



# ANALYSIS AND DESIGN OF MUFFLERS—AN OVERVIEW OF RESEARCH AT THE INDIAN INSTITUTE OF SCIENCE

M. L. MUNJAL

*Centre of Excellence for Technical Acoustics, Department of Mechanical Engineering,  
Indian Institute of Science, Bangalore 560 012, India*

*(Received 18 December 1995, and in final form 4 March 1997)*

The reciprocating motion of the piston(s) of engines and compressors and the associated intake and discharge of gases are responsible for noise radiation to the atmosphere that ranks as a major pollutant of the urban environment. Mufflers have been developed over the last seventy years based on electro-acoustic analogies and experimental trial and error. Passive mufflers based on impedance mismatch, called reflective or reactive mufflers, have been most common in the automobile industry. Mufflers based on the principle of conversion of acoustic energy into heat by means of highly porous fibrous linings, called dissipative mufflers or silencers, are generally used in heating, ventilation and air-conditioning systems. The author has been working in this area for over 27 years. Of late, he has been researching in the vibro-acoustics of hoses used in automotive climate control systems. The present paper gives an overview of the research findings of the author and his students in different aspects of active as well as passive mufflers in the last decade. The same are related to the contemporary state of the art. Finally, areas needing further research are indicated.

© 1998 Academic Press Limited

## 1. INTRODUCTION

Exhaust noise of automotive engines is the main component of noise pollution of the urban environment. With the ever increasing population density of vehicles on the road, this has become an important area of research and development. Most of the advances in the theory of acoustic filters and exhaust mufflers have come about in the last four decades, and the author of this article has been active in this area for about three decades. For a monograph on the acoustics of ducts and mufflers see reference [1]. This article, following the monograph in format, briefly reviews the work done by the author primarily during the last ten years (that is, subsequent to the drafting of the Monograph), independently or jointly with his colleagues and students, and relates it to state of the art of analysis and design of mufflers.

## 2. FREQUENCY DOMAIN ONE-DIMENSIONAL ANALYSIS

An acoustic filter is a one-dimensional dynamical system comprising distributed and lumped dynamical system comprising distributed and lumped elements, analogous to the electrical filters used in transmission line theory. While electro-acoustic analogies were known for a long time (see, for example Olson [2]), the analogic electrical circuits for engine exhaust mufflers were made use of by Davis [3]. These were adapted for computation of the insertion loss of mufflers by Sreenath and Munjal [4]. The state variables of acoustic

pressure and mass velocity were described as analogous to electromotive force (voltage) and current, and due consideration was given to the radiation impedance  $Z_0$  (of the atmosphere at the tail-pipe end), and source characteristics  $p_s$  and  $Z_s$ , analogous to the load impedance, open circuit voltage and internal impedance of the source, respectively.

Evaluation of insertion loss of an  $n$ -element filter for a sinusoidal signal by means of the classical method involves simultaneous solution of  $2n + 2$  algebraic equations. Therefore, a comprehensive transfer matrix method was developed for one-dimensional linear dynamical systems which would include mechanical systems (vibration isolators), electrical wave filters and acoustical filters.

In the case of engine mufflers, waves propagate in a moving medium. In order to account for the convective effect of incompressible mean flow, a new set of state variables, convective pressure  $p_c$  and convective mass velocity  $v_c$  were defined to replace the classical variables of acoustic pressure  $p$  and acoustic mass velocity,  $v$ . It turned out that the two sets of variables are related linearly to each other [5]. Through these convective state variables, the velocity ratio cum transfer matrix method was extended to the evaluation of a muffler with mean flow. This was significantly superior to the earlier method [6] that involved simultaneous solution of a large number of algebraic equations with complex coefficients.

Unlike a dissipative muffler, the energy leaving a reflective muffler at the radiation end is equal to that entering at the source end. It has been shown analytically [7] that a reflective muffler works by reducing the resistive component of the load impedance seen by the source, as compared to the atmospheric impedance that the source would see in the absence of the muffler. Thus a reactive muffler acts on the principle of impedance mismatch.

