Journal of Sound and Vibration (1998) **210**(3), 375–389

ON TRANSMISSION OF SOUND IN CIRCULAR AND RECTANGULAR NARROW PIPES WITH SUPERIMPOSED MEAN FLOW

E. Dokumaci

Dokuz Eylul University, Department of Mechanical Engineering, Bornova, Izmir, Turkey

(*Received* 26 *February* 1997, *and in final form* 14 *October* 1997)

The problem of transmission of sound in a narrow pipe carrying a mean flow has been dealt with in recent papers by solving the convected acoustic equations simplified in the manner of the Zwikker and Kosten theory. A significant difference between these studies appertains to the form of the axial mean flow velocity profile assumed in the analysis. Results have been presented previously for uniform and parabolic profiles. This paper presents a comparative study of these for pipes having circular and rectangular cross-sections. The solution of the governing equations for these cases has been presented in previous papers, except for the case of a rectangular pipe carrying a uniform mean flow, which is presented in this paper for the first time. The results indicate that the assumption of a uniform mean profile closely predicts the results based on a parabolic profile. An area in which the theories considered in this paper find application is the acoustical modelling of the honeycomb structure of a monolithic catalytic converter. Previous acoustic models proposed for the honecomb structure assume that the honeycomb pores to be circular. In the present paper a new model is presented for honeycomb structures with rectangular pores.

1. INTRODUCTION

The widespread use of automobile catalytic converters in recent years has stimulated a growing interest in the propagation of sound in a narrow pipe carrying a mean flow. Three independent papers [1–3], which appeared about the same time, have dealt with the problem by using the convected viscothermal acoustic equations simplified in the manner of the Zwikker and Kosten theory [4]. A significant difference between these studies appertains to the form of the mean flow velocity profile assumed in the analysis: Peat [1] and Astley and Cummings [3] assumed a steady Poiseuille type mean flow with a parabolic velocity distribution over the pipe cross-section, whereas in reference [2] a uniform velocity profile, which makes the problem amenable to a simple exact analytical solution, was assumed. The mean flow in the honeycomb pipes of a catalytic converter is expected to be laminar and, therefore, the assumption of a parabolic mean velocity profile appears to be the more realistic one. However, it was also expected that the assumption of a uniform mean flow velocity profile, with the man flow velocity taken equal to the average velocity over the pipe cross-sectional area, should give results that are not very much different from those one would get by using a parabolic mean velocity profile. Now that some results are available for the case of a parabolic mean velocity profile [1, 3], these can be compared with the corresponding results of reference [2] for the case of a uniform mean flow velocity profile. The present paper, as a sequel to reference [2], will present first this comparative study for narrow pipes with circular and rectangular cross-sections. The interest in the

0022–460X/98/080375 + 15 $25.00/0/sv971336

latter arises from the fact that the honeycomb pores in most catalytic converters are square rather than circular. For the case of a parabolic mean flow velocity profile, results are available in references [1] and [3] for a circular pipe, and in reference [3] for a square pipe. These will be compared with the corresponding results for the case of a uniform mean flow velocity profile having the same average velocity over the pipe cross-sectional area as the parabolic profile. The solution of the governing equations for this case has been presented in reference [2] for a circular pipe and will be presented in this paper, for the first time, for a rectangular pipe. The solution for the case of a narrow rectangular pipe with no mean flow has been presented previously by Roh *et al*. [4].

A typical catalytic converter consists of an inlet expansion, an outlet contraction, a honeycomb structure which may be one or two pieces and connecting pipe sections. The acoustic model of the whole catalytic converter is obtained by combining the acoustic two-port models of its constituent elements in series. Acoustical two-port models of all these elements except the honeyomb structure are well known in the literature. For the honeycomb structure, Glav, Boden and Abom [5] proposed an acoustic two-port which assumes the effect of mean flow on plane wave propagation in the honecomb pipes to be the same as in a wide pipe. Another two-port model for the honeycomb structure has been developed in reference [2] by using the proposed extension of the Zwikker and Kosten theory for the effect of a uniform mean flow. This model takes into account the viscothermal effects at inlet and outlet discontinuities of the honeycomb structure. Jeong and Ih [6] have fomulated an impedance matrix, which is based on a numerical solution of the governing narrow duct equations, for the wave transfer across a stack of narrow pipes carrying a Poiseuille type mean flow. These models assume that the honeycomb pores are circular. The present paper will present an extension of the two-port model of reference [2] for a honeycomb structure with rectangular pores.

2. ACOUSTIC EQUATIONS WITH UNIFORM MEAN FLOW

The basic equations employed in references [1–3] for acoustic wave propagation in a homogeneous narrow pipe come from the simplification, in the manner of the Zwikker and Kosten theory, of the linearized forms of the fluid dynamic continuity momentum and energy equations for a perfect gas. A recent examination, and its extension to pipes with arbitrary cross-sections, of the Zwikker and Kosten theory has been presented by Stinson [7] for the case of zero mean flow velocity. When an axial mean flow exists in the pipe, the formulation given in reference [7] is modified by the appearance of convective terms. Upon assuming the mean flow velocity profile to be uniform across the pipe cross-sectional area and neglecting axial temperature and pressure gradients, the convective equations can be expressed, with $\exp(-\mathrm{i}\omega t)$ time dependence assumed for the fluctuating quantities, where ω is the radian frequency, t is the time and i denotes the unit imaginary number, as follows.

The momentum equation is

$$\rho_0 (-\mathrm{i}\omega + v_0\, \partial/\partial x) v_x = -\partial p/\partial x + \mu \nabla_S^2 v_x, \qquad p = p(x). \tag{1}$$

The continuity equation is

$$(-\mathrm{i}\omega + v_0\, \partial/\partial x)\rho + \rho_0\, \nabla \cdot \mathbf{v} = 0. \tag{2}$$

The energy equation is (a perfect gas being assumed)

$$\rho_0\, c_p (-\mathrm{i}\omega + v_0\, \partial/\partial x) T = (-\mathrm{i}\omega + v_0\, \partial/\partial x) p + \kappa \nabla_S^2 T. \tag{3}$$

The state equation is

$$\rho = (p/RT_0) - (\rho_0\, T/T_0). \tag{4}$$

Here, x denotes the pipe axis, v_x is the axial component of the acoustic particle velocity $\mathbf{v}$; p, T and ρ are the acoustic pressure, temperature and density respectively, μ is the shear viscosity coefficient, κ is the thermal conductivity of the fluid, R is the gas constant, c_p is the specific heat coefficient at constant pressure, T_0 and ρ_0 denote the ambient temperature and density, respectively, v_0 denotes the axial mean flow velocity and ∇_S^2 denotes the Laplacian on the pipe cross-section. The boundary conditions relevant to equations (1)–(3) are that $\mathbf{v}$ and T have finite values on the pipe centre line and vanish on the pipe periphery. The solution of the boundary value problem thus formed is considered in the following sections for pipes having circular and rectangular cross-sections.

