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AN EMERGING TREND IN EXPERIMENTAL DYNAMICS: MERGING OF LASER-BASED THREE-DIMENSIONAL STRUCTURAL IMAGING AND MODAL ANALYSIS

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This paper describes the advances toward the implementation of a three-dimensional, six-degree-of-freedom structural dynamics measurement system. The result is a continuous description of the 3-D translational and rotational velocities on the surface of a vibrating prototype structure. This has been demonstrated on 2-D (flat plates) and 3-D structures such as compressor shells. The availability of this basic experimental measurement tool opens the door to many advances in experimental dynamics measurements. This paper will outline the basic approaches used to derive 3-D velocity or mobility response measurements from a vibrating structure. Next, applications of the technology will be discussed. Most important among these is the discussion of the development of a new experimentally based system modelling procedure, where the frequency response at *any* point and direction may be predicted without having measured that specific combination of forcing and response positions. This new modelling procedure will form the basis for the supplementation of experimental modal analysis as one knows it today. Other uses of this technology such as strain field display, stress field determination, acoustic radiation predictions from the structure, and power flow from sources to sinks in a structure.

1. INTRODUCTION

For more than three decades the engineering community has used an experimental technology to investigate the dynamic character of complex engineering structures. This technology is experimental modal analysis (EMA). In recent years the dynamics community has been trying to use the results of EMA to update finite element method (FEM) models. The prolem here is that finite element models can have up to 100 000 degrees-of-freedom (DOF) in the equations of motion. The analogue of this in EMA is usually between 32 and 240 measured DOF. In unusual cases the measured DOF may reach 450! Data acquisition and processing of 450 DOF may take up to three days with at least two experienced engineers in charge—very expensive. An additional problem is the fact that the EMA community has been using modal information in a maximum of three of the actual six structural DOF! That is, the EMA community, at best, measures the x, y, and z responses while neglecting all the rotational DOF, θ_x, θ_y and θ_z.

About five years ago the transducer industry developed adequate rotational accelerometers for use in experimental modal analysis. However, making six DOF measurements at each and every point is very burdensome timewise and is fraught with the increased probability of experimental faults or errors. Moreover, the assemblage of

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three translational/rotational combined accelerometers has many cross-coupling problems. The data, once taken, must be uncoupled by writing the kinematic equations that describe the rotational-to-translational motion as seen by the transducer in its measurement position. Thus, the measurement of 6 DOF has not become normal practice in EMA.

However, there is a need for a time-efficient, full-field, 6 DOF experimental method to characterize the dynamics of structures or structural components. Such a characterization would then be able to be used effectively in the updating of FEM models of the same structure. Moreover, if the experimental methodology were ideally formulated it would describe the modal 6-DOF shape of the structure at *exactly* the nodes of the underlying finite element model that is to be updated. No finite element condensation, interpolation, test/analysis model, or approximation would be necessary in the model updating procedures.

The first problem that has to be solved is the development of an easy and timely measurement procedure of a full 6-DOF set of response data from a dynamic structure.

Next, the need to measure many more experimental spatial DOF is evident. The disparity between a measurement of 450 spatial DOF and an analysis of 15 000–100 000 spatial DOF is too great to expect quality updating. For example, the poor modelling within a structure may be in a region of the structure that was not adequately documented by the experimental DOFs. Excessive deflection in the mis-modelled region of the structure would not be detected and would result in the updating algorithm having a much more difficult time changing the structural FEM model in a realistic fashion. The key words here are "in a realistic fashion". Of course, some updating routines would modify the FEM model so that the measured eigenvalues/vectors would coincide as closely as possible. However, the updated model may not correspond to the real structure and to the actual updating that should have been done to correct for the actual defect in the model. Use of the updated model to predict other modes than those used in the updating may produce poor results.

2. THREE-DIMENSIONAL LASER-BASED STRUCTURAL MEASUREMENTS (LSM)

Within the last six years a method of measurement called three-dimensional laser-based structural dynamic measurement [1–23] has been developed. Early work hinted that modes could be extracted with lasers [21, 22]. Next, Cafeo *et al.* [23] demonstrated the ability to extract angular information using lasers. In parallel with these efforts were a sequence of effort [1–20] which made 3-D laser-based structural dynamic measurements possible. This procedure uses a scanning laser Doppler vibrometer to measure thousands of structural responses at rates up to 60 points per second while preserving both magnitude and phase of response with respect to a reference forcing function which is necessary for any modal analysis operation. However, this structural measurement method was developed through obtaining a high spatial density measurement at a single excitation frequency. That is, a response shape of the structure driven from a specific excitation point and direction is obtained.