Derivation of transfer matrices involves equations of mass continuity, momentum balance, entropy change, and loss of stagnation pressure, with and without acoustic perturbations in a moving medium. While the transfer matrix of a uniform tube was derived earlier [5], those for simple area discontinuities (Figure 1(a) and 1(b), extended tube expansion chambers (Figures 1(c) to 1(f)) without or with flow reversals were derived by Panicker and Munjal [8, 9] making use of the basic equations developed by Alfredson and Davies [6]. A single common transfer matrix for all these area discontinuities was then derived by the author [1].

There is a coupling between viscous losses and flow losses for waves in a moving medium. Panicker and Munjal [10] derived closed-form expressions for aeroacoustic attenuation for progressive waves moving in either direction, and corroborated the same experimentally. These were used later to derive the transfer matrix of a tube with mean viscous flow [1].

Perforated element mufflers (Figures 1(g) to 1(n)) are known to be acoustically much more efficient and versatile than the unperforated ones (Figures 1(a) to 1(f)). However, these elements could not be analysed until 1979, when Sullivan [11, 12] presented a segmentation procedure for modelling the two-duct elements of figures 1(g) to 1(k), wherein the effect of perforations was lumped at a number of discrete points connected by unperforated uniform tubes. However, Rao *et al.* [13] developed a method of decoupling the otherwise coupled wave equations in the inner tube and the outer annular tube, and thus derived transfer matrices for the concentric tube resonator, the cross-flow expansion element and the cross-flow contraction element. This was later generalized as an eigenvalue problem [14, 15], and closed-form expressions for the transfer matrix parameters were derived for the two-duct reverse-flow elements of Figures 1(g) to 1(k) and the three-duct elements of figures 1(l) and 1(m). Recently, the open-ended cross-flow element of Figure 1(n) has also been analysed [16].

