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SEPARATION BETWEEN THE FLEXIBLE STRUCTURE AND THE MOVING MASS SLIDING ON IT

U. Lee

Department of Mechanical Engineering, Inha University, 253 Yonghyun-Dong, Nam-Ku, Inchon 402-751, Korea

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The problem of the dynamic response of an elastic structure due to a moving mass arises in many engineering applications. The interaction force between a moving mass and a structure obviously depends on the velocity of the moving mass and the flexibility of structure. Thus, in some situations, the interaction force may become zero to change its sign, which implies the onset of the separation between the moving mass and structure. Most of the investigations on this subject have missed or ignored the possibility of the onset of separation in solving the dynamic responses of structures excited by moving masses. Hence, this paper investigates the onset of the separation between the moving mass and beam, and then takes into account its effect in calculating the interaction forces and also in calculating the dynamic responses of the beams considered herein. Numerical tests show that the effects of separation become significant as the velocity of the moving mass and the mass ratio (M/ml) increase.

1. INTRODUCTION

For over 150 years since Stokes [1] first brought it to attention, the dynamic response of a structure excited by a moving mass has been the subject of numerous investigations. This subject has become more important and dynamic in both physics and engineering mainly due to increased speed of moving masses and structural flexibility. Typical examples include bridges, guideways, overhead cranes, rails, cable ways, road ways, gun-tubes, and so forth.

Irrespective of the many viewpoints and analytical methods proposed to solve the dynamic problems, most research can be grouped into two categories: the moving force problem and the moving mass problem. A large number of studies on the moving force problem is referred to in the state-of-the-art reviews, references [2, 3], and recently Olsson [4] has discussed the assumptions inherent in the moving force problem and the basic understanding of the moving force phenomena.

In the moving force problem, traditionally the magnitude of the moving force has been assumed to be constant by neglecting the inertia forces of a moving mass, mainly caused by strucure–moving mass interaction. However, in the moving mass problem, the physics is completely different from that in the moving force problem. There must be the interaction force between the moving mass and the structure during the time the mass travels along the structure. The interaction force considers contributions from the inertia of the mass, the centrifugal force, the Coriolis force, and the time-varying velocity-dependent forces, which arise due to the fact that the mass tends to follow the

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deformed shape of the structure, and also from gravity g. Hence, the velocity of the moving mass, structural flexibility, and the mass ratio of the moving mass and structure will be important factors that contribute to creating the interaction force. The interaction force is highly non-linear in nature in its local and convective derivatives and changes its position and magnitude with time. Thus it is usually represented as a concentrated force by use of the Dirac delta function in the dynamic equation of motion.

For instance, the most elaborate formulation of the interaction force between a moving mass and beam can be readily deduced from the transverse dynamic equation of a pipeline conveying internal flow (see reference [5]) as:

$$f(x, t) = M(g - \ddot{w} - 2v\dot{w}' - v^2 w'' - \dot{v}w' - v\dot{v}'w')\delta(x - vt), \tag{1}$$

where $w(x, t)$ is the transverse beam deflection for $0 \leqslant x < l$ and $0 \leqslant t \leqslant l/v$. M and v are the mass and velocity of the moving mass, respectively, and $\delta(x - vt)$ is the Dirac delta function. In equation (1), a prime and a dot indicate the derivatives with respect to x and t, respectively. The fifth and sixth terms on the right side of equation (1) are just due to the local and convective effects of the velocity change of the moving mass, respectively. The sixth term may be neglected in the case of the point moving mass while not in the case of a distributed moving mass similar to flowing fluid within a pipeline.

