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VIBRATION REDUCTION BY USING PERIODIC SUPPORTS IN A PIPING SYSTEM

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In this paper, the applicability of the periodic characteristics of wave stop and wave propagation bands are investigated for piping systems conveying fluid by employing the wave approach and are proved through experiments. The inviscid fluid dynamic forces acting on a pipe due to internal fluid flow are approximated by the plug-flow model with the slender-body theory. This paper shows that if the dominant frequency contents in the excitation loads are known, a proper design of periodic supports for reducing the vibration in those frequency bands is possible. Therefore, the periodic support design may be effective in vibration reduction in a piping system.

1. INTRODUCTION

In the design of a piping system, the pipe supports have very important design features in view of resisting system loads such as dead weights, thermal expansion, pump pulsation, water hammering and earthquakes. Piping systems such as the steam generator heat exchanger tubes, the main steam pipes and hot/cold leg pipes in a nuclear steam supply system, oil pipe lines, pump discharge lines, and marine risers, are supported in various ways, depending on environmental conditions and individual requirements. In general, the steam generator heat exchanger tubes and oil pipe lines are designed with periodic support patterns. Many components of engineering structures are supported in spatially periodic forms or constructed with an assembly of identical components, jointed end-to-end or side-by-side in an identical manner to form the whole structure. Such engineering structures which consist of periodic components include railway lines on equispaced sleepers, antenna systems, stiffened plates used in aircraft and ship panels, and truss structures of space stations. These periodic systems have special characteristics of free wave propagation such as wave stop and wave propagation frequency bands which have been known by many researchers. All cells of the periodic system act as band-pass and band-reject filters. For an infinite periodic system, the natural frequencies of the system do not exist in wave stop frequency bands and all natural frequencies appear in wave propagation frequency bands. But in the case of finite periodic systems, the periodic pattern may be changed at boundary regions. Therefore, the natural frequencies of the system may exist in wave stop frequency bands due to the vibration characteristics of boundary regions. Studies on various periodic systems have been done by many researchers [1–6]. D. J. Mead and his companions have published many papers since the 1970's in this field [1–4]. They used various methods such as the Galerkin type method, the Rayleigh–Ritz method, the transfer matrix method and the phased array method to calculate the wave propagation constants.

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Little has been done to utilize the characteristics of free wave propagation of the periodic system when designing support systems of pipes conveying fluid. As mentioned in the above paragraph, each cell of the periodic system has similar functions to the mechanical filters. Therefore, if one knows the frequency contents of excitation loads, one may adjust the pipe support design to avoid them. To apply the periodic characteristics to the pipe support design, various factors which can change the characteristics of the wave stop and propagation bands should be studied, such as the effects of pipe internal flow, support rigidity, and the periodic distance between each support.

In this paper, the uniform straight pipe element conveying fluid is formulated by the dynamic stiffness matrix using the wave approach. With these formulations, the uniform straight pipe sections can be expressed with a single pipe element and the analyses of the free wave propagation using the transfer matrix method can be very easily carried out. The dynamic stiffness method adopting the wave approach has been applied by many authors in various fields [7–10].

In the following section, the free wave propagation for the periodically supported, infinite piping system conveying fluid is formulated. In section 3, the formulations of the forced responses of a periodically supported, finite piping system are presented. In section 4, the characteristics of the free wave propagation of a periodic system and the frequency responses of a periodically supported, finite test piping system are examined both by analyses and experiments. For the substantiation of a periodic support design in piping systems, frequency response and power flow analyses are carried out for a complex three-dimensional piping system with partially periodic supports, and these results are compared with those of a non-periodic system.

2. FORMULATIONS OF AN INFINITE PERIODIC SYSTEM

2.1. UNIFORM PIPE ELEMENT CONVEYING FLUID

The differential equations governing the flexural vibrations of a single uniform pipe element can be expressed as

$$E(1 + \mathrm{j}\eta)Iy^{\mathrm{IV}}(x, t) + pA_f y''(x, t) + m_p \ddot{y}(x, t) = F_e(x, t) + F_f(x, t), \tag{1}$$

where the coefficient η represents the internal loss factor of the pipe and EI represents the flexural stiffness of the pipe. A list of notations is given in Appendix A. The pipe is subjected to internal pressure p and excitation force F_e. For excitation forces the subscript e means externally applied mechanical forces, and f means applied inviscid fluid dynamic forces acting on the pipe due to internal fluid flow. These inviscid fluid dynamic forces were derived according to the plug-flow approximation and slender-body theory. This slender-body theory is valid for a pipe with large length-to-radius ratios, where the diameter of the pipe is small compared to the wavelength of deformation and the small pipe deflections. From this approximation, the inviscid fluid dynamic forces for each vibration can be expressed,

$$F_f(x, t) = -m_f[\partial^2/\partial t^2 + 2V\partial^2/\partial x\,\partial t + V^2\partial^2/\partial x^2]y(x, t). \tag{2}$$

Substituting equation (2) into (1), and for the case of harmonic excitations, one can get the following equation of motion in the frequency domain.

$$E(1 + \mathrm{j}\eta)IY^{\mathrm{IV}}(x, \omega) + (m_f V^2 + pA_f)Y''(x, \omega) + \mathrm{j}2m_f\omega VY'(x, \omega) - (m_p + m_f)\omega^2 Y(x, \omega) = F_e(x). \tag{3}$$

The general solutions of equation (3) have the form as

$$Y(x) = \sum_{n=1}^{4} \mathbf{A}_n e^{\mathbf{k}_n x}. \tag{4}$$

The flexural wave numbers $\mathbf{k}$ are the four complex roots of the equation

$$E(1 + \mathrm{j}\eta)I\mathbf{k}^4 + (m_f V^2 + pA_f)\mathbf{k}^2 + \mathrm{j}2m_f\omega V\mathbf{k} - (m_p + m_f)\omega^2 = 0. \tag{5}$$