3. PIPES WITH A CIRCULAR CROSS-SECTIONAL AREA

In the case of a pipe with a circular cross-section, one has

$$\nabla_S^2 = \partial^2/\partial r^2 + \partial/r\,\partial r, \qquad \nabla\cdot\mathbf{v} = \partial v_x/x + \partial v_r/\partial r + v_r/r, \tag{5}$$

where r denotes the radial co-ordinate and v_r is the radial component of the particle velocity. For this case, the solution of equations (1)–(3) for the acoustic pressure can be expressed as [2]

$$p(x) = p^+(0)\exp(\mathrm{i}K^+kx) + p^-(0)\exp(\mathrm{i}K^-kx), \tag{6}$$

where the propagation constants $K^\mp$ are given by the roots of

$$\gamma + (\gamma-1)\mathrm{J}(\sigma\beta a) + (K/(1-KM))^2\mathrm{I}(\beta a) = 0, \tag{7}$$

where

$$\mathrm{I}(\xi) = \mathrm{J}(\xi) = \mathrm{J}_2(\xi)/\mathrm{J}_0(\xi), \qquad (\beta a)^2 = \mathrm{i}(1-KM)s^2. \tag{8, 9}$$

Here, a is the pipe radius, $M = v_0/c_0$ is the mean flow Mach number, $c_0 = \sqrt{(\gamma R T_0)}$ is the speed of sound, γ is the ratio of specific heat coefficients, $k = \omega/c_0$ is the wavenumber, $s = a\sqrt{(\rho_0\,\omega/\mu)}$ is the shear wavenumber, $\sigma^2 = \mu c_p/\kappa$ is the Prandtl number and J_n denotes a Bessel function of order n. Propagation constants $K^\mp$, which can be determined by simple iteration from equation (7), are given in reference [2], as a function of the shear wavenumber, for mean flow Mach numbers up to 0·3.

Peat [1] has presented several approximate analytical solutions for the same problem by assuming a parabolic mean flow velocity profile. The above formulation of the problem can be compared with the most general case considered in reference [1], namely, the non-isentropic solution, which is derived by assuming that the radial component of the acoustic velocity is zero and yields, for the wavenumbers, a cubic equation that can be expressed in the form of equation (7), with

$$\mathrm{J}(\sigma\beta a) = [4 - 1/(1-MK)]^2/6f(\sigma\beta a), \qquad \mathrm{I}(\beta a) = (3/2)f(\beta a), \tag{10}$$

where

$$f(\xi) = (12/\xi^2) + 1/(1-MK) - 3. \tag{11}$$

Here, M is to be interpreted as the average Mach number over the pipe cross-section.

The same problem with a parabolic mean flow velocity profile has also been solved by Astley and Cummings [3] by using a finite element discretization over the pipe cross-section. This solution is in principle similar to the variational approach of Peat [1], but it is more accurate because a nine-noded isoparametric finite element mesh is used to approximate the variation of the axial component of acoustic particle velocity and acoustic temperature over the pipe cross-section, instead of a simple parabolic shape function assumed in reference [1].

Shown in Figures 1 and 2 are the attenuation and phase velocity ratio characteristics, of the waves propagating in a circular pipe with $2a = 1$ mm, $T_0 = 1000$ K, $\gamma = 1{\cdot}4$, $\sigma^2 = 0{\cdot}7$, $\mu = 4{\cdot}15 \times 10^{-5}$ Ns/m^2, $\rho_0 = 0{\cdot}35$ kg/m^3 and $M = 0{\cdot}2$, which is the pipe considered by Astley and Cummings [3] as being typical of the pipes in a honeycomb structure of a catalytic converter. The attenuation and the phase speed ratio of the pressure waves travelling in the $\mp x$ directions have been computed by using the expressions $A^{\mp} = \mp 8{\cdot}686 k K_I^{\mp}$ dB/m and $\phi^{\mp} = \mp 1/K_R^{\mp}$, respectively, were the subscripts R and I refer to the real and imaginary parts of a complex quantity, respectively. The finite element results of reference [3] are reproduced to the accuracy attainable from the published curves, and the corresponding results of Peat's non-isentropic formulation [1] were computed by the present author from equations (7) and (10). The present results, for which a uniform mean flow velocity profile is assumed, have been computed from equations (7) and (8), by using, as the mean flow Mach number, the average mean flow Mach numbers of references [1–3]. As can be seen from Figure 1, the attenuation which the present formulation yields for the waves travelling in the $+x$ direction is not distinguishable from the finite element results of reference [3], while Peat's formula yields increasingly inaccurate results for frequencies greater than 1000 Hz. For the phase velocity, however, Peat's formula gives better agreement with the finite element results, but the maximum deviation of the present results is less than 5%. For the waves travelling in the $-x$ direction

