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VIBRATION AND STABILITY OF ROTATING PLATES WITH ELASTIC EDGE SUPPORTS

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1. INTRODUCTION

In typical rotating plate vibration texts and handbooks it is assumed that the plate considered is annular with the free outer edge. However, there are other configurations of this problem referring to the support. In Figure 1 an elastic circular plate the edge of which is welded to a rigid cylinder rotating at constant angular speed is shown. The plate considered can be the lid or the base of a rotating compartment. The elasticity of the welded seam, which will be modelled by linear and torsion springs uniformly distributed around the edge of the plate, will be taken into account.

The present work is concerned with the effect of elastic edge supports on axisymmetric vibrations of rotating circular plates. More generalized research into vibrations of rotating circular plates would include non-axisymmetric modes as well.

2. PROBLEM FORMULATION

The radius and thickness of the plate are R and h, and E and ν are Young's modulus and the Poisson ratio. The material of the plate is assumed to be homogeneous and isotropic, with mass density ρ. The radial and tangential stresses in the plate due to rotation, σ_r and σ_c respectively, are given by [1]

$$\frac{\mathrm{d}\sigma_r}{\mathrm{d}r} + \frac{\sigma_r - \sigma_c}{r} + \rho r \omega^2 = 0. \tag{1}$$

The stress–strain relations in the polar co-ordinate system, from Hooke's law, are

$$\varepsilon_r = \frac{\sigma_r - \nu\sigma_c}{E}, \qquad \varepsilon_c = \frac{\sigma_c - \nu\sigma_r}{E}, \tag{2, 3}$$

which, along with the strain–displacement relations for ε_r and ε_c, yield

$$\frac{\mathrm{d}\sigma_c}{\mathrm{d}r} - \nu\frac{\mathrm{d}\sigma_r}{\mathrm{d}r} = \frac{1+\nu}{r}(\sigma_r - \sigma_c). \tag{4}$$

The governing equations for the axisymmetric deflection of a circular rotating plate in terms of w and σ_r become

$$D\nabla^4 w + \rho h \frac{\partial^2 w}{\partial t^2} = \frac{h}{r}\frac{\partial}{\partial r}\left(r\sigma_r \frac{\partial w}{\partial r}\right), \tag{5}$$

where w is the transverse deflection of the plate and $D = Eh^3/12(1-\nu^2)$.

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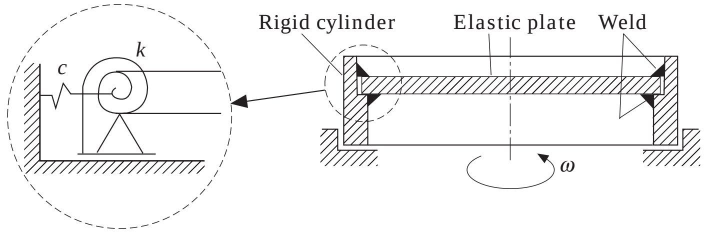

Figure 1. A circular plate supported elastically around its edge.

For a plate with an elastically restrained outer edge, with rotational and in-plane stiffnesses k and c, the boundary conditions are

$$r = R;\ w = 0, \qquad k\frac{\partial w}{\partial r} + D\left(\frac{\partial^2 w}{\partial r^2} + \frac{v}{r}\frac{\partial w}{\partial r}\right) = 0, \qquad cu + h\sigma_r = 0, \tag{6a–c}$$

where u is the radial displacement.

3. METHOD OF SOLUTION

It is convenient at this stage to introduce some dimensionless parameters:

$$\lambda = \frac{\rho\omega^2 R^4}{Eh^2}, \qquad y = \frac{w}{h}, \qquad x = \frac{r}{R}, \qquad \tau = t\sqrt{\frac{D}{h\rho R^4}}, \qquad K = \frac{kR}{D}, \qquad C = \frac{cR}{E}. \tag{7}$$

After solving equations (1) and (4) simultaneously, and applying the boundary condition (6c) one obtains

$$\sigma_r = \rho R^2\omega^2[A - B(r/R)^2], \tag{8}$$

where

$$A = \frac{3 + v + C(1 - v^2)}{8[1 + C(1 - v)]}, \qquad B = \frac{3 + v}{8}. \tag{9, 10}$$

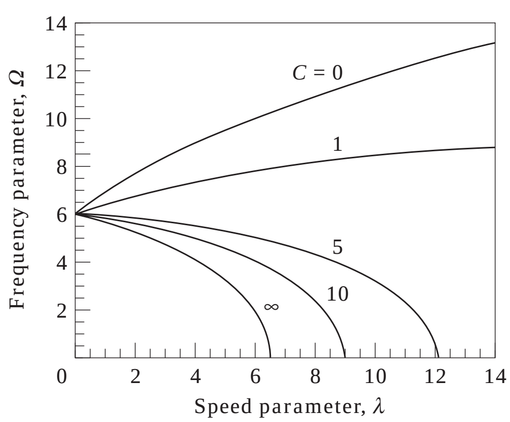

Figure 2. The frequency parameter Ω in the fundamental mode for $K = 1$ ($v = 0{\cdot}3$, $M = 5$).