Because of the difficulties in treating the interaction force between the moving mass and structure, which is sufficiently complex, many authors have studied some simplified versions of the moving mass problem in the past. Many simplifying assumptions on the interaction force between the moving mass and structure have been introduced in calculating the dynamic responses of the structure excited by the moving mass, and most of methods used in the past are restricted to following cases. First, the inertia of a moving mass is completely neglected and therefore the problem is reduced to the moving force problem with constant force magnitude (references [2, 3, 6–10]). This assumption basically requires that the mass of the moving mass be smaller than the mass of the structure, and that the mass inertia be sufficiently small in comparison with its gravitational effect. Secondly, the coupling terms in the interaction force, such as the Coriolis and centrifugal forces are neglected (references [11, 12]). This assumption requires that the velocity of a moving mass and its time rate of change be relatively small. Lastly, the velocity of a moving mass is assumed to be constant. Thus only the force terms related to the local and convective derivatives of the moving mass velocity are neglected (references [13–19]). Ting *et al*. [20] seems to be the first to include an inertia term related to the local derivative of moving mass velocity in their analyses.

Because of many difficulties arising in mathematical operations, many analytical methods have been proposed in the last decasdes. The finite element method which is powerful due to its versatility was first applied to the moving force problem in reference [21] and then in references [22–29] was applied to the moving mass problem. The method of an expansion of the eigenfunctions in series has been utilized in the references [4, 11, 12, 16]. Cifuentes [17] introduced a combined finite element–finite difference technique based on a Lagrangian multiplier formulation that allows one to express the compatibility condition at the beam–mass interface. Ting *et al*. [20] and Sadiku and Leipholz [18] formulated the equation of motion for the system in the form of an integro–differential equation utilizing influence functions, which mathematically are Green functions. The former study used the finite difference method, and the latter study used the eigenfunctions expansion method.

In most of investigations on the moving mass problem, to the author's knowledge, it has been implicitly assumed that the moving mass travels on the structure, always being in contact with it. Under this assumption, the interaction force acting on the beam due to the moving mass has been represented by the function as given in equation (1). However this is not always true in view of the physics if a moving mass simply just sits on the upper surface of the structure and then slides along it. It is obvious from equation (1) that the interaction force between the mass and structure depends on both the velocity of the moving mass and the flexibility of structure. This fact is obviously different from the moving force problem in which the magnitude of moving force is assumed to be independent of the velocity of moving force and the structural flexibility. Thus, certainly the magnitude of the interaction force in the moving mass problem varies with time and can become zero at an instant to change its sign whilst traveling on the structure. This simply implies the onset of the separation between the moving mass and structure. After the onset of separation, the interaction force between the mass and structure must be zero until the moving mass recontacts the structure. The case of the separation of a moving two-axle system and the following impact was studied by Frýba [2, 29]. Hutton and Counts [16] and Ting *et al*. [20] have investigated that their numerical results give slightly larger values in comparison with the experimental results by Ayre *et al*. [30]. Even though there have been very few publications on experimental work to confirm their investigation, the author judges that this discrepancy may be in part due to the onset of the separation between the moving mass structure. To the author's surprise, so far most of investigators have missed or ignored the possibility of the onset of separation in solving the dynamic responses of a structure excited by the moving mass. Thus, the purpose of this paper is to investigate the onset of separation between the moving mass and beam analytically. The effect of separation is then taken into account in calculating the interaction forces and the dynamic responses of the beams considered herein.

2. PROBLEM DEFINITION AND FORMULATION

The moving mass problem considered here is illustrated in Figure 1. The beam shown in Figure 1 is the Bernoulli–Euler beam subjected to a single mass M at a constant velocity v. Hence, the reaction force exerted by the moving mass on the beam is deduced from equation (1) to be

$$f(x,t) = M[g - \ddot{w} - 2v\dot{w}' - v^2 w'']\delta(x - \zeta), \tag{2}$$

where $\zeta = vt$ indicates the present position of the moving mass with time. The equation of motion of the transverse beam displacement $w(x,t)$, neglecting the effect of the rotary inertia, shear and structural damping on the flexural motion of the beam can be written as

$$EIw'''' = F(x,t) \tag{3}$$

with

$$F(x,t) = -m\ddot{w} + f(x,t), \tag{4}$$

where, EI is the beam stiffness and $m(x)$ is the mass of beam per unit length.

