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TRANSVERSE VIBRATIONS OF A CIRCULAR PLATE WITH A CONCENTRIC SQUARE HOLE WITH FREE EDGES

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1. INTRODUCTION

Free vibrations of a rectangular (and in particular square) plate with a circular hole have been studied by several researchers [1]. On the other hand, the geometrically opposite configuration, a circular plate with a concentric square perforation, has not apparently been considered in the open literature. This situation is also of technological interest since it appears in certain mechanical systems.

The present study deals with the determination of the lower natural frequencies of vibration of the structural element shown in Figure 1. In this case, the edges of the hole are free and the outer boundary is either simply supported or clamped. The problem is solved by two independent techniques. (1) The Rayleigh–Ritz method using polynomial co-ordinate functions which identically satisfy the boundary conditions at the outer edge [2]. (2) The finite element method (making use of the SAMCEF system [3]).

2. SOLUTION BY MEANS OF THE RAYLEIGH–RITZ METHOD

The displacement amplitude is approximated using the expression

$$W(r,\theta) = W_\alpha(r,\theta) = \sum_{j=0}^{J} A_{0j}(\alpha_{0j}r^4 + \beta_{0j}r^2 + 1)r^{2j} + \sum_{k=1}^{4}\sum_{j=0}^{J} A_{kj}(\alpha_{kj}r^4 + \beta_{kj}r^2 + 1)r^{j+k}\cos k\theta \qquad (1)$$

where α's and β's of each co-ordinate function are determined substituting each functional relation in the governing boundary conditions at the external contour, i.e., for the simply supported edge,

$$W(a,\theta) = 0, \qquad \left.\frac{\partial^2 W}{\partial r^2} + \mu\left(\frac{1}{r}\frac{\partial W}{\partial r} + \frac{1}{r^2}\frac{\partial^2 W}{\partial \theta^2}\right)\right|_{r=a} = 0, \qquad (2)$$

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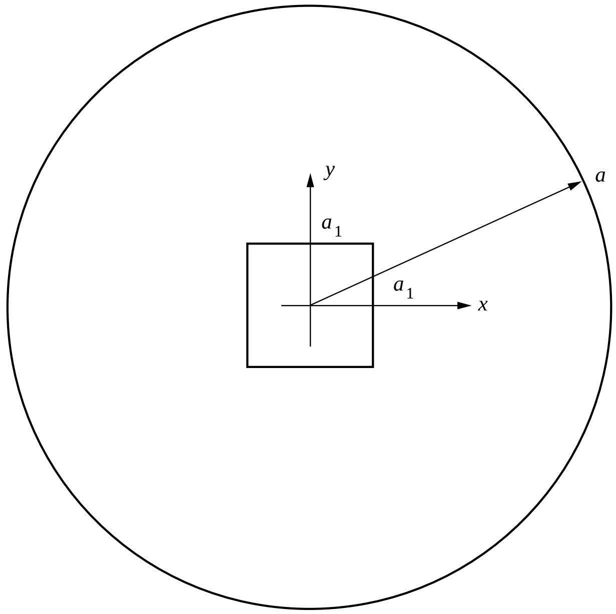

Figure 1. Vibrating system under study.

and for the clamped edge,

$$W(a, \theta) = \frac{\partial W}{\partial r}(a, \theta) = 0. \tag{3}$$

Substituting equation (1) in the well-known energy functional $J[W]$ corresponding to the thin elastic plate under investigation, and requiring that

$$\frac{\partial J}{\partial A_{kj}}[W] = 0 \tag{4}$$

one obtains, finally, a homogeneous linear system of equations in the A_{kj}'s.

The non-triviality requirement yields a secular determinant whose roots constitute approximations to the lower natural frequencies of transverse vibration.

3. THE FINITE ELEMENT SOLUTION

Numerical results for the present problem have been obtained by using the finite element code SAMCEF [3]. Hybrid plate elements of triangular and rectangular shape (element types 55 and 56 of the SAMCEF library) have been used.

These are displacement based elements which superimpose a Marguerre membrane and a hybrid plate, and allow the modelling of thin shells or plates. For the plate behavior, they follow Kirchhoff's theory, and possess displacement connectors at the element vertices and equilibrium connectors at the mid-sides. For the case in which $a_1/a = 0{\cdot}1$, half of the plate domain was subdivided into 661 elements.

4. NUMERICAL RESULTS

The eigenvalues, $\Omega_i = \sqrt{\rho h/D}\,\omega_i a^2$, have been determined for $\mu = 0{\cdot}30$ and $a_1/a = 0{\cdot}10$ and 0·20.

TABLE 1

Frequency coefficients of the system shown in Figure 1

	$a_1/a = 0{\cdot}10$		$a_1/a = 0{\cdot}20$	
	Analytical method*	Finite element	Analytical method	Finite element
Simply supported				
Ω_1	4·84	4·82	4·47	4·64
Ω_2	13·90	13·85	13·65	13·27
Ω_3	25·59	25·32	25·34	24·50
Ω_4	29·15	29·37	27·46	32·20
Ω_5	39·97	39·98	39·97	39·46
Ω_6	48·84	47·58	45·09	43·83
Clamped				
Ω_1	10·09	10·12	9·65	10·49
Ω_2	21·28	21·10	20·72	19·94
Ω_3	34·90	34·33	34·40	33·08
Ω_4	38·96	39·41	36·90	44·22
Ω_5	51·03	50·88	51·03	50·04
Ω_6	61·40	59·18	55·65	55·11

* The analytical results have been obtained using 40 polynomials.

Table 1 depicts values of the frequency coefficients for simply supported and clamped outer boundaries computed: (1) analytically, and (2) by means of the finite element method.

The agreement is reasonably good from an engineering viewpoint. Rather surprisingly the eigenvalues determined using the Rayleigh–Ritz method and polynomial approximations were, in general, somewhat lower than the values obtained using the finite element code. For the simply supported case, both methods predict fundamental frequencies lower than the frequency corresponding to the solid plate ($\Omega_1 = 4{\cdot}9351$). This is also the case when the plate is clamped for $a_1/a = 0{\cdot}10$. However, when $a_1/a = 0{\cdot}20$, the analytical method yields a result lower than the value of Ω_1 corresponding to a solid plate ($\Omega_1 = 10{\cdot}2158$), while the finite element technique predicts a slightly higher value.

The frequency coefficients presented in this study must be considered as first order approximations. This is in view of the fact that due to the presence of the re-entrant corners of the square perforation, an in-plane stress field is generated as the plate vibrates in its normal modes.

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