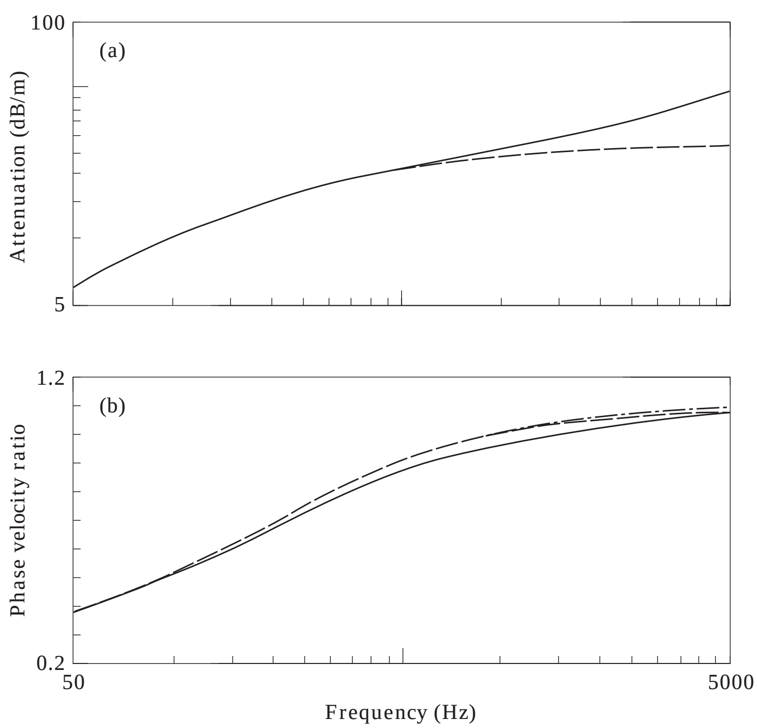

Figure 1. Transmission characteristics of the sound wave travelling with the mean flow ($M = 0{\cdot}2$) in a circular pipe of diameter 1 mm. (a) Attenuation, dB/m; (b) phase velocity ratio. ——, Uniform mean flow profile; – –, parabolic profile [1]; – - –, parabolic profile [3].

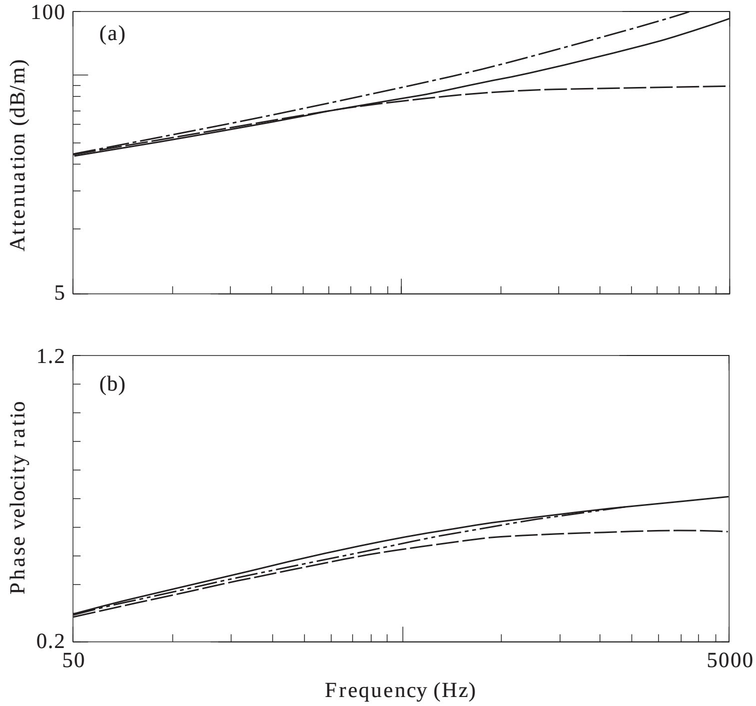

Figure 2. Transmission characteristics of the sound wave travelling against the mean flow ($M = 0{\cdot}2$) in a circular pipe of diameter 1 mm. (a) Attenuation, dB/m; (b) phase velocity ratio. ——, Uniform mean flow profile; – –, parabolic profile [1]; – - –, parabolic profile [3].

(Figure 2), the present formulation underestimates the finite element results for attenuation with an error less than 10%, and the phase velocity by less than 5%. Peat's formula yields increasingly inaccurate results for frequencies greater than 500 Hz, say, for both the attenuation and phase velocity ratio.

4. PIPES WITH A RECTANGULAR CROSS-SECTIONAL AREA

The pores in a honeycomb structure of an automobile catalytic converter are approximately square in cross-section. Therefore, a more realistic acoustic model of a catalytic converter requires the knowledge of propagation constants for a rectangular pipe. For a rectangular pipe, the Laplacian on the cross-section and the divergence of the particle velocity $\mathbf{v}$ are given by

$$\nabla_S^2 = \partial^2/\partial y^2 + \partial^2/\partial z^2, \qquad \nabla \cdot \mathbf{v} = \partial v_x/\partial x + \partial v_y/\partial y + \partial v_z/\partial z, \tag{12}$$

respectively, where y and z denote the transverse co-ordinates, with the pipe cross-sectional area lying in $0 \leqslant y \leqslant 2a$, $0 \leqslant z \leqslant 2b$, and v_y and v_z are the components of the particle velocity in the y and z directions. Solution of equations (1)–(3) can then be sought in the form

$$p = A\exp(\mathrm{i}Kkx), \qquad v_x = H(y, z)p, \quad T = F(y, z)p, \tag{13}$$

where A denotes a constant. Substituting these in equations (1)–(3) and using equation (4) to express the acoustic density in terms of the acoustic temperature and the particle velocity, the following equations result:

$$\partial^2 H/\partial y^2 + \partial^2 H/\partial z^2 + \beta^2 H = \mathrm{i}Kk/\mu, \tag{14}$$

$$\partial^2 F/\partial y^2 + \partial^2 F/\partial z^2 + \beta^2\sigma^2 = \mathrm{i}(1 - KM)\omega/\kappa, \tag{15}$$

$$\partial v_y/\partial y + \partial v_z/\partial z = (\mathrm{i}\omega(1/p_0 - F/T_0)(1 - MK) - \mathrm{i}KkH)p. \tag{16}$$

Here β is as defined by equation (9) with a representing now that side of the pipe cross-section with respect to which the shear wavenumber s is defined. The solution of equation (14) can be expressed in the form of a double Fourier series [4]:

$$H(y, z) = \sum_{m,n} a_{mn} \sin(m\pi y/2a)\sin(n\pi z/2b), \qquad m, n = 1, 3, 5, \ldots. \tag{17}$$

The coefficients a_{mn} can be determined by substituting equation (17) in equation (14) and averaging the resulting equation over the pipe cross-sectional area. This process gives

$$a_{mn} = \mathrm{i}(16Kk/\pi^2\mu)/mn\beta^2\alpha_{mn}(\beta a), \tag{18}$$

where

$$\alpha_{mn}(\xi) = 1 - \pi^2(m^2 + n^2a^2/b^2)/4\xi^2. \tag{19}$$

The solution of equation (15) can be expressed similarly as

$$F(y, z) = \sum_{m,n} b_{mn} \sin(m\pi y/2a)\sin(n\pi z/2b), \qquad m, n = 1, 3, 5, \ldots, \tag{20}$$

where

$$b_{mn} = \mathrm{i}(16\omega(1 - KM)/\pi^2\kappa)/mn\sigma^2\beta^2\alpha_{mn}(\sigma\beta a). \tag{21}$$

An eigen-equation for K can now be derived by substituting equations (17) and (21) into equation (16) and applying the boundary conditions on v_y and v_z after integration. The resulting eigen-equation comes out in the form of equation (7) with

$$\mathrm{I}(\xi) = \mathrm{J}(\xi) = -(64/\pi^4)\sum_{m,n} 1/m^2n^2\alpha_{mn}(\xi), \qquad m, n = 1, 3, 5, \ldots. \tag{22}$$

The propagation constants, K^+ and K^-, for the waves travelling in the $+x$ and $-x$ directions, respectively, can then be determined from equation (7) by simple iteration.