To obtain the dynamic stiffness matrix of the pipe element, the equation (5) may be used by obtaining the displacements and elastic restoring forces at the ends of the pipe element. First, one can express the displacements at the ends of each pipe element with state vectors as

$$\begin{Bmatrix} Y_L \\ Y'_L \\ Y_R \\ Y'_R \end{Bmatrix}_e = \begin{bmatrix} 1 & 1 & 1 & 1 \\ \mathbf{k}_1 & \mathbf{k}_2 & \mathbf{k}_3 & \mathbf{k}_4 \\ e^{\mathbf{k}_1 L} & e^{\mathbf{k}_2 L} & e^{\mathbf{k}_a 3 L} & e^{\mathbf{k}_4 L} \\ \mathbf{k}_1\, \mathrm{e}^{\mathbf{k}_1 L} & \mathbf{k}_2\, \mathrm{e}^{\mathbf{k}_2 L} & \mathbf{k}_3\, \mathrm{e}^{\mathbf{k}_3 L} & \mathbf{k}_4\, \mathrm{e}^{k_4 L} \end{bmatrix}_e \begin{Bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \\ \mathbf{A}_3 \\ \mathbf{A}_4 \end{Bmatrix}_e, \tag{6}$$

where the subscripts L and R represent left and right end state vectors of the pipe element and e represents a single uniform straight pipe element in local co-ordinates. Equation (6) can be expressed concisely as

$$\mathbf{W}_e = [\mathbf{D}_1]_e \mathbf{A}_e. \tag{7}$$

From the Euler beam theory, the elastic restoring shear force and bending moment can be obtained respectively from the equations as

$$S(x) = -E(1 + \mathrm{j}\eta)IY'''(x), \qquad M_b(x) = E(1 + \mathrm{j}\eta)IY''(x). \tag{8, 9}$$

One can obtain the elastic restoring force state vectors using equations (8, 9) at the ends of the pipe element as

$$\begin{Bmatrix} -S_L \\ -M_{bL} \\ S_R \\ M_{bR} \end{Bmatrix} = E(1 + \mathrm{j}\eta)I \begin{bmatrix} \mathbf{k}_1^3 & \mathbf{k}_2^3 & \mathbf{k}_3^3 & \mathbf{k}_4^3 \\ -\mathbf{k}_1^2 & -\mathbf{k}_2^2 & -\mathbf{k}_3^2 & -\mathbf{k}_4^2 \\ -\mathbf{k}_1^3 e^{\mathbf{k}_{a1} L} & -\mathbf{k}_2^3 e^{\mathbf{k}_2 L} & -\mathbf{k}_3^3 e^{\mathbf{k}_3 L} & -\mathbf{k}_4^3 e^{\mathbf{k}_4 L} \\ \mathbf{k}_1^2 e^{\mathbf{k}_1 L} & \mathbf{k}_2^2 e^{\mathbf{k}_2 L} & \mathbf{k}_3^2 e^{\mathbf{k}_3 L} & \mathbf{k}_4^2 e^{\mathbf{k}_4 L} \end{bmatrix} \begin{Bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \\ \mathbf{A}_3 \\ \mathbf{A}_4 \end{Bmatrix}. \tag{10}$$

Equation (10) can be expressed concisely as

$$\mathbf{F}_e = [\mathbf{D}_2]_e \mathbf{A}_e. \tag{11}$$

From equation (11) one can represent the coefficient vectors

$$\mathbf{A}_e = [\mathbf{D}_1]_e^{-1} \mathbf{W}_e, \tag{12}$$

and substituting equation (12) into equation (11) one obtains the equation

$$\mathbf{F}_e = [\mathbf{D}_2]_e [\mathbf{D}_1]_e^{-1} \mathbf{W}_e = [\mathbf{D}]_e \mathbf{W}_e. \tag{13}$$

The matrix $[\mathbf{D}]_e$ in equation (13) is a well known dynamic stiffness matrix and is a function of the frequencies. When the pipe internal fluid flow V is not zero, the dynamic stiffness matrix $[\mathbf{D}]_e$ becomes a non-symmetric matrix. This dynamic stiffness matrix, derived using the wave approach, is the exact solution because the exact solution forms of equation (4) are used in formulations [7]. The vector $\mathbf{F}_e$ is the elastic restoring force which acts at the ends of the pipe element and $\mathbf{W}_e$ is the displacement vector.

2.2. PIPE SUPPORT ELEMENTS

The dynamic stiffness matrix representing the concentrated mass, the spring and the damper elements can be easily obtained from the conditions of the force equilibrium. These elements can be applied to construct the model of the pipe supports and valves and so forth.

The conditions of the force equilibrium in translational linear spring-damper elements can be expressed with the relative displacements at the two end points 1, 2 in the frequency domains as

$$\mathrm{j}\omega D[\mathbf{W}_1(\omega) - \mathbf{W}_2(\omega)] + K[\mathbf{W}_1(\omega) - \mathbf{W}_2(\omega)] = F(\omega), \tag{14}$$

where $\mathbf{W}_1$ and $\mathbf{W}_2$ represent the displacement vectors at the end points, 1 and 2 and K and D are the spring and damping constant respectively. The dynamic stiffness matrix for translational and torsional spring-damper elements can be represented as

$$\begin{Bmatrix} \mathbf{F}_1 \\ \mathbf{F}_2 \end{Bmatrix} = \begin{bmatrix} (\mathrm{j}\mathbf{D} + \mathbf{K}) & -(\mathrm{j}\mathbf{D} + \mathbf{K}) \\ -(\mathrm{j}\mathbf{D} + \mathbf{K}) & (\mathrm{j}\mathbf{D} + \mathbf{K}) \end{bmatrix} \begin{Bmatrix} \mathbf{W}_1 \\ \mathbf{W}_2 \end{Bmatrix}. \tag{15}$$

Similarly the rotational spring–damper elements are expressed as

$$\begin{Bmatrix} \mathbf{F}_1 \\ \mathbf{F}_2 \end{Bmatrix} = \begin{bmatrix} (\mathrm{j}\mathbf{D}_1\omega + \mathbf{K}_1) & \phi \\ \phi & (\mathrm{j}\mathbf{D}_2\omega + K_2) \end{bmatrix} \begin{Bmatrix} \mathbf{W}_1 \\ \mathbf{W}_2 \end{Bmatrix}. \tag{16}$$

The conditions of the force equilibrium for the concentrated mass element can be expressed in the frequency domains as

$$\mathbf{F}(\omega) = -M\omega^2\mathbf{W}(\omega). \tag{17}$$

Therefore, the dynamic stiffness matrix of the pipe supports can be obtained by using the equations (15) and (16) and assembled with the pipe element of equation (13).