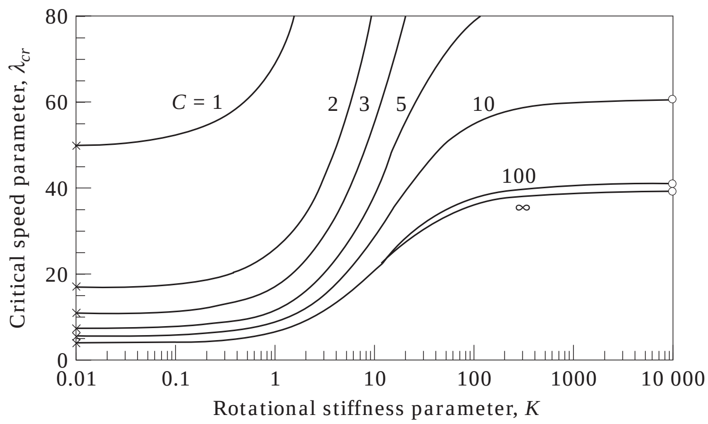

Figure 3. The critical speed parameter λ_{cr}. $\times$, $K = 0$; $\bigcirc$, $K = \infty$.

Then, substituting equation (8) into equation (5) and reducing the resulting equation to the dimensionless form yields

$$\nabla^4 y + \frac{\partial^2 y}{\partial \tau^2} - 12(1 - \nu^2)\frac{\lambda}{x}\frac{\partial}{\partial x}\left[x(A - Bx^2)\frac{\partial y}{\partial x}\right] = 0. \tag{11}$$

The dimensionless forms of boundary conditions (6a, b) are

$$y|_{x=1} = 0, \qquad (K + \nu)\frac{\partial y}{\partial x}\bigg|_{x=1} + \frac{\partial^2 y}{\partial x^2}\bigg|_{x=1} = 0. \tag{12a, b}$$

Solution of equation (11) is obtained by application of the Galerkin method. To this end, the solution is assumed in the form

$$y = \sum_{i=0}^{M} C_i R_i(x) \sin \Omega\tau, \tag{13}$$

where C_i are unknown constants, R_i are functions chosen to satisfy the boundary conditions of the plate (12a, b), $\Omega = \omega_N R^2\sqrt{\rho h/D}$ is a frequency parameter with ω_N being

TABLE 1

Critical speed parameter λ_{cr} ($\nu = 0{\cdot}3$, $M = 5$)

	K			
C	0	1	10	∞
0	∞	∞	∞	∞
0·5	213·883	265·136	636·950	2741·29
0	50·2593	69·8367	200·342	657·048
10	5·3708	8·9626	29·1151	60·0450
100	4·0374	6·7392	21·2646	40·6947
∞	3·9098	6·5231	20·4860	38·8533

the radian frequency of free vibration. In the present analysis it is assumed that R_i is a polynomial in x of the form

$$R_i = \left[1 - 2\,\frac{K + 4i + v + 3}{K + 4i + v + 5}\,x^2 + \frac{K + 4i + v + 1}{K + 4i + v + 5}\,x^4\right]x^{2i}. \tag{14}$$

Substituting equation (13) into equation (11) and applying the Galerkin procedure (multiplying both sides by $xR_j(x)$ and integrating from $x = 0$ to $x = 1$) leads to the equations

$$\sum_{i=0}^{M} C_i(A_{ij} - \Omega^2 B_{ij}) = 0, \qquad j = 0, 1, \ldots, M, \tag{15}$$

where

$$A_{ij} = \int_0^1 \left\{x\nabla^4 R_i - 12(1 - v^2)\,\frac{\mathrm{d}}{\mathrm{d}x}\left[x(A - Bx^2)\,\frac{\mathrm{d}R_i}{\mathrm{d}x}\right]\right\}R_j\,\mathrm{d}x, \qquad B_{ij} = \int_0^1 R_i R_j x\,\mathrm{d}x, \quad (16, 17)$$

which in turn leads to the frequency equation

$$\det\,[A_{ij} - \Omega^2 B_{ij}] = 0, \qquad i, j = 0, 1, \ldots, M. \tag{18}$$

The frequency equation (18) in Ω^2 can be solved by a numerical method. The lowest value of Ω provides the fundamental frequency.

4. RESULTS

The results are presented in Figures 2 and 3 and Table 1.

In Figure 2 is shown the effect of λ on the frequency parameter Ω for different values of in-plane stiffness parameter C for $K = 1$, when the plate is vibrating in the fundamental mode. In Figure 3 the critical speed parameter λ_{cr} depending on the rotational stiffness K is shown for different values of the in-plane stiffness parameter C (1, 2, 3, 5, 10, 100 and ∞). The critical speed λ_{cr} is defined as the rotational speed at which the natural frequency of the fundamental mode vanishes. In Table 1, λ_{cr} is presented for different values of rotational stiffness parameter K and in-plane stiffness parameter C.

5. CONCLUSIONS

In this note the following results have been obtained.

1. The fundamental frequency of vibration of the rotating plate, depending on the rotation speed (parameter λ) and the in-plane spring stiffness (parameter C) on the plate edge, have been determined.

2. The critical value of the parameter λ, depending on the stiffness of the springs, has been determined. The values of the stiffness of the torsional springs do not change the qualitative nature of the buckling problem. For $C = 0$, the rotating plate does not buckle.

REFERENCES

1. S. TIMOSHENKO and J. N. GOODIER 1951 *Theory of Elasticity*. New York: McGraw-Hill.
2. A. W. LEISSA 1969 *Vibration of Plates* (NASA SP 160). Washington, D.C.: U.S. Government Printing Office.