Astley and Cummings [3] have presented, assuming a parabolic mean flow velocity profile, the attenuation and phase velocity ratio characteristics of a square duct of side $2a = 1$ mm, the other salient properties of the pipe being the same as those of the circular duct considered in section 3. A nine-noded isoparametric finite element mesh was used to model the variation of the acoustic temperature and axial component of particle velocity over the pipe cross-section. The results of reference [3] are reproduced in Figure 3, to the accuracy obtainable from the published curves, together with the results of the present formulation. As can be seen, the deviation of the latter from the finite element results is less than 10% for the attenuation and less than 5% for the phase speed ratio in the frequency range considered.

In the case of zero mean flow velocity, the foregoing rectangular pipe formulation reduces to that by Roh *et al*. [4], who presented numerical results only for square pipes. The aspect ratio, a/b, may have considerable effect on the acoustic propogation constants. In Figure 4 are given the real and imaginary parts of the wavenumbers $K^{\mp}$ as functions of the shear wavenumber for aspect ratios of 0·1 to 10. For aspect ratios less than 0·1, the characteristics lie close to the $a/b = 0{\cdot}1$ curve. The presence of mean flow may modify these characteristics considerably. As an example which is also relevant to the present study, shown in Figures 5 and 6 are the propagation constants of a square pipe carrying a uniform mean flow of Mach numbers $M = 0{\cdot}1$, 0·2 and 0·3. For other aspect ratios, the non-zero mean flow characteristics display similar deviations from the corresponding zero mean flow characteristics.

5. ACOUSTIC MODELLING OF A HONEYCOMB STRUCTURE

For the honeycomb structure of a catalytic converter, an acoustic two-port embodying the transmitted waves in the honeycomb pipes as well as the sudden area discontinuities at the inlet and outlet of the honeycomb structure, has been presented in reference [2] for a honeycomb structure with circular pores and uniform mean flow. Now that the validity of approximating the mean flow velocity profile in the honeycomb pipes by a uniform profile has been established, it appears to be worthwhile to extend this model to the more realistic case of a honeycomb structure with rectangular pores.

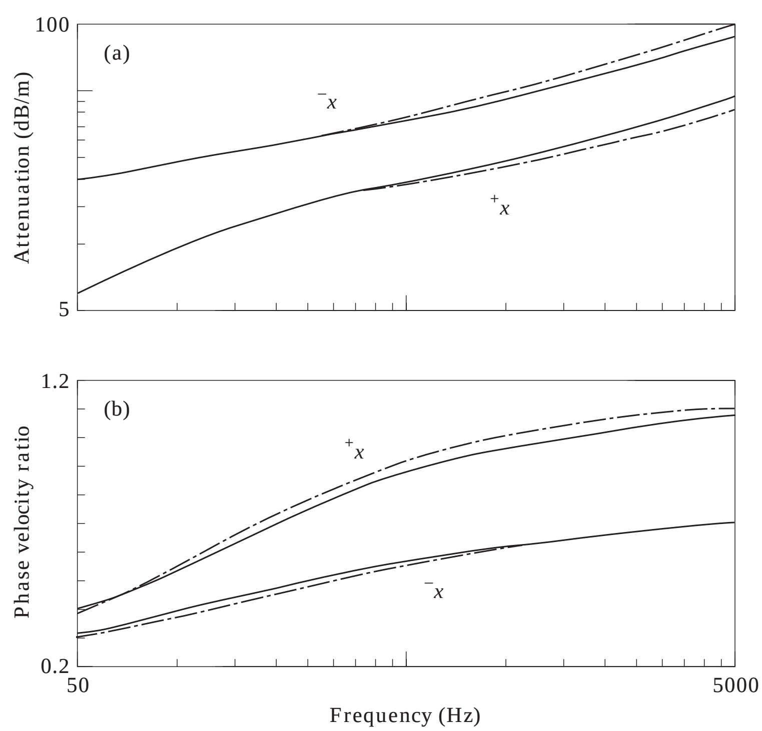

Figure 3. Transmission characteristics of the sound waves in a square pipe of side 1 mm with $M = 0{\cdot}2$. (a) Attenuation, dB/m; (b) phase velocity ratio. ——, Uniform mean flow profile; – - –, parabolic profile [3] ($+x$ and $-x$ refer to waves travelling with and against the mean flow, respectively).

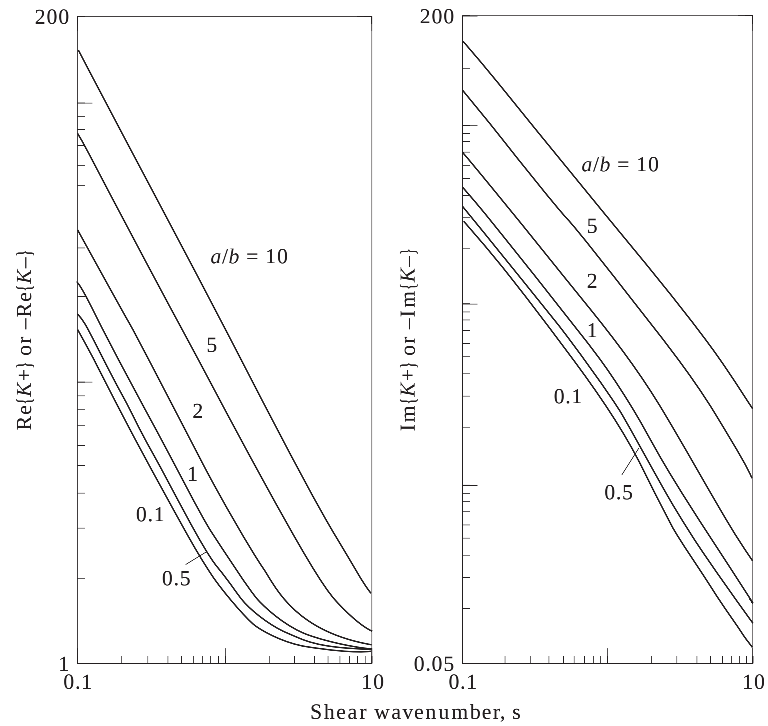

Figure 4. The effect of the aspect ratio on the real and imaginary parts of the propagation constant $K^{\mp}$ for a rectangular pipe with zero mean flow.