2.3. TRANSFER MATRIX OF THE PERIODIC CELL

The transfer matrix of a periodic cell of a periodically supported piping system, as shown in Figure 1, can be easily obtained from the dynamic stiffness matrix. The dynamic stiffness matrix, derived using the wave approach, is very useful in getting the transfer matrix of a periodic cell because it uses exact formulation, unlike the finite element method which

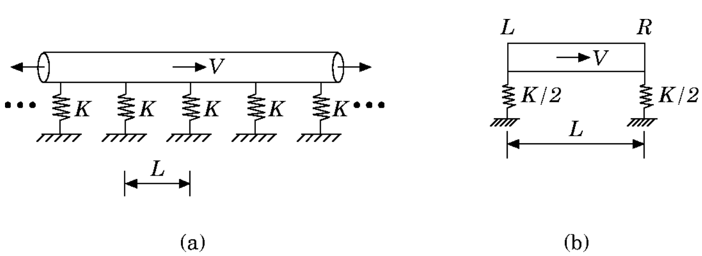

Figure 1. Periodically supported (a) infinite piping system and (b) a single periodic cell.

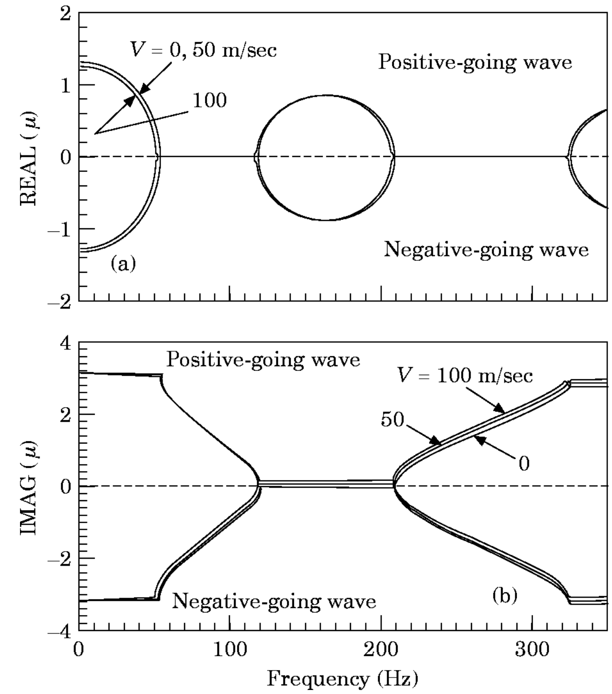

Figure 2. Effects of pipe internal fluid velocities in free wave propagation: $D_o = 0{\cdot}1$ m, $D_i = 0{\cdot}09$ m, $K = 1 \times 10^{10}$ N/m, and $L = 2$ m.

adopts matrix condensations. To obtain the transfer matrix, the dynamic stiffness matrix of a periodic cell is partitioned into left and right end state vectors as

$$\begin{Bmatrix} -\mathbf{F}_L \\ \vdots \\ \mathbf{F}_R \end{Bmatrix} = \begin{bmatrix} \mathbf{K}_1 & \cdots & \mathbf{K}_2 \\ \vdots & \vdots & \vdots \\ \mathbf{K}_3 & \cdots & \mathbf{K}_4 \end{bmatrix} \begin{Bmatrix} \mathbf{W}_L \\ \vdots \\ \mathbf{W}_R \end{Bmatrix}. \tag{18}$$

In equation (18), the negative values of $\mathbf{F}_L$ are introduced to satisfy the force compatibility. A further partial inversion of the dynamic stiffness matrix leads to the following transfer matrix as

$$\begin{Bmatrix} \mathbf{W}_R \\ \vdots \\ \mathbf{F}_R \end{Bmatrix} = \begin{bmatrix} -\mathbf{K}_2^{-1}\mathbf{K}_1 & \cdots & -\mathbf{K}_2^{-1} \\ \vdots & \vdots & \vdots \\ (\mathbf{K}_3 - \mathbf{K}_4\mathbf{K}_2^{-1}\mathbf{K}_1) & \cdots & -\mathbf{K}_4\mathbf{K}_2^{-1} \end{bmatrix}_n \begin{Bmatrix} \mathbf{W}_L \\ \vdots \\ \mathbf{F}_L \end{Bmatrix}. \tag{19}$$

Equation (19) can be represented compactly as

$$\mathbf{Z}_R = [\mathbf{T}(\omega)]\mathbf{Z}_L, \tag{20}$$

where $[\mathbf{T}(\omega)]$ is the transfer matrix of a periodic cell.

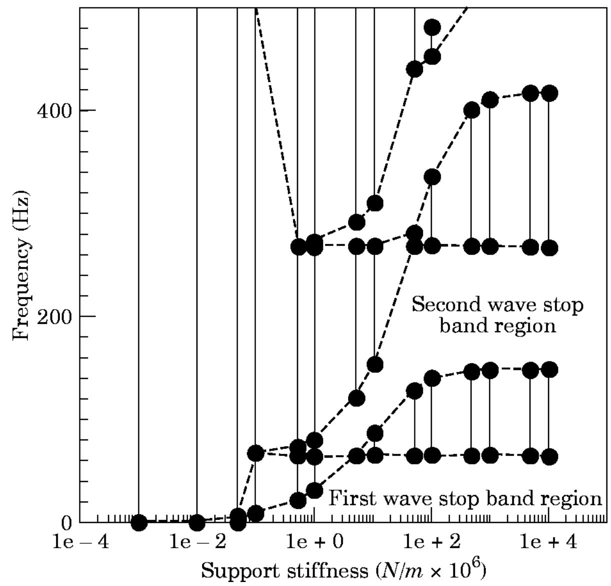

Figure 3. Effects of support stiffness in free wave propagation bands: $L = 2$ m and no pipe internal fluid; ●–●, propagation bands.

2.4. FREE WAVE PROPAGATION CONSTANTS

The free wave propagation analysis of infinitely periodically supported piping systems conveying fluid can be further carried out by using the Floquet theorem. The right and left end state vectors in a single periodic cell can be represented as

$$\mathbf{Z}_{i+1} = \mathbf{C}(\omega)\mathbf{Z}_i, \tag{21}$$

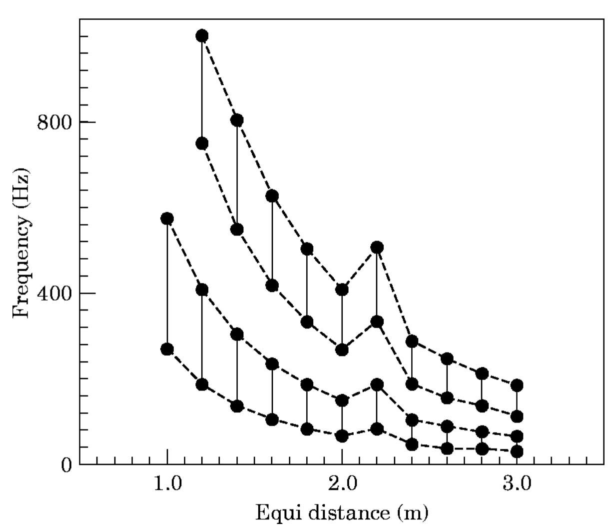

Figure 4. Effects of the equidistance of periodic system in free wave propagation: ●–● wave propagation bands.