Consider the resonance vibration of a flat square thin (18 in × 18 in × $\frac{1}{8}$ in steel free–free) plate where the excitation is tuned for a phase-resonance dwell (excitation at a plate resonance) on a well separated mode of the plate. Figure 1 shows the real part of the response since at phase resonance the imaginary part is essentially zero. The left velocity plot is the original measured data at 45 582 points. Note that lasers, like other instruments, have noise and their idiosyncratic difficulties. The left edge of the plate has a "noise spike" caused by the laser beam "falling off the edge of the plate." In addition,

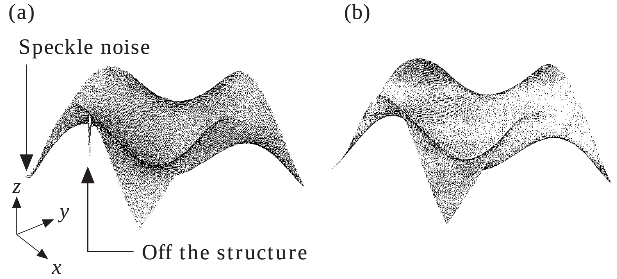

Figure 1. Plate vibration response (213 × 214 points): raw data and smoothed. (a) original plate, (b) smoothed plate.

the upper left corner has some small noise spikes that are caused by speckle-induced dropout (the laser receives insufficient light to detect the instantaneous velocity). The laser data can be functionalized 2-dimensionally by many techniques. Here the discrete Fourier transform (DFT)–inverse discrete Fourier transform (IDFT) method is used to functionalize the space [2, 4]. The right plot labelled "Smoothed plate" is the velocity shape that is generated by limiting the number of Fourier terms in the spatial reconstruction. Thus, the Fourier transform is being used as a noise filter. It must be pointed out that the smoothed plot is a reconstructed image evaluated at exactly the same points at which the data were originally taken. This, of course, is not a limitation since the outward velocity is represented as a continuous spatial functional. The velocity could be evaluated at any point within the plate boundary.

Since the continuous spatial functions shown in the smoothed version of Figure 1 are alternately described in the spatial frequency domain, the partial spatial derivatives of the velocity field can be taken in the spatial frequency-domain form by multiplying each Fourier term by $j\omega$, where ω is the spatial frequency associated with the particular spatial frequency and direction [4]. Inverse transform of the appropriately "differentiated" Fourier transform will result in the slopes of the velocity curves in the various directions. These operations were used to generate Figure 2. The left plot is the angular velocity about the y-axis whereas the right plot is the angular velocity about the x-axis. Note that one now has two sets of 45 582 DOF in addition to the original. The current representation has a total of 136 746 DOF! At this point one has not even attempted to extract the in-plane velocities and the out-of-plane drilling DOF. Notice that even with this work the measurements have exceeded the limits of most conventional FEM modelling systems, 100 000 DOF. It should be obvious that the problem of spatial aliasing that plagues conventional modal analysis is not a problem when using this technique.

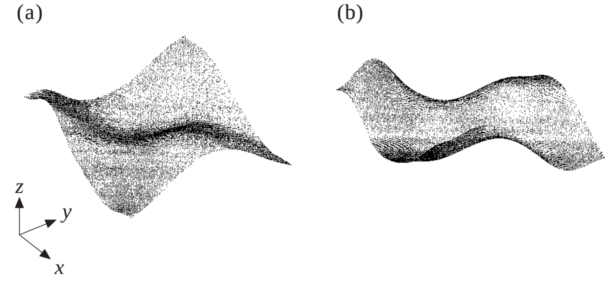

Figure 2. Extracted plate angular velocities: (a) $w_y = \mathrm{d}\dot{z}/\mathrm{d}x$; (b) $w_x = \mathrm{d}\dot{z}/\mathrm{d}y$.

Now, if three or more of these structural velocity profiles from different vantage (view) points are obtained of the same structural component driven the same way, then one may extract an orthogonal triad of velocity response at each measurement point on the structure [11, 14, 15, 17, 18]. It should be pointed out that data does not have to be taken at the same point for each laser view point. There must be sufficient data over the structural element as seen from three or more viewpoints.