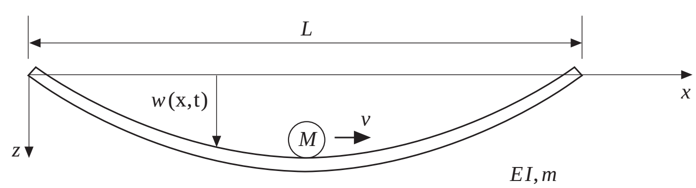

Figure 1. Beam with a moving mass sliding on it.

The partial differential equation of motion (3) can be transformed into the form of an integro–differential equation [31] as

$$w(x,t)=\int_0^l C(x,\alpha)F(\alpha,t)\,\mathrm{d}\alpha, \tag{5}$$

where, $C(x,\alpha)$ is the structural influence function which indicates the static deflection at x due to the unit force applied at $x=\alpha$. The structural influence function satisfies the reciprocal theorem $C(x,\alpha)=C(\alpha,x)$. Substituting equation (4) into equation (5), the general motion of the beam can be rewritten as

$$w(x,t)+\int_0^l C(x,\alpha)m(\alpha)\ddot{w}(\alpha,t)\,\mathrm{d}\alpha = C(x,\zeta)M[g-\ddot{w}(\zeta,t)-2v\dot{w}'(\zeta,t)-v^2w''(\zeta,t)]. \tag{6}$$

The normal modes $\phi_n(x)$ of the beam without the moving mass must satisfy the equations

$$\phi_n(x)=\omega_n^2\int_0^l C(x,\alpha)m(\alpha)\phi_n(\alpha)\,\mathrm{d}\alpha, \tag{7}$$

and its boundary conditions. In equations (7), ω_n are the natural frequencies of the beam. The normal modes $\phi_n(x)$ are also orthogonal functions satisfying the relation

$$\int_0^l m(x)\phi_n(x)\phi_m(x)\,\mathrm{d}x=\bar{m}_n\,\delta_{mn}, \tag{8}$$

where $\bar{m}_n$ is the modal mass and δ_{mn} is the Kronecker delta. The general solution of equation (6) can be represented in terms of $\phi_n(x)$ as

$$w(x,t)=\sum_{n=1}^{N}\phi_n(x)q_n(t), \tag{9}$$

where $q_n(t)$ are the generalized co-ordinates to be determined. Introducing solution (9) into equation (6), multiplying the result by $\phi_m(x)$, and then integrating over the interval $0\leqslant x\leqslant l$ yields

$$\bar{m}_n(\ddot{q}_n+\omega_n^2q_n)=M\phi_n(\zeta)\,[g-\sum\phi_m(\zeta)\ddot{q}_m-2v\sum\phi_m'(\zeta)\dot{q}_m-v^2\sum\phi_m''(\zeta)]$$

$$(n=1,2,\ldots,N). \tag{10}$$

In deriving equations (10), the Dirac delta integral property

$$\int_0^l f(x)\delta(x-\zeta)\,\mathrm{d}x=f(\zeta);\qquad (0<x,\zeta<l), \tag{11}$$

and the relations of equations (7) and (8) are applied. Now, equations (10) can be represented in the form of matrix differential equation as follows:

$$[\mathbf{M}]\{\ddot{\mathbf{q}}\}+[\mathbf{C}]\{\dot{\mathbf{q}}\}+[\mathbf{K}]\{\mathbf{q}\}=\{\mathbf{P}\}, \tag{12}$$

with

$$[\mathbf{M}] = \mathrm{diag}[\bar{m}_n] + M\,\mathrm{diag}\,[\phi_n(\zeta)]\,[\mathbf{\Phi}(\zeta)], \qquad [\mathbf{C}] = 2Mv\,\mathrm{diag}\,[\phi_n(\zeta)]\,[\mathbf{\Phi}'(\zeta)],$$