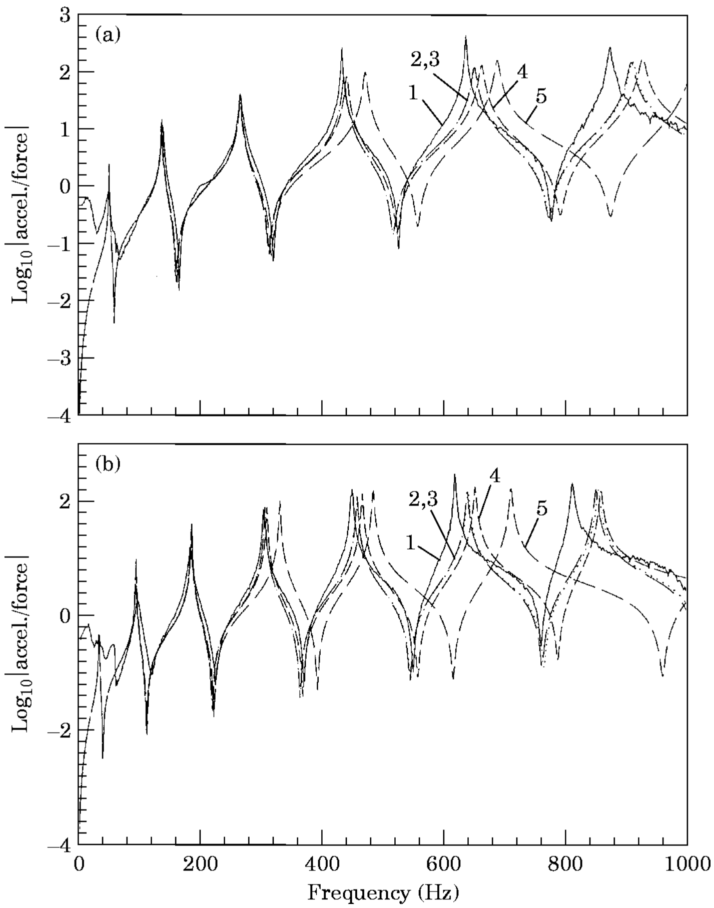

Figure 5. Frequency response functions of the piping system at driving point: (a) without pipe internal fluid; (b) with pipe internal fluid. Key: ——, 1 experimental; –·–, 2 wave approach; . . . . , 3 FEM (EL = 16); – – –, 4 FEM (EL = 10); — — —, 5 FEM (EL = 6).

where $\mathbf{C}(\omega)$ is a complex constant of a function of frequency. From equation (21), let $\mathbf{Z}_R = \mathbf{Z}_{i+1}$ and $\mathbf{Z}_L = \mathbf{Z}_i$, so that the complex eigenvalue problem for $\mathbf{C}(\omega)$ can be obtained as

$$\{[\mathbf{T}(\omega)] - \mathbf{C}(\omega)\}\mathbf{Z}_i = 0. \tag{22}$$

When solving the complex eigenvalue problem of equation (22), the number of eigenvalues obtained is found to be equal to the degrees of freedom of the periodic cell. These eigenvalues are generally complex and when fluid velocity in the pipe is zero, they occur in $\mathbf{C}(\omega)$ and $1/\mathbf{C}(\omega)$ pairs. When the fluid velocity exists in the pipe, such pairs do not exist. The equivalent propagation constants, $\mu = \mu_R + j\mu_I$, can be obtained from these complex eigenvalues by the simple equation;

$$\mathbf{C}_i(\omega) = \exp(\mu_{Ri} + j\mu_{Ii}), \qquad i = 1, 2, \ldots, n \tag{23}$$

where n is a number of the degrees of freedom in a single periodic cell. Equation (23) can be simply solved by a logarithmic exponential procedure.

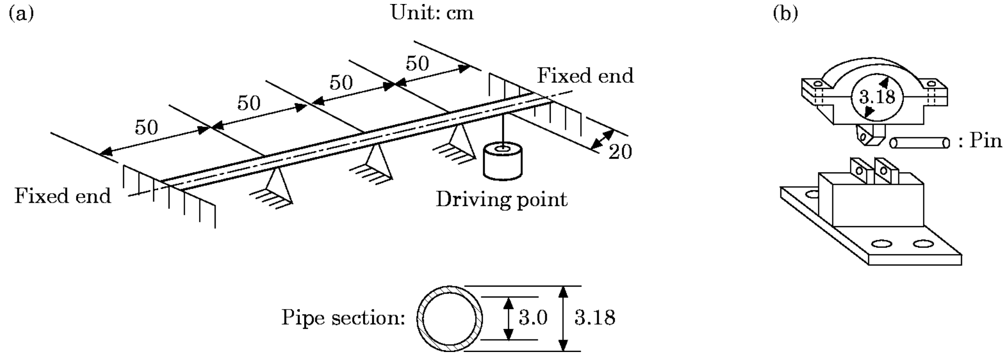

Figure 6. Layout of the periodically supported test piping system used in experiments: (a) lay dimensions; (b) configurations of the hinged support.

3. FORMULATIONS FOR THE FREQUENCY RESPONSES OF FINITE PERIODIC SYSTEMS

Combining the dynamic stiffness matrix of each pipe element and pipe support element, the whole piping system matrix can be constructed using the global assembly techniques used in finite element methods. To assemble the dynamic stiffness matrix of each element in the local co-ordinate system into the global co-ordinate system, one may introduce the