Moreover, in the process of the development of these orthogonal triads the dynamic responses are surfacewise functionalized so that the 3-D response may be determined at any point on the surface, measured or not [8, 13, 18]. Each orthogonal triad may be rotated into the element co-ordinate system so that an out-of-plane velocity and two in-plane velocities are obtained at each point.

All this capability does not come without effort. Each laser view of the structure must have the laser registered relative to the structure so that the viewpoint of each measurement is known. This way magnitude, phase, and vector direction of each measurement velocities are known. The registration involves accurate layout of known points on the structure. This can take up one-half hour, but once done it does not have to be repeated. Next, the laser beam is directed to a registration point, the mirror deflections recorded, and the beam moved to the next registration point. This is repeated for a minimum of four registration points at each laser viewing position. Registration for each viewing position may take 15 min. The data from each view point are taken at rates from 3–60 points/s. There are a minimum of three scan view points needed for 3-D velocity extraction.

For example, if three viewpoint scans of 10 000 points each were taken of a structure at a rate of 10 points/s, then the total testing time would be made up of 30 min for layout of the registration points, 45 min (3 × 15 min) for all registrations, and 51 min for all the scans (3 scans × 17 min each). Adding another 30 min for laser movement logistics, one would have a total time of 2 h 36 min for the measurement of 30 000 structural responses. This would yield a continuous functional relationship for $\dot{x}$, $\dot{y}$, $\dot{z}$, $\dot{\theta}_x$, $\dot{\theta}_y$, and $\dot{\theta}_z$.

Since these methods are not limited to the 2-D structures [20] used here and since 3-D structures may result in "hidden" regions of the structure, more viewpoints will be needed to "see" all points of interest. Moreover, certain points may not be viewable at all because of reentrant angles, etc. Thus, with the advantages come limitations.

The extraction of othogonal triads of velocity response solves half of the problem—the determination of the translational responses at any place on the structure. One obtains 3 out of 6 DOF when one uses the method! Taking a hint from the derivatives taken in Figure 1 to find angles, then from mechanics the curl of the velocity field will yield twice the angular velocities of the measurement points [9, 12]. Now, through the use of this idea all 6 DOFs will be known functionally and can be evaluated at each measurement or other point on the structure.

If one considers the bottom-clamped plate shown in Figure 3 and if one develops methodologies that allows the laser to determine its spatial position relative to the plate's co-ordinate system, then the laser may be positioned at multiple positions in space so that different known viewpoints can be had of the structure. The position determination or "registration algorithm" is of prime importance in the laser pose (position of the laser co-ordinate system relative to the known test structure co-ordinate system both translationally and rotationally) determination [7]. The plate was excited at 266·25 Hz (not at resonance) and six different, spatially well separated, non-coplanar vantage points were used to view the plate. At a set of reference points on the structure the orthogonal triad of in-plane and out-of-plane structural velocities are extracted. This is different from Figures 1 and 2 where only out-of-plane velocities were measured. Here three translational degrees of freedom are determined at each point. The real and imaginary forms of this

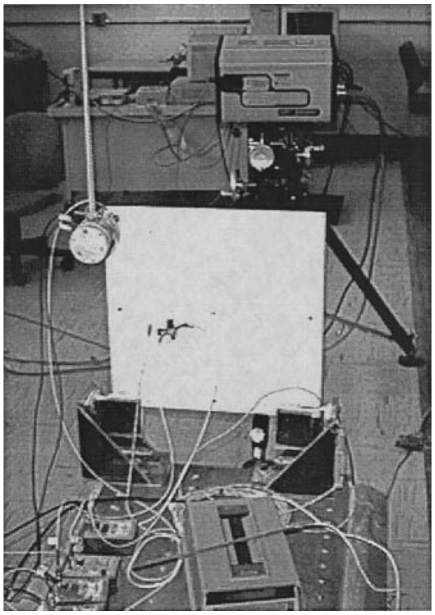

Figure 3. Example: bottom clamped plate.

mobility (velocity/force) result is shown in Figure 4. At this point the spatial field is fully functionalized so many more points could be displayed, if desired. Remember the functionalization of the x, y and z mobilities is done in the spatial frequency domain. The curl of the velocity field can be appropriately taken in the spatial frequency domain, halved, and then transformed via IDFT to the spatial representation of the angular mobilities shown in Figure 5. Notice that the z angular mobility, the out-of-plane drilling DOF, is near zero as it should be at such low frequencies. High z angular mobility would