$$[\mathbf{K}] = \mathrm{diag}\,[m_n\,\omega_n^2] + Mv^2\,\mathrm{diag}\,[\phi_n(\zeta)]\,[\mathbf{\Phi}''(\zeta)], \qquad \{\mathbf{P}\} = Mg\{\phi_1(\zeta), \ldots, \phi_N(\zeta)\}^{\mathrm{T}}, \quad (13)$$

where,

$$[\mathbf{\Phi}(\zeta)] = \begin{bmatrix} \phi_1(\zeta) & \ldots & \phi_N(\zeta) \\ \phi_1(\zeta) & \ldots & \phi_N(\zeta) \\ \vdots & & \vdots \\ \phi_1(\zeta) & \ldots & \phi_N(\zeta) \end{bmatrix}. \quad (14)$$

In equations (13), diag [ ] indicates the diagonal matrix and the matrices $\mathbf{\Phi}'$ and $\mathbf{\Phi}''$ are the first and second derivatives of the matrix $\mathbf{\Phi}(x)$ with respect to x, respectively. Equation (12) is a set of coupled ordinary differential equations and, in this study, the fifth order Runge–Kutta scheme is employed to solve it. The dynamic responses of the beam can be readily determined by introducing the solutions of equation (12) into equation (9).

When the separation between the moving mass and beam occurs, the interaction force of equation (2) must be forced to be zero. That is, during separation $t_1 \leqslant t \leqslant t_2$, the foregoing dynamic equation of motion must be replaced by the followng two equations:

$$w(x, t) + \int_0^l C(x, \alpha)m(\alpha)\ddot{w}(\alpha, t)\,\mathrm{d}\alpha = 0, \qquad \ddot{z} = g \quad (15)$$

with the initial conditions specified as

$$w(x, t_1) = w_1(x), \qquad \dot{w}(x, t_1) = \dot{w}_1(x), \qquad z(t_1) = w_1(\zeta_1), \qquad \dot{z}(t_1) = \dot{w}_1(\zeta_1), \quad (16)$$

where $\zeta_1 = vt_1$ and z represents the motion of moving mass (positive downward) due to gravity. Equations (16) represents the motion of the moving mass due to the gravity during separation. When two separate solutions of equations (15) become equal again at $t = t_2$, which implies that the moving mass starts recontacting the beam, then the equation of motion (6) must be solved in sequence by using the solutions of equations (15) at $t = t_2$ as the new initial conditions for the dynamic response of the beam after $t = t_2$. As studied in Frýba [2], the separation and the following impact may be very important in practice

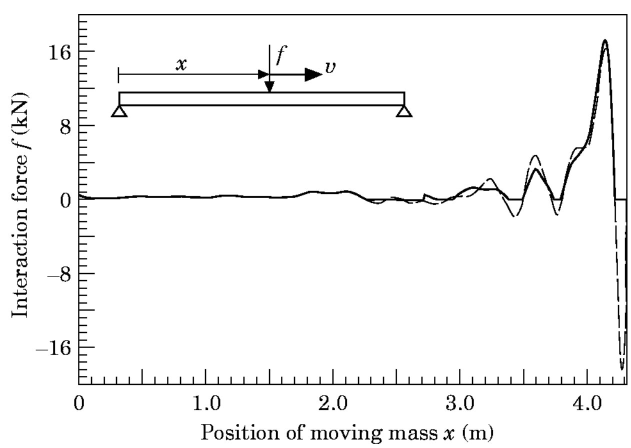

Figure 2. The interaction force (positive downward) acting on the simply supported beam when $v = 50$ m/s. Key: —, separation considered; — — —, separation not considered.