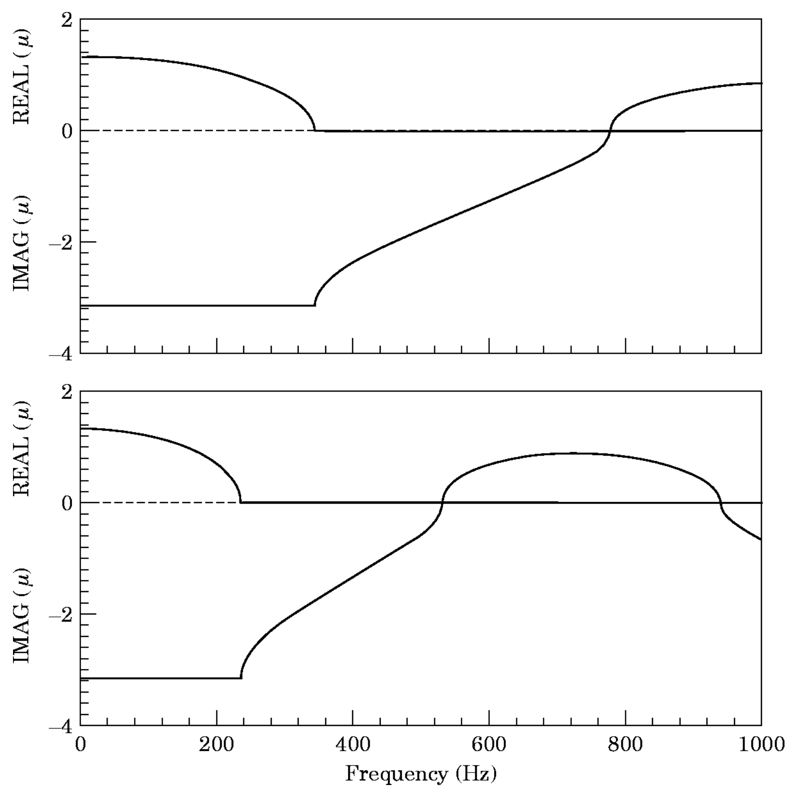

Figure 7. Simulation results of the free wave propagation characteristics in the periodically supported test piping system: (a) without pipe internal fluid; (b) with pipe internal fluid.

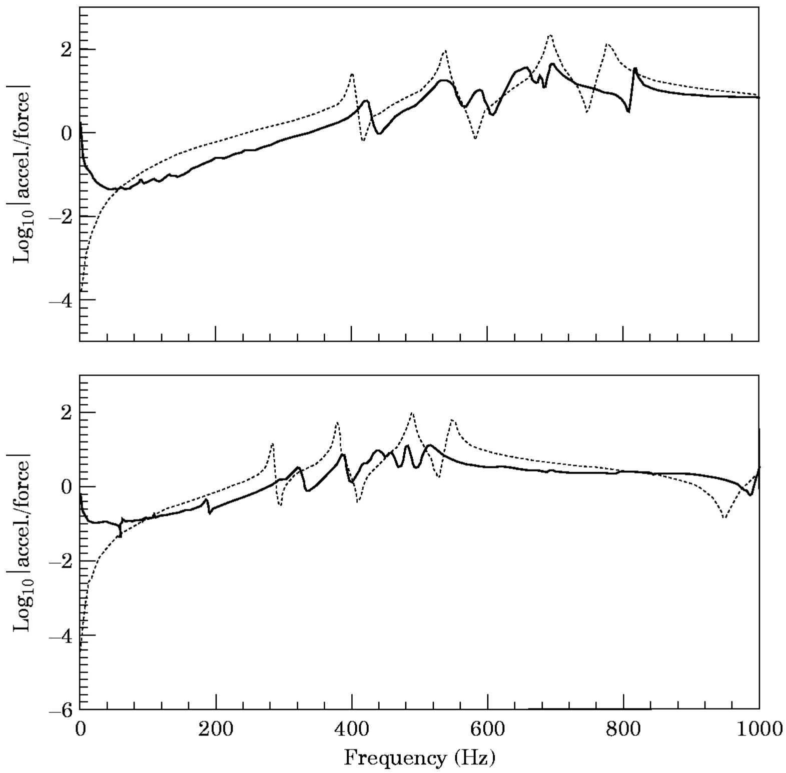

Figure 8. Frequency response functions of the driving point of the periodically supported test piping system: (a) without pipe internal fluid; (b) with pipe internal fluid. Key: ——, experimental; . . . . . , wave approach.

co-ordinate transformation matrix [**Tc**], so that the dynamic stiffness matrix can be transformed into the global co-ordinate system as

$$[\mathbf{Tc}]^{\mathrm{T}}\mathbf{F}_e = [\mathbf{Tc}]^{\mathrm{T}}[\mathbf{D}]_e[\mathbf{Tc}]\mathbf{W}e, \tag{24}$$

or, compactly

$$\mathbf{F}_g = [\mathbf{D}]_{eg}\mathbf{W}_g, \tag{25}$$

where the subscript g represents the global co-ordinate system and eg represents the element matrix in the global co-ordinate system. Therefore, the global assembled dynamic stiffness matrix of the piping system can be written compactly as

$$\mathbf{F}_g = [\mathbf{D}]_g\mathbf{W}_g. \tag{26}$$

The state vectors in equation (26) must include all the points such as wave reflection points, disturbance points, material property change points, driving points, termination points, geometric discontinuity points, and response points. When applying the boundary conditions in equation (27), all rows and columns in the dynamic stiffness matrix which are constrained in displacements can be eliminated. The system equation, after considering boundary conditions, may be written as

$$\mathbf{F}_g = [\mathbf{D}]_{b.c.}\mathbf{W}_g. \tag{27}$$

After obtaining the displacement solutions from equation (27), the reaction forces in the displacement constrained boundary nodes can be calculated from equation (26). From the displacement solutions, the wave coefficient vectors for each pipe element can be calculated from equation (12). Therefore, the responses at any locations of the piping system can be obtained from equation (4).

4. EXAMPLES

4.1. CHARACTERISTICS OF THE FREE WAVE PROPAGATION

Sensitivity analyses are done to investigate the characteristics of the free wave propagation of a periodically supported piping system due to the pipe's internal fluid velocity, support stiffness, and equi-distance. These sensitivity analyses can give important information to determine the wave stop bands, where the excitation is not well propagated. The data used in these analyses are: $D_0 = 0{\cdot}1$ m, $D_i = 0{\cdot}09$ m, $E = 2{\cdot}08 \times 10^{11}$ N/m^2, $m_p = 8{\cdot}0 \times 10^3$ kg/m^3, $m_f = 1{\cdot}0 \times 10^3$ kg/m^3, and $\eta = 0$. Figure 2 shows the pipe's internal fluid velocity effects on free wave propagation in a periodically supported piping system where the support stiffness, K_0, is $1{\cdot}0 \times 10^{10}$ N/m and the equidistance, L_p is 2 m. In this figure, one can see that the pipe's internal fluid velocity affects both the positive-going and the negative-going waves. Therefore, the wave speeds differ in two directions. As fluid velocity increases, the speed difference grows, and the wave propagation bands move slightly toward the lower frequency region. These effects of the pipe's internal flow are