In the context of the theory described in the previous section, the wave transfer relationship across a rectangular pipe of length L is conveniently expressed in the form of a scattering matrix, namely,

$$\begin{bmatrix} p^+(L) \\ p^-(L) \end{bmatrix} = \begin{bmatrix} e^{iK^+kL} & 0 \\ 0 & e^{iK^-kL} \end{bmatrix} \begin{bmatrix} p^+(0) \\ p^-(0) \end{bmatrix}, \tag{23}$$

where the wavenumbers $K^{\mp}$ are computed from equations (7) and (22). The honeycomb pipes will be assumed to be all identical. Then, equation (23) determines the wave transfer in the honeycomb structure.

The inlet (outlet) of the honeycomb structure can be modelled as a sudden area contraction (expansion) and applying the mass and energy equations to a control volume enclosing the discontinuity. First, consider the contraction discontinuity at the inlet to the honeycomb structure. Upon assuming quasi-static conditions, the mass and energy equations can be expressed, respectively, as

$$\int_{S_1} (\rho_{0_1} v_{x_1} + \rho_1 v_{0_1})\, dS_1 = \int_{S_3} (\rho_{0_3} v_{x_3} + \rho_3 v_{0_3})\, dS_3, \tag{24}$$

$$\int_{S_1} \left(T_{0_1} s_1 + \frac{p_1}{\rho_{0_1}} + v_{0_1} v_{x_1} \right) \rho_{0_1} v_{0_1}\, dS_1 = \int_{S_3} \left(\frac{p_3}{\rho_{0_3}} + v_{0_3} v_{x_3} \right) \rho_{0_3} v_{0_3}\, dS_3, \tag{25}$$

where s denotes entropy, S cross-sectional area, and the subscripts 1 and 3 refer to the downstream and upstream sections of the discontinuity, respectively. Note that $S_1 = 4nab$, where n denotes the number of the honeycomb pipes. Conditions upstream of the discontinuity are assumed to be plane and isentropic. The axial component of the particle velocity, v_x, can be expressed as

$$\rho_0\, cv_x\,(x, y, z) = h^+(y, z)p^+(x) + h^-(y, z)p^-(x), \tag{26}$$

where, for isentropic propagation $h^{\mp}(y, z) = \mp 1$ and, from the second of equations (13), for viscothermal propagation,

$$h^{\pm}(y, z) = \frac{K^{\pm}}{1 - MK^{\pm}}\frac{16}{\pi^2}\sum_{m,n}\frac{\sin\,(m\pi y/2a)\,\sin\,(n\pi z/2b)}{mn\alpha_{mn}\,(\beta^{\pm}a)}, \qquad m, n = 1, 3, \ldots . \tag{27}$$

Here, $\beta^{\mp}$ denote the values of β evaluated for $K^{\mp}$. Similarly, by using the state equations for a perfect gas it can be shown that the density and entropy fluctuations can be expressed as

$$c_0^2\,\rho(x, y, z) = g^+(y, z)p^+(x) + g^-(y, z)p^-(x), \tag{28}$$

$$\rho_0\, T_0\, s(x, y, z) = e^+(y, z)p^+(x) + e^-(y, z)p^-(x), \tag{29}$$

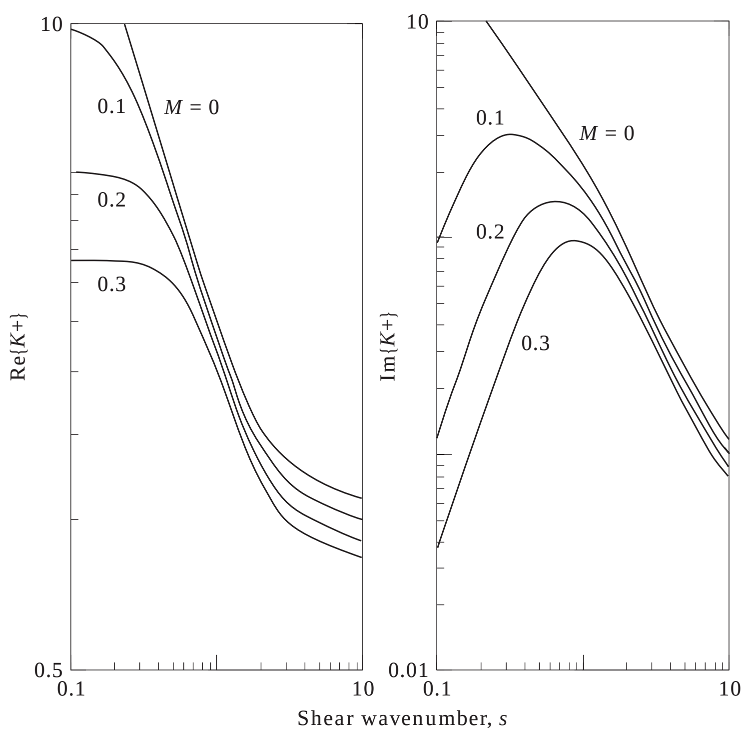

Figure 5. The effect of the mean flow Mach number on the propagation constant K^+ for a square pipe.