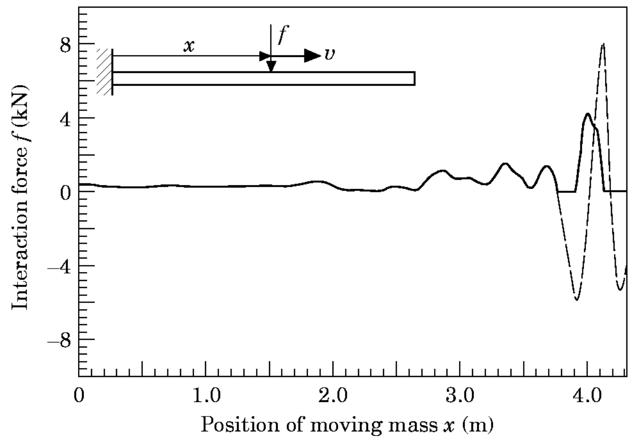

Figure 3. The interaction force (positive downward) acting on the cantilevered beam when $v = 50$ m/s. Key as for Figure 2.

for highway and railroad bridges. If impact occurs, a higher number of natural modes of vibration must be taken into account in the analysis. In the present study, however the effects of the impact as well as the elasticity between the moving mass and beam are not considered for simplicity.

3. NUMERICAL RESULTS AND DISCUSSION

Some moving mass problems are considered using the solution procedure described in the previous section. The properties of the Bernoulli–Euler beam considered herein are $l = 4{\cdot}318$ m, $M = 20{\cdot}245$ kg/m, and $EI = 63000$ Nm^2, which was also used by Ting *et al.* [20]. To confirm the accuracy of the computer algorithm developed for solving the equation (12) prior to further calculations and discussion, the exactly same problem considered by Akin and Mofid [12] was revisited retaining only the inertia force of the mass in order to reduce the present equation of motion to their simplified one. Obviously the present numerical results proved to be almost identical to the numerical results obtained by them.

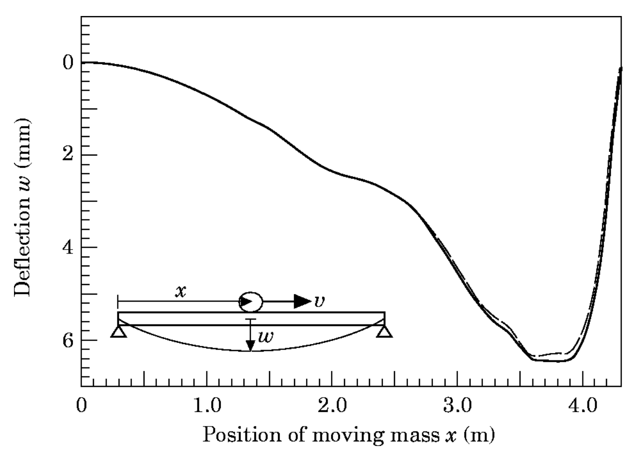

Figure 4. Deflection of the simply supported beam at the present position of moving mass when $v = 50$ m/s. Key as for Figure 2.

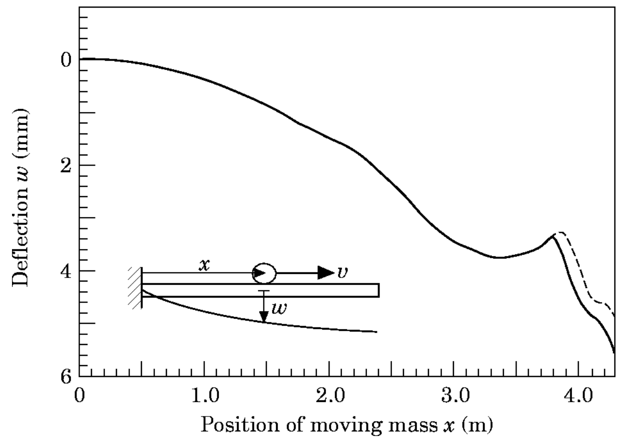

Figure 5. Deflection of the cantilevered beam at the present position of moving mass when v = 50 m/s. Key as for Figure 2.

In the present numerical calculations, a sufficient number of natural modes of vibration are taken into account so that the vibration responses converge enough within 0·1%.