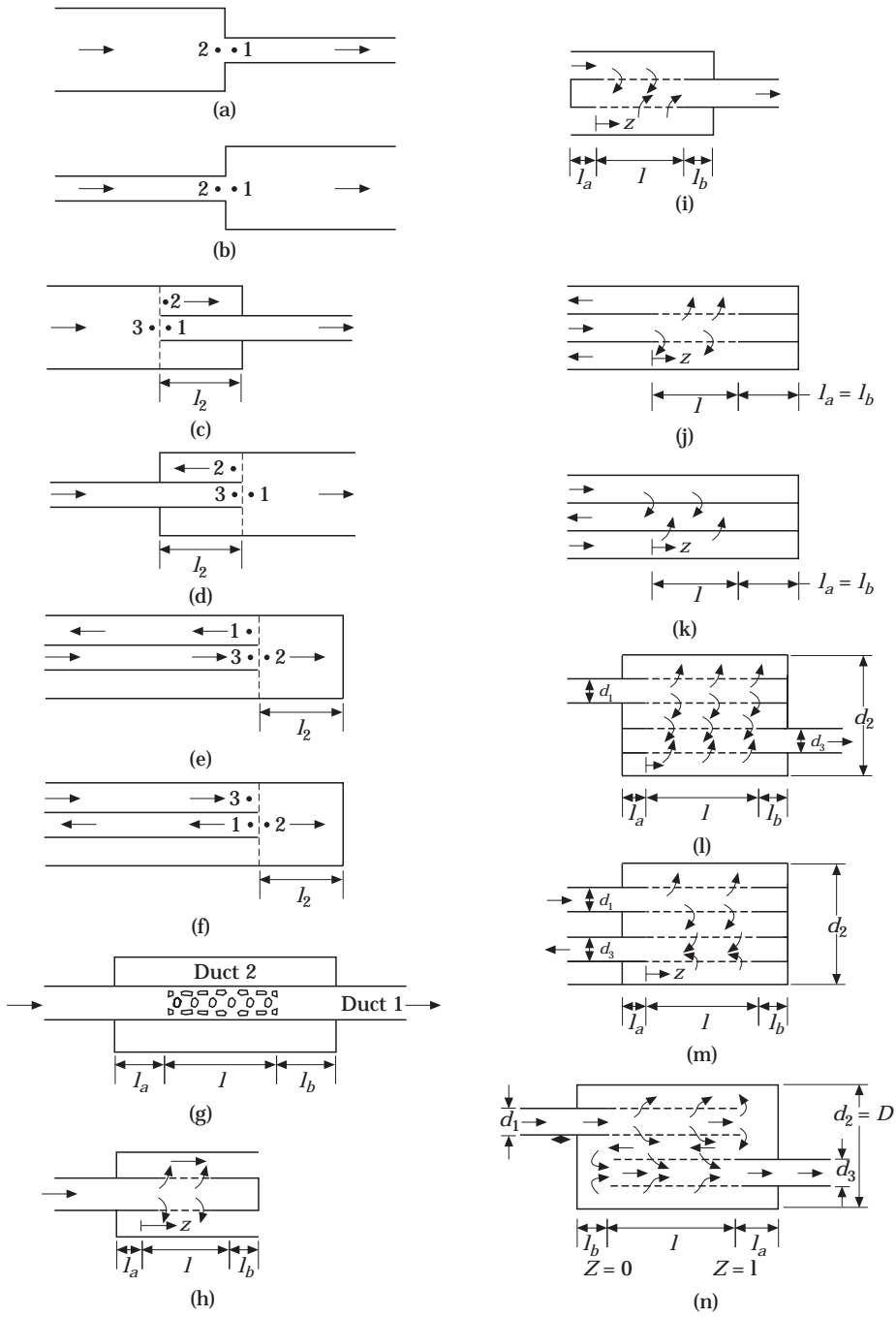


Figure 1. Different basic muffler elements. (a) Sudden contraction, (b) sudden expansion, (c) extended outlet, (d) extended inlet, (e) reversal expansion, (f) reversal contraction, (g) concentric tube resonator, (h) cross-flow expansion, (i) cross-flow contraction, (j) perforate reversal expansion, (k) perforate reversal contraction, (l) cross-flow, closed-end, three-duct chamber, (m) reversed-flow, closed-end, three-duct chamber, and (n) cross-flow, open-end, three-duct chamber.

Exhaust systems of IC Engines are characterized by a mean temperature gradient along every constituent pipe. A theoretical study, involving the solution of the inhomogeneous wave equation by means of Green's functions, on propagation of plane waves in the presence of a hot meanflow in a uniform pipe, yielded the fourpole parameters of such a pipe [17]. Peat analysed the same problem later as a perturbation problem [18].

### 3. TIME-DOMAIN ONE-DIMENSIONAL ANALYSIS

Unlike the frequency domain analysis discussed in the preceding section, where one deals with harmonic acoustic perturbations on mean values, here the equations are solved numerically in the time domain. There are several methods for this; the primary ones being the finite difference method and the method of characteristics. In the method of characteristics, the basic equations do not have to be linearized; hence the name finite wave analysis. In the analysis of hot exhaust systems with heat transfer and aeroacoustic losses, there are three variables; viz., the forward pressure wave, the reflected pressure wave, and the entropy wave. In their classic paper, Benson *et al.* [19] proposed a scheme of calculation where the two pressure variables were calculated by the fixed frame (or mesh) method while the entropy variable was calculated by means of the moving frame (or wave diagrams) method. Gupta and Munjal [20, 21] presented a computational improvement where all the three variables were calculated by the fixed frame (or mesh) method. Thence, they developed numerical methods for prediction of source characteristics of a single-cylinder internal combustion engine [22]. In contrast, Jones [23] developed the moving-frame or the wave-diagram method for analysing waves in an exhaust pipe.

### 4. AEROACOUSTIC MEASUREMENTS

Measurements are required for supplementing the analysis by providing certain basic data or parameters that cannot be predicted precisely, for verifying the analytical or numerical predictions, and also for evaluating the overall performance of a system configuration in order to check if it satisfies the design requirements. The convective and dissipative effects of the moving medium affect the acoustic measurements in several ways. The standard impedance tube method [24] was made use of in an ingenious way to measure the impedance of perforates with grazing flow as required in the analysis of concentric tube resonators. These parametric measurements led to a fairly general empirical formula for the grazing flow impedance of perforated plates [25]. However, this method is cumbersome as it involves discrete frequency measurements. Chung and Blaser [26, 27] developed a random excitation, two microphone, transfer function method. The same has now been extended to incorporate the effects of moving medium and damping [28].