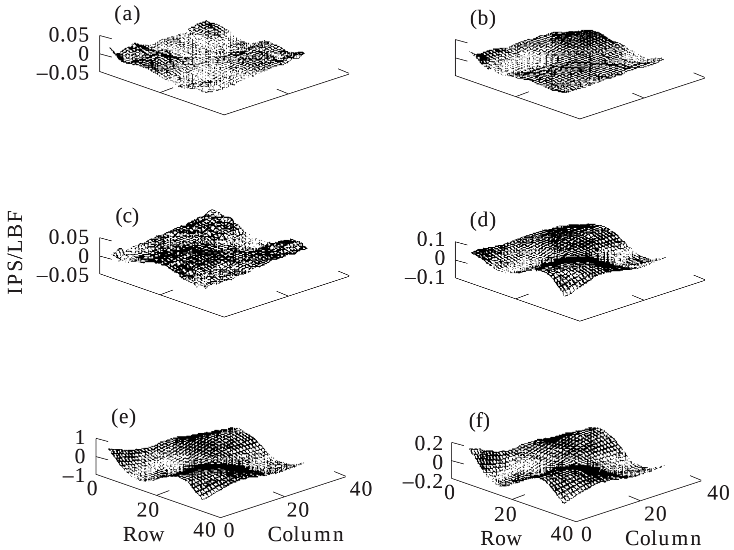

Figure 4. Complex 3-D translational mobilities of a bottom edge clamped plate: (a) mobility **x**-real, (b) mobility **x**-imaginary, (c) mobility **y**-real, (d) mobility **y**-imaginary, (e) mobility **z**-real, (f) mobility **z**-imaginary.

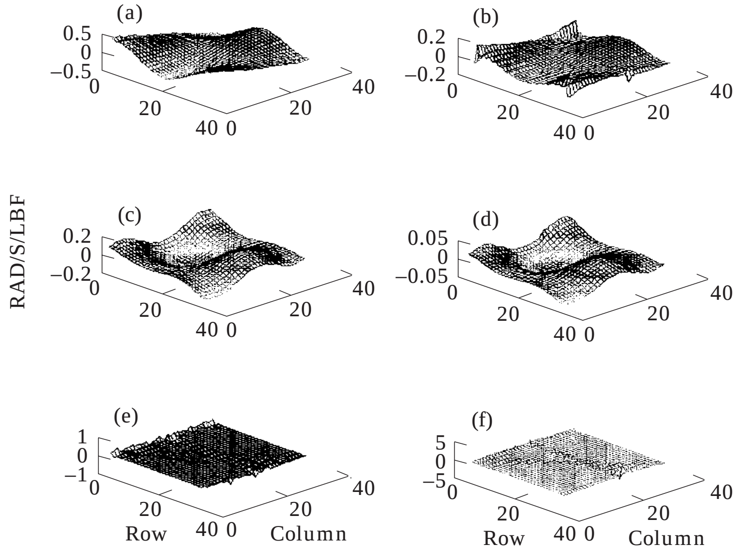

Figure 5. Complex 3-D rotational mobilities of a bottom edge clamped plate: (a) angular mobility **x**-real, (b) angular mobility **x**-imaginary, (c) angular mobility **y**-real, (d) angular mobility **y**-imaginary, (e) angular mobility **z**-real, (f) angular mobility **z**-imaginary.

mean that the material in the plane of the plate is rotating about the z-axis. These kinds of modal activity occur at frequencies at least 10 times the first bending mode frequency [9]. The other angular mobilities are just the slopes of mobilities in Figure 4. These angular mobility profiles are shown in Figure 5. Thus, one has 6 degrees-of-freedom of mobility at each reference point in these figures.

4. ADVANCES IN EXPERIMENTAL MODAL ANALYSIS (EMA)

The above demonstration of the 3-D measurement capabilities of properly implemented laser measurement technology has a major limitation; it looks at the system velocity or mobility response at one frequency! Usually one needs to know more about the system dynamics than only at one forced response frequency. The measurements are of the forced response DOFs, not the mode shapes of the structure. Experimental modal analysis is built on a foundation of modal properties such as modal frequency, modal damping, and mode shape. If one wants to take advantage of the years of work in modal analysis, then one must conform to these modal analysis conventions.