With and without taking into account the effect of the separation between the moving mass and beam, the interaction forces for the simply supported and cantilevered beams when v = 50 m/s are shown in Figures 2 and 3. When the effect of the separation is not taken into account, the interaction force certainly can have its direction upward in some regions, which physically means the onset of the separation between the moving mass and beam. The solid lines indicate the interaction force calculated by forcing the function $f(x, t)$ of equation (2) being zero during the separation. Figures 2 and 3 also show that the effect of separation on the interaction force is apparent in the region near the arrival end. Accordingly, as shown in Figures 4 and 5 when v = 50 m/s, the effect of separation on the beam deflection is also apparent in the region near the arrival end. The discrepancy between the beam deflections with and without considering the separation can be also observed from Figures 4 and 5. Numerical tests show that the effect of separation on the dynamic response of the beam becomes important as the velocity of the moving mass increases. Recalling that Hutton and Counts [16] and Ting *et al.* [20] reported that their

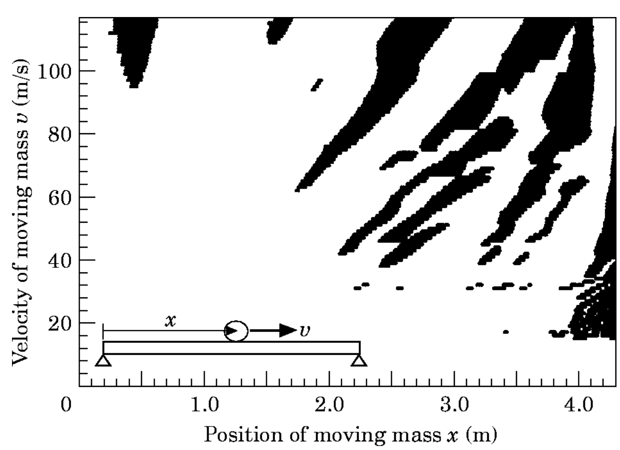

Figure 6. Separation region with respect to the velocity of moving mass, in which the separation between the mass and simply supported beam can occur. Key: ■, Separation; □, contact.

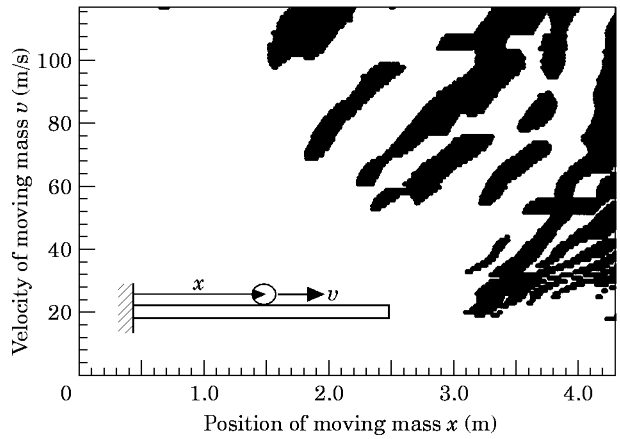

Figure 7. Separation region with respect to the velocity of moving mass, in which the separation between the mass and cantilevered beam can occur. Key as for Figure 6.

numerical results calculated without considering the effect of separation gave slightly larger values compared with the experimental results by Ayre *et al*. [30] and also observing that the present numerical results calculated with consideration of the separation (solid lines) are slightly smaller than those without considering the separation (dotted lines), the author can conclude that the discrepancy between the numerical results and the experimental results has arisen in part due to the neglect of the separation between the moving mass and beam. Thus, for accurate prediction of the dynamic response of a structure excited by a moving mass, the effect of separation must be taken into account in the analysis.

Figures 6 and 7 show the separation region along the beam in which the separation between the moving mass and beam can occur. Similarly, Figures 8 and 9 show the time duration in which the separation can occur. The figures show that there exists no separation region at all below certain value of the velocity of moving mass. However, the separation region seems to widen and also move toward the departure end as the velocity of the moving mass increases. Thus the onset of separation also seems to make an earlier start as the velocity of moving mass increases, which is shown in Figures 8 and 9.