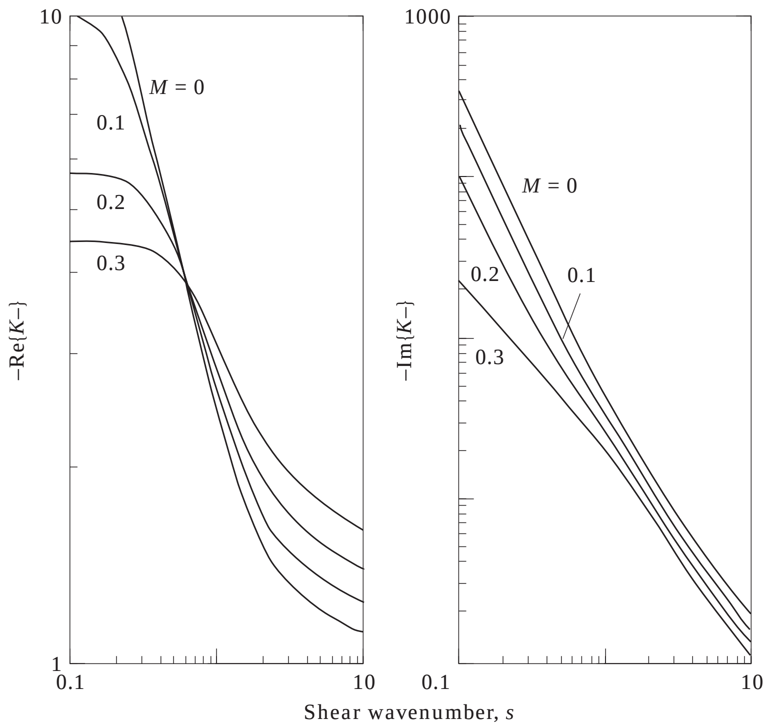

Figure 6. The effect of the mean flow Mach number on the propagation constant K^- for a square pipe.

where, for isentropic propagation $g^{\pm}(y, z) = 1$ and $e^{\pm}(y, z) = 0$, and, for viscothermal propagation,

$$g^{\pm}(y, z) = \gamma - (\gamma - 1)\frac{16}{\pi^2}\sum_{m,n}\frac{\sin(m\pi y/2a)\sin(n\pi z/2b)}{mn\alpha_{mn}(\sigma\beta^{\pm}a)}, \qquad m, n = 1, 3, \ldots, \tag{30}$$

$$e^{\pm}(y, z) = -1 + \frac{16}{\pi^2}\sum_{m,n}\frac{\sin(m\pi y/2a)\sin(n\pi z/2b)}{mn\alpha_{mn}(\sigma\beta^{\pm}a)}, \qquad m, n = 1, 3, \ldots. \tag{31}$$

Upon assuming plane sound waves upstream of the contraction discontinuity, and substituting equations (26), (28) and (29) for the downstream acoustic quantities, the mass and energy equations, equations (24) and (25), can be integrated to obtain, respectively, the following relationships between the upstream and downstream sound pressure components:

$$S_1[(h^+_{m_1} + M_1 g^+_{m_1})p^+_1 + (h^-_{m_1} + M_1 g^-_{m_1})p^-_1] = S_3[(1 + M_3)p^+_3 - (1 - M_3)p^-_3], \tag{32}$$

$$(1 + e^+_{m_1} + M_1 h^+_{m_1})p^+_1 + (1 + e^-_{m_1} + M_1 h^-_{m_1})p^-_1 = (1 + M_3)p^+_3 + (1 - M_3)p^-_3. \tag{33}$$

Here

$$h^{\pm}_m = -K^{\pm}\mathrm{J}(\beta^{\pm}a)/(1 - K^{\pm}M), \qquad g^{\pm}_m = 1 - (\gamma - 1)e^{\pm}_m,$$

$$e^{\pm}_m = -1 - \mathrm{J}(\sigma\beta^{\pm}a). \tag{34}$$

Subscript m is used here to indicate a cross-sectionally averaged value. In matrix notation, equations (32) and (33) yield the following scattering matrix for the wave transfer at the inlet of the honeycomb structure:

$$\begin{bmatrix} S(h_m^+ + Mg_m^+) & S(h_m^- + Mg_m^-) \\ 1 + e_m^+ + Mh_m^+ & 1 + e_m^- + Mh_m^- \end{bmatrix}_1 \begin{bmatrix} p^+ \\ p^- \end{bmatrix}_1 = \begin{bmatrix} S(1+M) & -S(1-M) \\ 1+M & 1-M \end{bmatrix}_3 \begin{bmatrix} p^+ \\ p^- \end{bmatrix}_3. \quad (35)$$

The wave transfer at the outlet of the honeycomb structure can be determined similarly. By again using the subscripts 1 and 3 to refer to the downstream and upstream sections of the discontinuity, respectively, and assuming plane wave propagation and isentropic conditions downstream, it can be shown that the wave transfer at the discontinuity is described by equations (32) and (33), or equation (35), but with the subscript 1 replaced by 3 and 3 replaced by 1 (the effect of jet formation at the outlet is neglected in the present study).

The acoustic two-port for the whole catalytic converter can now be obtained by combining the inlet expansion, outlet contraction and straight pipe sections with the honeycomb structure in series by using the usual transfer matrix scheme. This process is capable of modelling commercial monolithic catalytic converters. To demonstrate the effect of the pore geometry, however, it suffices to consider the simple catalyst considered in reference [2]. This consists of a 200 mm long honeycomb structure consisting of 2000 square cross-section pipes of side 1 mm, placed in a uniform pipe of 80 mm diameter, giving a frontal porosity of 40%. Temperature is assumed to be 600°C throughout, with $\gamma = 1{\cdot}4$, $R = 287$ J/kg K and $p_0 = 10^5$ Pa. A uniform mean flow of Mach number $M = 0{\cdot}025$ is assumed to exist in the 80 mm diameter pipe and the transmission loss of the honeycomb is calculated assuming an anechoic termination.