However, it is possible to avoid conventional modal analysis and use the complete frequency response functions directly as a source of a model. Changes in the FRF-based model can be made to predict new system responses when such changes are made to the actual physical system [24, 25]. However, measurement of the full FRF at thousands of points on a structure even with a laser is a daunting task. Another approach must be found.

One now needs an expedient method for the extraction of mode shape information. Of course, one could take the measurements as in the 3-D Structural Measurement methodology, but instead of exciting the structure with a single harmonic force one could excite it with random or burst random excitation.

One would then Fourier transform, average, and curve-fit extracted modal responses at each measurement point. Now fast Fourier transforms (FFTs) are fast, but not that fast. Thousands of points measured from three or more directions can mean

750 000–1 000 000 FFTs and thousands of modal response extractions. This would be a very time consuming task. One needs to time compress this operation further or find a new extraction procedure.

5. HIGH-SPEED EXPERIMENTAL MODAL ANALYSIS (HSEMA)

In 1987 Van Der Auweraer *et al*. [26, 27] suggested that the number of frequency-domain points could be reduced if one expanded the number of data taken around resonance and reduced the number of data taken around saddles and anti-resonance regions. In 1992 Kochersberger (28–30) followed by Moser *et al*. [31] in 1993 extended these ideas so that a new approach for the determination of modal properties and mode shapes was borne where a minimum overdetermined set of spectral response lines were used to extract some of the modal properties.

First, the structure is investigated via conventional FRF measurement. However, these frequency responses are to be taken with the utmost care via step-sine or swept-sine measurements. In the case of the step-sine it is proposed that all the measurements be made at the center of each FFT frequency cell. This guarantees that the data are periodic in the time-domain so FFT windowing functions are not required. Thus, the leakage phenomenon is avoided. The use of a single sine excitation results in the full utilization of the shaker capability giving the best possible signal-to-noise (s/n) ratio. There are few, if any, problems with low coherence data and noise-induced biasing. This careful determination of the base FRFs used in the global natural frequency and damping determinations results in a very high quality basis for the modal space model. One or several measurements can be used to extract this information.

The next problem is to extract the residue or mode shape component at each of the structure's response points. Before extracting appropriate residues one needs to account for the residual effects above and below the modes of interest. The modelling of the residuals above and below the frequency band of interest can take many forms. The simplest is usually a residual stiffness and a residual mass to model the upper residual and the lower residual, respectively. More complex models of the residuals can utilize one or more poles or zeros above and below the frequency band of interest.

Kochersberger [28] proposed and showed success in extraction of mode shapes by exciting a structure with a multi-sine signal. This multi-sine signal was composed of the sum of a number of pure-tone harmonic signals such as shown in Figure 6. The goal was to get the steady-state response to these forcing functions. One designs the frequencies to be exactly periodic in the Fourier analyzer's time window. Moreover, one does not have to use averaging to deal with noise because the s/n ratio is very high. Very high quality data is produced.

The idea is to select a limited number of pure harmonic forcing functions at which to excite the structure. One would have to measure the mobilities at a number of frequencies equal to the number of desired modes plus the number of degrees of freedom used in the residual model plus one or two additional for overdetermination. These excitations are applied simultaneously to the structure as a multi-sine signal. The multi-sine response is digitized. Next, a discrete Fourier transform (DFT) is carried out, at each spectral line associated with the multi-sine inputs. This means that, if 2048 time points are acquired, but only 8–12 spectral lines are analyzed, instead of the 2048 spectral lines that are analyzed by a "standard" FFT. Moreover, the elimination of the FFT opens the possibility of the use of a regressive Fourier series (RFS) analysis that will yield some statistical information about the responses as well.