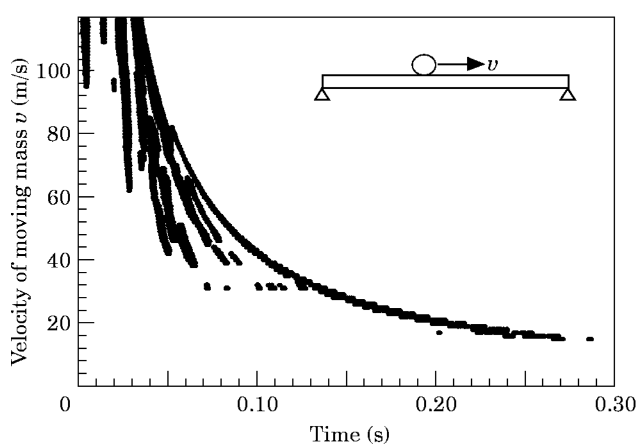

Figure 8. Time duration with respect to the velocity of moving mass, in which the separation between the mass and simply supported beam can occur. Key as for Figure 6.

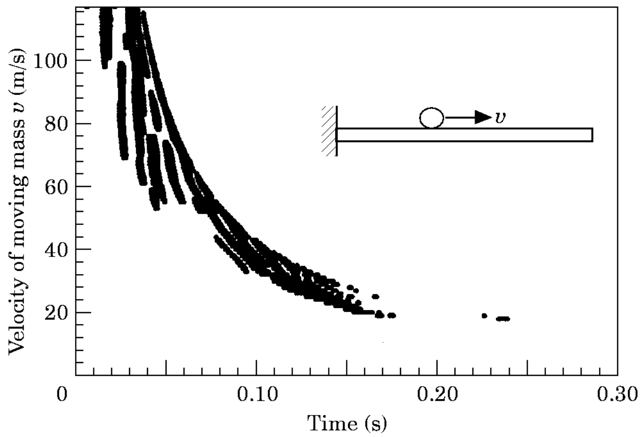

Figure 9. Time duration with respect to the velocity of moving mass, in which the separation between the mass and cantilevered beam can occur. Key as for Figure 6.

4. CONCLUSIONS

The paper has considered the onset of the separation between a moving mass and beam by using the integro–differential equation of motion and modal analysis method. Numerical tests show that the separation can occur more easily and has a significant effect on the dynamic responses of the beam especially at high velocity of the moving mass. Thus, one may conclude that the separation phenomenon must be taken into account in the analysis for accurate prediction of the dynamic responses of a structure due to a moving mass as the velocity and weight of the moving mass increase. To complete the issue of the separation phenomenon, however the effects of the impact occurring after separation and the elasticity between a structure and moving mass should be further investigated by both theoretical and experimental approaches, which is the on-going research topic of the present author.

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APPENDIX: NOTATION

$C(x, \zeta)$	structural influence function
$[\mathbf{C}]$	damping matrix defined in equation (13)
EI	flexural rigidity of beam
$f(x, t)$	interaction force between a moving mass and beam
$F(x, t)$	function defined in equation (4)
g	gravitational acceleration
$[\mathbf{K}]$	stiffness matrix defined in equation (13)

l	length of beam
$m(x)$	mass per unit length of beam
$\bar{m}_n$	modal mass
M	mass of a moving mass
$[\mathbf{M}]$	mass matrix defined in equation (13)
$\{\mathbf{P}\}$	generalized forces vector defined in equation (13)
$q_n(t)$	generalized co-ordinates
t	time
v	velocity of a moving mass
w	transverse deflection of beam
x	axial co-ordinate along the beam
$\delta(x)$	delta function
δ_{mn}	Kronecker delta
$\zeta = vt$	present position of a moving mass
ϕ_n	natural modes of vibration
ω_n	natural frequencies