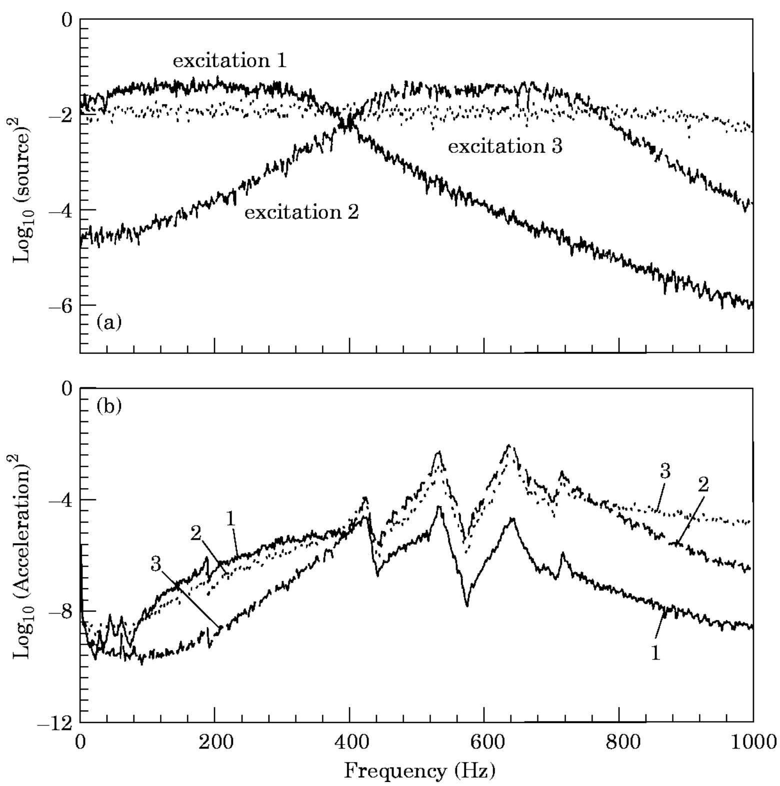

Figure 9. Frequency response functions of the driving point for the band-limited random excitations, without pipe internal fluid: (a) power spectrum of excitation source signal; (b) acceleration power spectrum (1, by excitation 1; 2, excitation 2; 3, excitation 3).

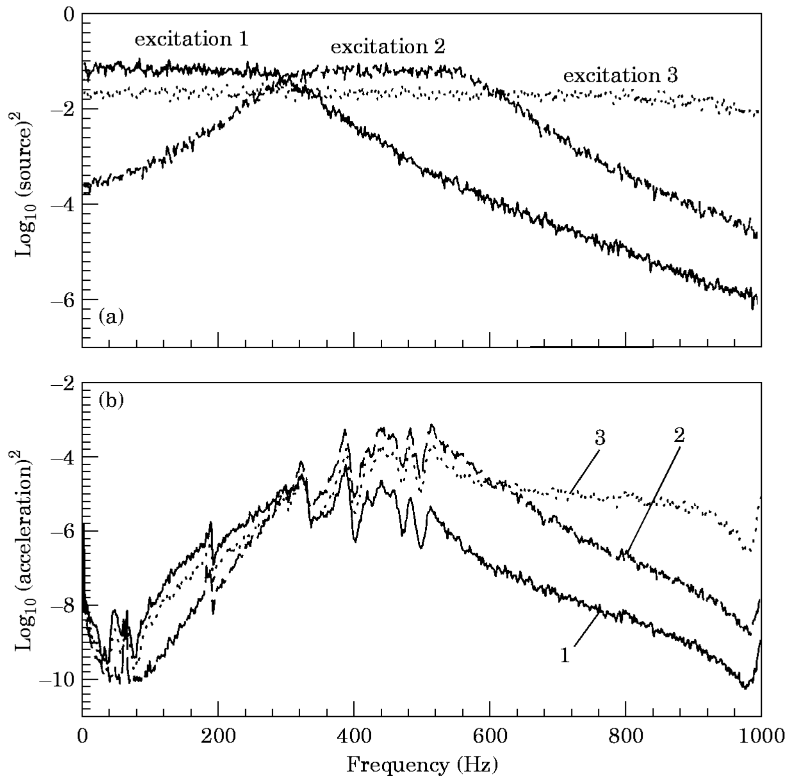

Figure 10. Frequency response functions of the driving point for the band-limited random excitations, with pipe internal fluid: (a) power spectrum of excitation source signal; (b) acceleration power spectrum. Key as Figure 9.

similar to those of its internal loss factors, but the internal loss factors are affected by flow velocity only near the bounding frequencies in wave propagation bands. The effects of internal loss factors are investigated by X. M. Zhang and W. H. Zhang [11].

Figure 3 plots the effect of support stiffness. One can see that the support stiffness affects the wave propagation bands considerably. As the support stiffness decreases, the wave propagation bands move towards the lower frequency region. When the stiffness, K_0, is less than $1{\cdot}0 \times 10^4$ N/m, the periodic pipe supports no longer show wave stop bands.

The equidistance effect is shown in Figure 4. In this case the support stiffness, K_0, is fixed at $1{\cdot}0 \times 10^9$ N/m. Figure 4 shows that as the equidistance increases, the wave propagation bands move towards the lower frequency region. The effects of equidistance show wide variation with the wave propagation bands. Therefore, equidistance can be one of the most important design variables in the periodic support design of piping systems.

4.2. RESPONSE TESTS OF PERIODICALLY SUPPORTED PIPING SYSTEM

In order to verify the formulations of the wave approach, the frequency response results are compared with those of experiments and those from the finite element formulations. The piping system used has a 2 m long-uniform straight pipe having a fixed–fixed end boundary condition. The data used in the analyses and tests are: $D_0 = 0{\cdot}0318$ m, $D_i = 0{\cdot}03$ m, $E = 2{\cdot}08 \times 10^{11}$ N/m^2, $m_p = 8{\cdot}0 \times 10^3$ kg/m^3, $m_f = 1{\cdot}0 \times 10^3$ kg/m^3, $\eta = 0{\cdot}005$, and $V = 0{\cdot}0$ m/s. Figure 5 shows frequency response functions with and without pipe internal fluid. An impact hammer and accelerometer are used to measure frequency response functions. Figure 5 shows that in low frequency regions, all three plots (wave approach, finite element approach [12] and test results) are close to one another. But in high frequency regions, the test results differ slightly from those obtained by simulation. This is due to the uncertainties of the fixed–fixed boundary condition, the uniformity of straightness and the material properties of the pipe. However, it can be observed that as the number of finite elements increases, the results determined by finite element analyses are similar to those obtained by the wave approach. This means that the wave approach gives an exact solution.