Presented in Figure 7 is the transmission loss characteristics of this simple catalyst as computed by using the theory presented in section 4. Also shown are the transmission loss characteristics which have been computed, for 1 mm^2 circular pores, by using the two-port formulations presented in references [2] and [5]. It is seen that, in this case, the use of

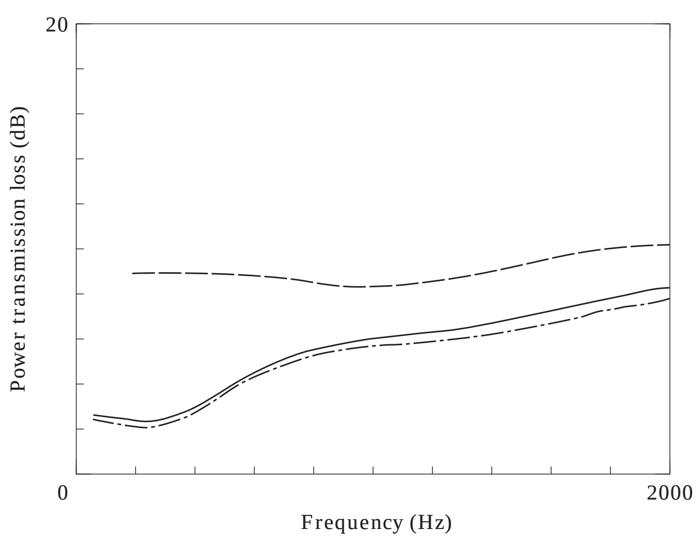

Figure 7. The transmission loss of the simple catalyst. — —, No overlapping acoustic boundary layers [5]; ——, present theory, square pores; – - –, present theory, circular pores.

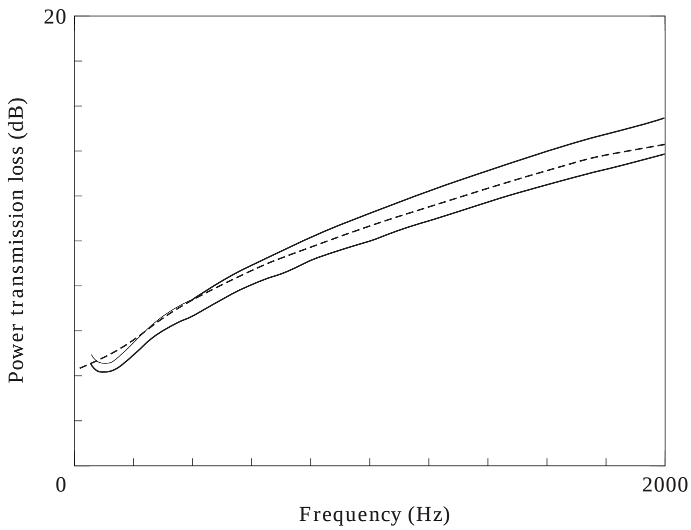

Figure 8. The transmission loss of the simple catalyst test in reference [6]. - - -, Reference [6], circular pores; ——, present theory, square pores (upper curve); ——, present theory, circular pores with an equivalent hydraulic radius (lower curve).

circular geometry with the present theory underestimates the transmission loss of the honeycomb structure with square pores with less than 0·5 dB deviation in the frequency range considered. The theory of reference [5], on the other hand, predicts a relatively higher transmission loss than the present theory for circular pipes. Unfortunately, the data plotted in Figures 4, 5 and 6 of reference [2] were not relevant to the case under consideration and this mistake went unnoticed in the manuscript. The curve given in Figure 7 of the present paper for the case of circular pores, is the correct form of the corresponding curve given in Figure 4 of reference [2] (the correct forms of Figures 5 and 6 of reference [2] are given in the Appendix as Figures A1 and A2, respectively).

A similar simple catalyst has been investigated recently by Jeong and Ih [6], who have shown that the measured transmission loss of the catalyst can be predicted reasonably closely by using the theory based on a parabolic mean flow velocity profile. The test catalyst consisted of a 610 mm long honeycomb structure with square pores of side 0·91 mm and of porosity 68%, placed in a uniform pipe of 80 mm diameter. The test was carried out at room temperature (17°C) and the average mean flow Mach number was 0·068. In Figure 8 is shown, to the accuracy attainable from the published characteristic, the predicted transmission loss of reference [6] for this simple catalyst model. A Runge–Kutta based numerical integration was used for solving the governing viscothermal equations, with the radial component of the particle velocity included in the continuity equation, for a circular pipe. The square pores of the test catalyst were modelled as circular ones by invoking the concept of hydraulic radius. Also shown in Figure 8 are the transmission loss characteristics computed for this simple catalyst by using the present theory for two cases of a honeycomb structure, namely, one with square pores of side 0·91 mm and one with circular pores of an equivalent hydraulic radius, 0·513 mm. It is seen that, while there is a deviation ranging from 0·5 dB to 3 dB in the frequency range considered between the square pipe and circular pipe models of the catalyst based on the present theory, the present circular pipe model underestimates the prediction of reference [6], which is based on a parabolic mean flow velocity profile, by only about 0·5 dB in the frequency range considered.

6. CONCLUSIONS

The data presented in this paper for sound transmission in a narrow pipe indicates that the assumption of a uniform mean flow velocity profile yields results that are not significantly far from those based on a parabolic mean velocity profile. Mean flow in a rectangular duct with a parabolic velocity profile does not satisfy the associated ambient energy and state equations and the actual mean flow velocity profiles which prevail in the honeycomb pores of a catalytic converter may in fact be different from a parabolic one; however, with the adoption of a uniform mean flow velocity profile, the sound transmission characteristics can still be modelled with reasonable accuracy by using the simple analytical solutions presented in this paper.

Strictly speaking, of the modes of propagation reported in references [1, 3], only the least attenuated ones have been considered in this paper. Peat [1] has reported one higher order mode, which is predicted from the cubic equation that results when equations (10) and (11) are substituted in equation (7), and Astley and Cummings [3] have reported as many higher order modes as there are number of degrees of freedom in the finite element model. These higher order modes were found to exist only when there is mean flow, propagate with the mean flow and attenuate much more rapidly than the least attenuated modes considered in the present study. In the numerical solution presented in reference [6], the higher order modes were also extracted by assuming a distribution for the axial particle velocity. In the present theory, all acoustic variables are determined analytically by the governing equations and the present author's attempts to extract such higher order modes from the present circular or rectangular pipe solutions have been abortive. However, if the variational procedure of reference [1] is applied to the present basic acoustic equations with uniform mean flow, equations (1)–(3), the resulting eigen-equation for the approximate wavenumbers becomes of the form of equation (7), with

$$I(\xi) = \mathrm{J}(\xi) = 1/4[(2/\xi^2) - 1/3]. \tag{23}$$

Equation (7) then yields a cubic equation for the wavenumber K, the two roots of which are approximations to the exact wavenumbers predicted by equations (7) and (8), and the third has the character of the higher order modes described in references [1, 3]. This indicates that, in the context of the present theory, the higher order modes are peculiar to the approximate discrete model of the system and are superfluous as far as the exact nature of the fundamental mode propagation is concerned. Similarly, Ih, Park and Kim [8], who have recently derived, by neglecting the radial component of the particle velocity, an exact analytical solution for a circular pipe with a parabolic mean velocity profile, have reported no higher order modes, although their basic acoustic modes with and against the mean flow agreed with those of reference [1].