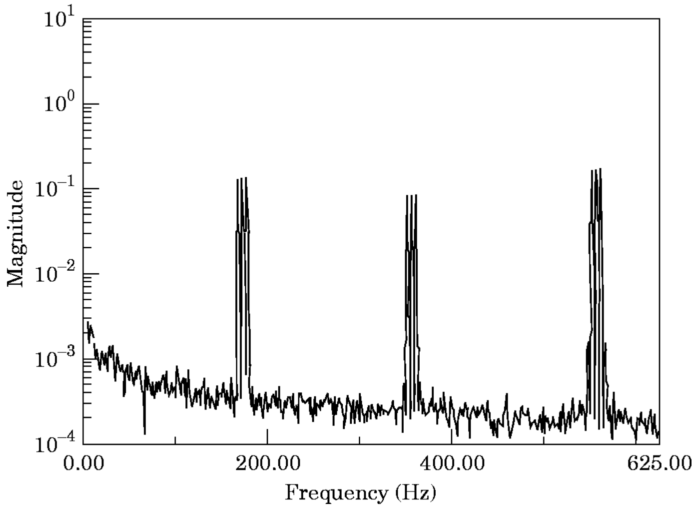

Figure 6. Typical multi-sine excitation-nine spectral lines.

Next, these responses are placed in a set of equations that must be solved in a least squares sense for the modal residues and the FRF residuals. The result will be the development of the *n* complex (real) mode shape components at that measurement point corresponding to the *n* global natural frequencies. If one repeats this at all the measurement points on the structure, then one has a full modal description of the test structure.

The result is an expedient method for extraction of the mode shape components at each point as the laser takes the data. So instead of getting a single response in a known direction one now has *n* modal responses. The processing time for these operations should approach the data-taking time for the next set of time series. This means that the data taking rates can approach 60 points/s, if no additional overhead is incurred. The result provides on a mode-by-mode basis the information necessary to extract the 6-DOF at each measurement point over the entire surface that has been measured.

6. THE MARRIAGE OF HIGH-SPEED EMA WITH 3-D LASER STRUCTURAL MEASUREMENTS

If one reviews the shortcomings of EMA and of LSM one will quickly find that a marriage will result in each technology complementing the other so that the shortcomings of each are eliminated. The new high-speed EMA methodologies have allowed time compression of the mode shape extraction while the laser-based structural imaging has provided a means by which the full 6 DOF description of the structural mobilities can be determined in a very time-efficient manner. Moreover, the LSM methodologies have provided a means through surface functionalization to determine the mobilities at any point on the structure.

7. NEW OPPORTUNITIES IN EXPERIMENTAL MEASUREMENTS AND MODELLING

Given the spatially based modal modelling system described above, one can think of numerous research and development avenues that will be opened in the future. Remembering that one has the modal characteristics anywhere on the structure and as such one can determine the response at any point or points on the stucture to any force or combination of forces at any frequency in the frequency range of the analysis. Thus, one has the phasewise response in all 6 DOF all over the surface of the structure. What can one do with that information?

The most obvious to persons in the discipline of experimental modal analysis is "experimental structural modification." With all the rotations and all the translations one now has the ability to add realistic physical modifications via analytical processing of the full 6 DOF modal model using models of discrete or continuum beams, gussets, and/or plate modifications. The predictions of the new system response will be computed in seconds from the experimentally based modal model. They will breath new life in the old experimental structural modification methodology.

Next, with the full-surface response descriptions one can consider the development of strain distributions over the surface. The natural extension is the prediction of the surface stress distributions. The use of the outward normal of vibration along with a boundary element code can predict the acoustic intensities, the acoustic sound pressures at points in space, or the sound power over the space. Even more challenging are the possibilities of measuring the source-to-sink power flows in a structure. One could measure also the 6 DOF, 3-D operating shape of a running machine.

It can be easily seen that the development of a full 6 DOF, 3-D modal modelling system will open many experimental doors to the engineering community. One is on the verge of an experimental data explosion. These methods will be tapped by many when the price of this technology drops. Currently the laser systems needed to make the measurements are too expensive for medium-sized industrial firms. But future price reductions are possible, if pressure is applied.

8. SUMMARY

The marriage of the three-dimensional laser-based structural dynamics measurement system with multi-sine excitation methodologies will provide a unique database from which one with be able to develop a new technology; three-dimensional structural modal modelling system: six degrees-of-freedom. This new technology will be able to utilize the decades of work in modal analysis because one can cast the laser solution into the same format as current FEM modal models. However, one might overload current routines with huge amounts of data. A strange situation: experimental methods could give say 300 000 DOF to the finite element community that currently can barely reach 100 000 DOF in some of the larger FEM models.

Once the new methodologies are available then there will be an expansion of its use into many fields such as power flow, stress distributions measurement, experimental structural modifications, etc. The future use of these full-field measurements is bright.

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