To verify the characteristics of the periodically supported piping system, the frequency responses are measured for the test system described above. Figure 6 shows the periodically supported test piping system used has three hinged supports. The simulation results from the free wave propagation characteristics of the system with and without pipe internal fluid are compared in Figure 7. Figure 7 shows the first wave stop band is in the frequency range of 0·0–344·0 Hz, and the first wave propagation band is from 345–778 Hz when there is no pipe internal fluid. But when fluid is filled inside the pipe, the frequency bands are shifted to lower frequencies, as expected. Figure 8 plots the driving frequency response function, showing resonance peaks only in the wave propagation bands. It can also be observed that all the responses within the wave stop band are significantly reduced

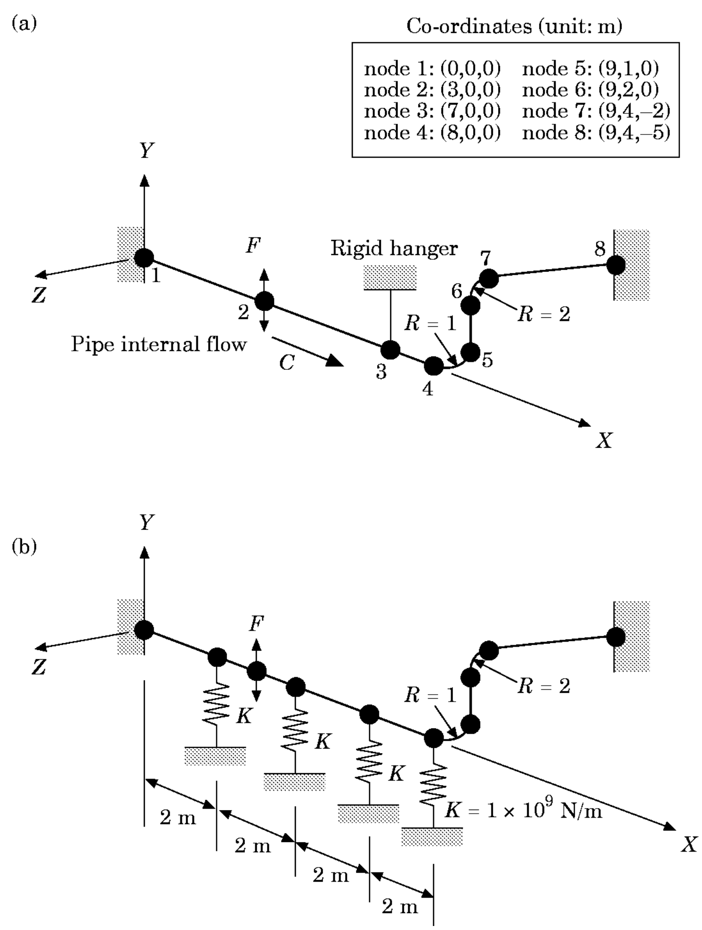

Figure 11. Layout of three dimensional piping system: (a) non-periodic support system; (b) periodic support system. ●, nodal locations.

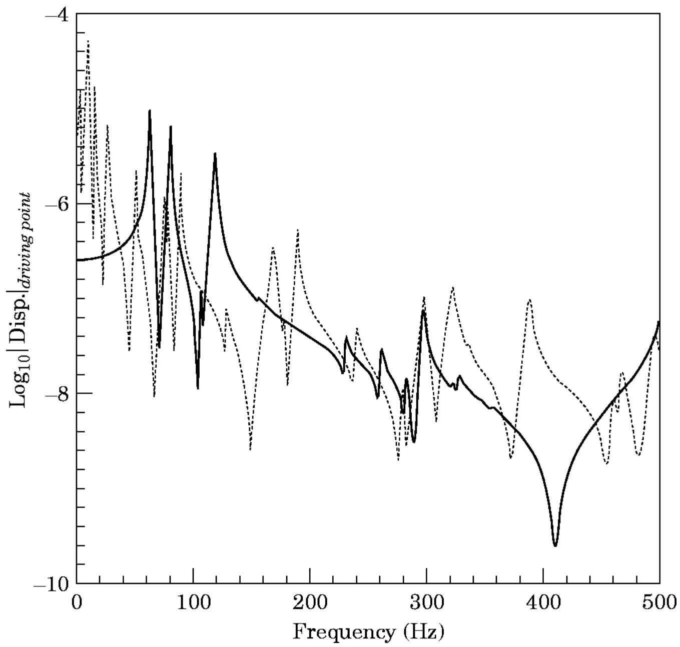

Figure 12. Comparison of frequency responses: ——, periodic; . . . . , non-periodic.

compared with those calculated from a piping system without supports, which are plotted in Figure 5. From these results it can be said that if the excitation frequencies are within wave stop bands, the responses can be dramatically suppressed using periodically placed supports. The periodic supports can be regarded as mechanical filters. Figures 9 and 10 show the driving point frequency responses for various band-limited random excitations.

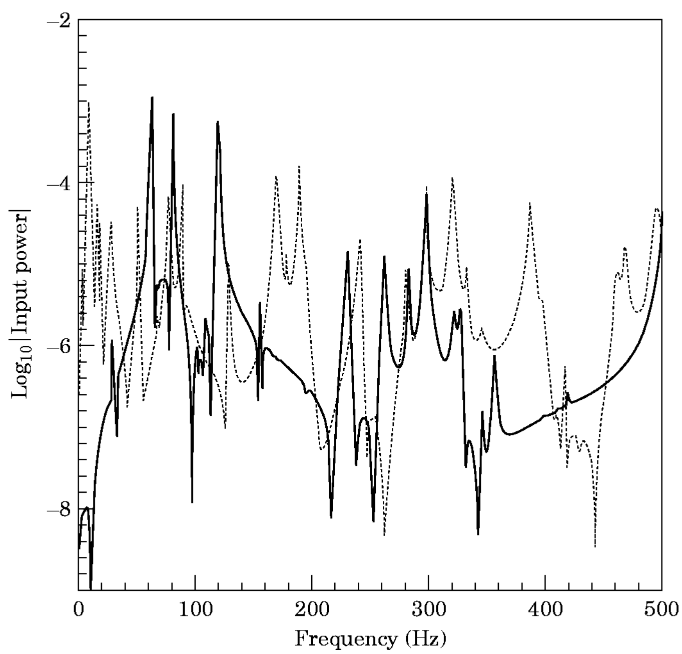

Figure 13. Comparison of input power flow: Key as Figure 12.

