure 3.

the effective jet is increased by an increase in primary temperature, but the magnitude of the effect depends on a subtle combination of primary velocity and temperature.

Finally, in the context of this data we note for the heated flows a distinct tendency to overprediction at the lowest frequencies. In fact, it appears that either the effective or fully mixed jet contribution alone would account for the measured levels. It is hypothesized that this is due to some "double accounting" of the fully mixed and effective jets at these very low frequencies which becomes important when the levels are comparable. However, since this occurs only at frequencies below about 300 Hz at this model scale, corresponding to frequencies of the order of 10 Hz at a typical engine scale, it is of little practical importance.

6.2. ANGULAR DEPENDENCIES

In this section, we shall compare data with prediction at angles of 40°, 90° and 120° to the jet axis, again using the velocity ratio of 0·79 so that each source makes some contribution to the overall prediction. The results obtained for a primary jet temperature of 600 K are shown in Figures 4(a), 4(b) and 4(c). We see that at small angles to the jet axis, 40°, the effective jet dominates the region of the spectral peak, while at the larger angles the fully mixed and effective jet contributions become progressively more similar. The higher frequencies result from contributions from the effective jet with the secondary jet becoming dominant at the highest frequencies. The overall agreement is within 1 dB except at the very lowest frequencies, as reviewed above.

Similar comments apply for a primary jet temperature of 800 K—see Figures 5(a), 5(b) and 5(c)—except that the effective jet has now become the principal contributor over the majority of the spectrum.

6.3. EFFECT OF VELOCITY RATIO

The final comparison of data with prediction is for the much lower velocity ratio of 0·63. This is shown for a primary jet temperature of 600 K in Figures 6(a), 6(b) and 6(c) and for 800 K in Figures 7(a), 7(b) and 7(c). The effective jet contribution is now dominant

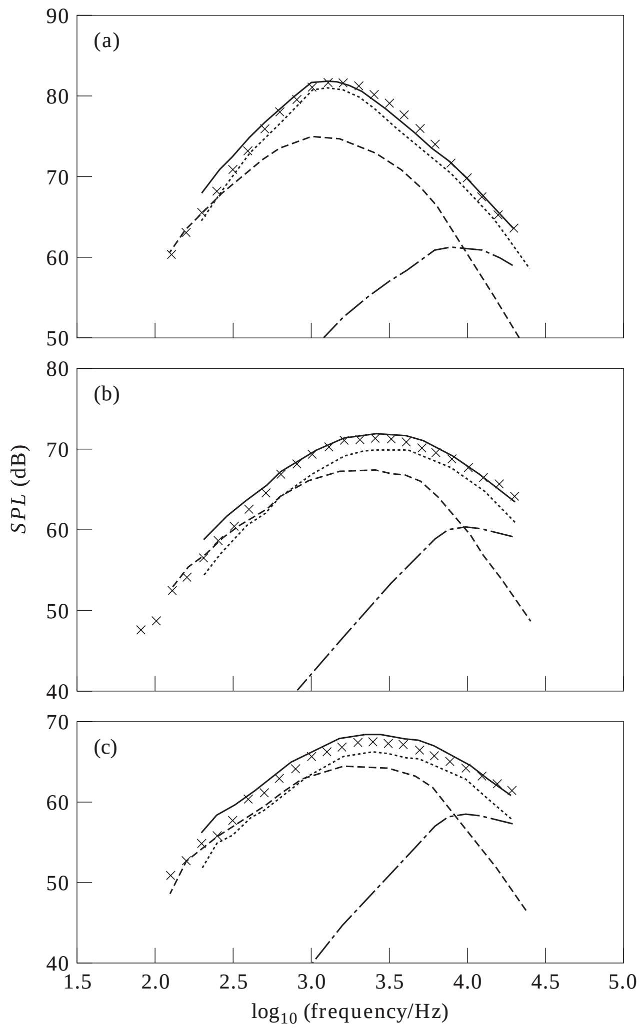

Figure 7. A comparison of data with the prediction at three angles for $V_p = 265$ m/s, $\lambda = 0{\cdot}63$, $T_P = 800$ K. (a) $\theta = 40°$; (b) $\theta = 90°$; (c) $\theta = 120°$. Key as Figure 3.

over the majority of frequencies, particularly for the latter temperature. The comparison does therefore lend credence to the model for the interaction zone noise proposed in earlier sections of this paper. A more extensive range of comparisons than those contained here may be found in reference [13].

7. CONCLUSIONS

The principal objective of the work presented in this paper was to extend a previously proposed model [1] for coaxial jet noise prediction to the case of heated primary flows.

It has been argued that this extension is entirely straightforward for spectral contributions due to both the secondary jet shear layer and fully mixed jet. However, the prediction of noise from the interaction zone, modelled by the effective jet, has proved more challenging for heated flows. Specifically, it has been shown that the presence of dipole sources in heated flows, in addition to the quadrupole sources associated with isothermal flows, requires special consideration in the adaptation of single-jet prediction methods to obtain the noise from this interaction region.

The necessary considerations have been described and quantified to form an extended prediction method. This has been tested against a systematic database with primary jet temperatures up to 800 K. Comparison between data and prediction indicate agreement of order ± 1 dB in one-third octave spectral levels over a wide range of jet operating conditions and angles of observation. The nature of the aggreement also strongly suggests that the premise of this method is firmly based on suitable physical principles.

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