Most authors who have dealt with the case of a parabolic mean flow velocity profile have neglected the radial component or, in the case of a rectangular cross-section pipe, the in-plane components, of the particle velocity in the continuity equation. In reference [3], the in-plane velocity component(s) was removed by integrating the continuity equation over the pipe cross-sectional area. The numerical solution presented in reference [6] shows that the effect of this term is small but not imperceivable. The present theory retains the in-plane velocity terms in the acoustic continuity equation. In fact, it is the inclusion of these terms that enables the problem to be solved as a well-posed boundary value problem. If the in-plane component of the acoustic velocity is assumed to be zero, then the acoustic continuity equation can be satisfied only in the its cross-sectionally averaged form. In this case, however, it may be of interest to note that cross-sectional averaging of the continuity equation happens to be mathematically equivalent to the application of the boundary conditions on the in-plane component of the particle velocity and, therefore, gives the same eigen-equation for the wavenumbers as the solution in which the in-plane component of the particle velocity is taken into account.

REFERENCES

1. K. S. Peat 1994 *Journal of Sound and Vibration* **175**, 475–489. A first approximation to the effects of mean flow on sound propagation through cylindrical capillary tubes.
2. E. Dokumaci 1995 *Journal of Sound and Vibration* **182**, 799–808. Sound transmission in narrow pipes with superimposed uniform mean flow and acoustic modelling of automobile catalytic converters.
3. R. J. Astley and A. Cummings 1995 *Journal of Sound and Vibration* **188**, 635–657. Wave propagation in catalytic converters: formulation of the problem and finite element solution scheme.
4. H. S. Roh, W. P. Arnott, J. M. Sabatier and R. Raspet 1991 *Journal of the Acoustical Society of America* **89**(6), 2617–2624. Measurement and calculation of acoustic propagation constants in arrays of small air-filled rectangular tubes.
5. R. Glav, H. Boden and M. Abom 1988 *Proceedings of Inter-Noise* **'88**, 1261–1266. An acoustic model for automobile catalytic converters.
6. K.-W. Jeong and J.-G. Ih 1996 *Journal of Sound and Vibration* **198**, 67–79. A numerical study on the propagation of sound through capillary tubes with mean flow.
7. M. Stinson 1991 *Journal of the Acoustical Society of America* **89**, 550–558. The propagation of plane sound waves in narrow and wide circular tubes, and generalization to uniform tubes of arbitrary cross-sectional shape.
8. J.-G. Ih, C.-M. Park and H.-J. Kim 1996 *Journal of Sound and Vibration* **190**, 163–175. A model for sound propagation in capillary ducts with mean flow.

APPENDIX: ERRATA

Figures A1 and A2 of this appendix are the correct forms of Figures 5 and 6 of reference [2], respectively. Figure A1 shows the effect of increasing the porosity of the honeycomb for zero mean flow, the porosity being increased by increasing the number of the circular honeycomb pipes while their cross-sectional area is kept constant at the value of 1 mm^2.

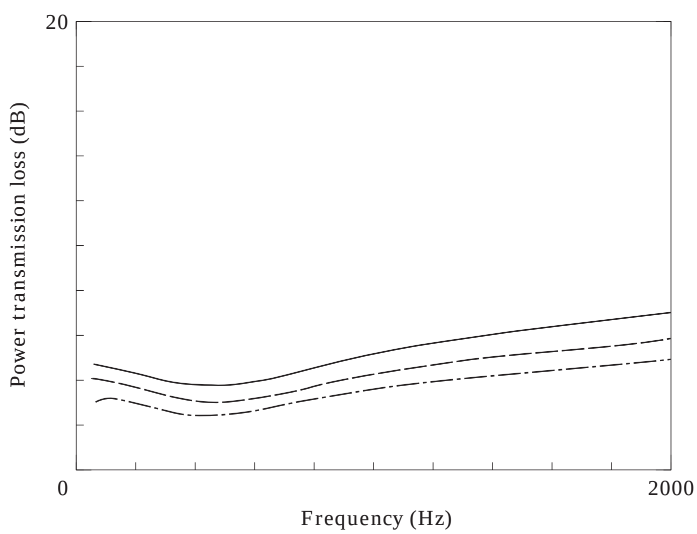

Figure A1. The effect of the honeycomb porosity on the transmission loss of the simple catalyst with circular pores (correction to Figure 5 of reference [2]). ——, 40% – – – 60%; – - –, 80%.

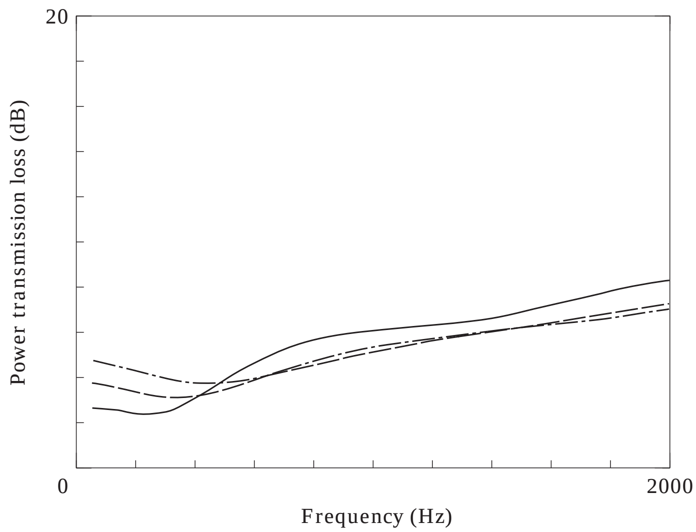

Figure A2. The effect of the mean flow Mach number on the transmission loss of the simple catalyst with circular pores (correction to Figure 6 of reference [2]). ——, $M = 0$; – – –, $M = 0{\cdot}1$; – - –, $M = 0{\cdot}2$.

These characteristics are modified slightly when flow is present. The effect of the mean flow on the transmission loss of the most porous (80%) honeycomb is shown in Figure A2.