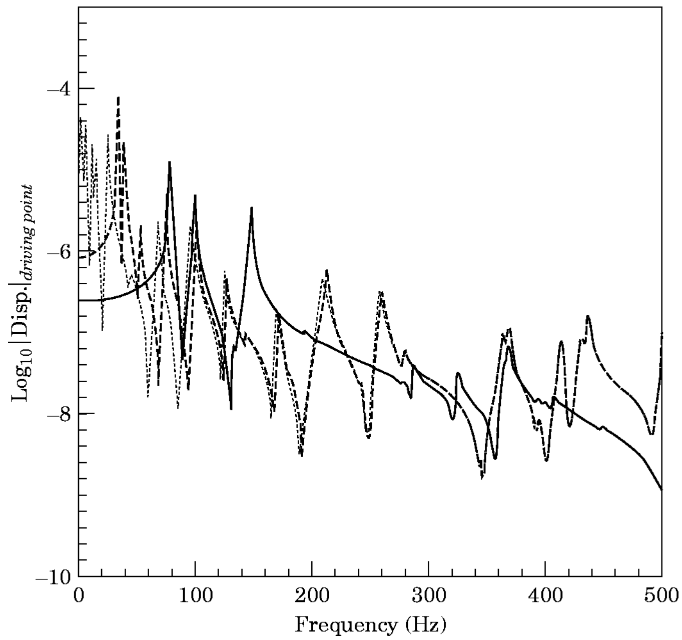

Figure 14. Effects of support stiffness on frequency responses in case of no pipe internal fluid: K values (N/m); . . . . , 1×10^3; – – –, 1×10^6; ——, 1×10^9.

When the dominant frequency contents of the excitation are within the wave stop band, as shown in Figures 9(a) and 10(a), the responses can be significantly reduced and no resonance peaks are observed. This means that all the modes in the wave stop band can be eliminated by proper design of periodic supports.

4.3. THREE DIMENSIONAL PIPING SYSTEM PARTIALLY SUPPORTED BY PERIODIC SPRINGS

The characteristics of the periodic system as a mechanical filter are applied to the actual three dimensional piping system conveying fluid. The main purpose of this example is to substantiate the usefulness of the periodic support design in reducing the vibration responses of an actual piping system. The non-periodic support system is designed to be supported by a general rigid hanger at 7 m from the original co-ordinate as shown in Figure 11(a). Figure 11(b) shows the partially supported piping system by periodic springs. The data used in the analyses are: $D_0 = 0{\cdot}1$ m, $D_i = 0{\cdot}09$ m, $E = 2{\cdot}08 \times 10^{11}$ N/m^2, $m_p = 8{\cdot}0 \times 10^3$ kg/m^3, $m_f = 1{\cdot}0 \times 10^3$ kg/m^3, $\eta = 0{\cdot}0$, and $V = 50$ m/s.

For the periodically supported system, the data used for analyses are the support stiffness $K_0 = 1 \times 10^9$ N/m, the equidistance of periodic supports $L_p = 2$ m, and the number of periodic cell $N_p = 4$. The first free wave stop band for this periodic system is from 0–60 Hz, as shown in Figure 4. Therefore, when the dominant excitation frequency contents are within this wave stop band, the vibration responses can be significantly reduced.

Figure 12 shows the comparison of the frequency responses at driving point between the non-periodic system and the periodic system. From these results, one sees that the resonance responses can be fully eliminated in the first wave stop band from 0–60 Hz by using the characteristics of a periodic system as a mechanical filter.

Figure 13 shows the comparison of the total active input power flow between the non-periodic system and the periodic system, which is defined as:

$$0{\cdot}5\mathrm{Re}\left(\sum_{m=1}^{6} \mathrm{j}\omega W_m^{\mathrm{H}} F_m\right),$$

where m is the degree of freedom at each nodal point, W is the displacement, F is a restoring force, and the superscript H denotes the complex conjugate transpose. From these results, one can see that there are no input power resonance peaks in the first wave stop band for a periodically supported system. Therefore, when the input source has a dominant energy in the wave stop frequency bands, one can easily filter out input energy using the periodic support design.

To substantiate the effects of periodic support stiffness as shown in Figure 3, more parametric analyses are carried out for an actual piping system as shown in Figure 11(b). Figure 14 shows the frequency responses at driving point with changes of support stiffness in the case of no pipe internal fluid. From these results, it can be seen that the resonance frequency bands are in good agreement with the free wave propagation bands, as shown in Figure 3. When periodic support stiffness is lower than about $1{\cdot}0 \times 10^6$ N/m, the periodic system has no wave stop bands.

5. CONCLUSIONS

This paper studied the applicability of the periodic support characteristics to vibration reduction of a piping system by using the formulations of the wave approach. The support rigidity and its equidistance significantly affect the characteristics of free wave propagation of a periodically suported piping system. The results from both analysis and test show that a proper design of periodically placed supports can reduce piping system vibration.

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APPENDIX A: NOTATION

A_f	cross-sectional area of fluid
$\mathbf{A}_n$	wave coefficient vectors
$C(\omega)$	complex constants
D	damping value
$[\mathbf{D}_1]e$, $[\mathbf{D}_2]e$	displacement and force coefficient matrix of a single pipe element
$[\mathbf{D}]_g$	global assembled dynamic stiffness matrix
D_i, D_0	inner and outer diameter of pipe
E	elastic Young's modulus
$\mathbf{F}_e$, $\mathbf{F}_g$	element force vectors in local and global co-ordinate systems
$\mathbf{F}_L$, $\mathbf{F}_R$	force vectors of left and right end side
I	moment of inertia of pipe cross-section
j	complex value, $\sqrt{-1}$
k	general wave numbers of flexural vibration
K	spring constant
$\mathbf{K}_1$, $\mathbf{K}_2$, $\mathbf{K}_3$, $\mathbf{K}_4$	sub-matrix of dynamic stiffness matrix
L_p	equidistance between periodic supports
m_p, m_f	mass of pipe and fluid per unit length
M	concentrated mass
M_b	bending moment
p	internal pressure in pipe
S	shear force
$[\mathbf{Tc}]$	co-ordinate transformation matrix
V	velocity of pipe internal fluid
$\mathbf{W}_e$, $\mathbf{W}_g$	displacement vectors in local and global co-ordinates systems
$\mathbf{Z}_L$, $\mathbf{Z}_R$	state vectors of left and right end side of pipe element
η	internal loss factor of pipe
ρ_p	polar moment of inertia of pipe per unit length
ω	angular frequency