A new method has been developed for experimentally determining the matrix parameters that define the transmission properties with or without gas flow. This method is conceptually similar to the existing two-load method [29], but involves the placement of an acoustic source at two separate locations. The form of the equations relating the four parameters in terms of six measured transfer functions or pressure ratios is the same for both the methods. However, the result of theoretical uncertainty analysis and many comparative tests have shown that substantial improvements can be expected with this new two-source location method, particularly under flow conditions [30]. More recently, Kim and Prasad have developed a modified transfer function method for acoustical source characterization in duct systems which has been shown to work even with low signal to noise ratio [31, 32].

### 5. LINED DUCTS AND DISSIPATIVE MUFFLERS

Mufflers used in heating, ventilation and air-conditioning (HVAC) ducts, industrial fans, ventilation and access openings of acoustic enclosures, intake and exhaust ducts of power stations, cooling tower installations, gas turbines, and jet-engine test cells, act not only by muffling the sources through successive reflections of sound by means of impedance mismatching but also by dissipating the incident sound energy as heat. These mufflers are, in fact, primarily dissipative mufflers with the advantage of providing definite attenuation of sound over a wide range of frequencies. In order to integrate a lined duct with other muffler elements, source characteristics and radiation impedance, the transfer matrix has been derived for a rectangular duct [33] as well as a circular duct [1] lined on the inside with an acoustically absorptive material. The lining has been assumed to be locally reacting, represented by its normal impedance. The backing airgap, if any, can be accounted for in the wall impedance by means of the transfer matrix approach as in the case of multi-layer barriers and enclosures excited by normally incident plane waves. The assumption of locally reacting linings is violated in the case of the axially uninterrupted linings which allow axial wave propagation. Analysis of ducts with bulk reacting lining as well as locally reacting lining has now been done incorporating the effect of a thin protective layer [34].

### 6. THREE-DIMENSIONAL NUMERICAL ANALYSIS OF MUFFLERS

Three-dimensional effects are a primary source of discrepancies between the measured values of automotive muffler performance and those predicted by the plane wave theory at higher frequencies. The author presented a simple collocation method, making use of compatibility conditions for acoustic pressure and particle velocity at a number of equally spaced points in the planes of the junctions (or area discontinuities) to generate the required number of algebraic equations for evaluation of the relative amplitudes of various modes (eigenfunctions), the total number of which is equal to the area ratio [35].

3-D analysis of mufflers with more complex shapes would, however, require use of the boundary element method (BEM) or the finite element method (FEM). Researchers in FEM have often restricted themselves to two-dimensional analysis with simplifying assumptions. Recently, the variational approach with Hermite polynomial shape functions and the Galerkin weighted residual approach with isoparametric elements has been presented for a truly 3-D analysis of simple expansion chamber mufflers [36]. Heavy strain on the core memory and computational efficiency of the computer is reduced by means of the recursive substructuring principle for the repetitive segments of the expansion chamber muffler. Making use of this and the transfer matrix method, a finite element computer program has been developed for the 3-D analysis of the extended-tube expansion chamber as well as simple expansion chamber mufflers [37].

It is well-known that a sudden area discontinuity (Figures 1(a) to 1(f)) generates higher order, evanescent acoustic waves even at low frequencies within the plane wave limit. This is attributable to the inertance effect, which has been evaluated through FEM as a function of the radius ratio, frequency and the offset distance [38].

### 7. BREAKOUT NOISE

Most of the research work on mufflers has dealt with wave propagation along the axis. However, walls of most commercial mufflers and HVAC systems are not rigid. Compliance of wall results in a coupling of the waves inside with those outside leading to breakout

noise or shell noise. This has been considered recently resulting in a rigorous model [39] and a relatively much simpler and faster impedance model [40]. Now, it is possible to calculate not only axial TL but also transverse TL and the net TL.

8. ACTIVE NOISE CONTROL IN A DUCT

Both reflective as well as dissipative mufflers generally show poor performance at low frequencies, and for wideband responses, they would be very large and expensive. These limitations of passive mufflers have given rise to the idea of active attenuation, which consists in sensing the undesired noise in the exhaust pipe and reintroducing the signal through a loudspeaker.

Assuming plane waves, and making use of transfer matrices and electroacoustic analogies, a standing wave analysis of the active noise control system in a duct (Figure 2) has been presented incorporating the characteristics of the primary source as well as

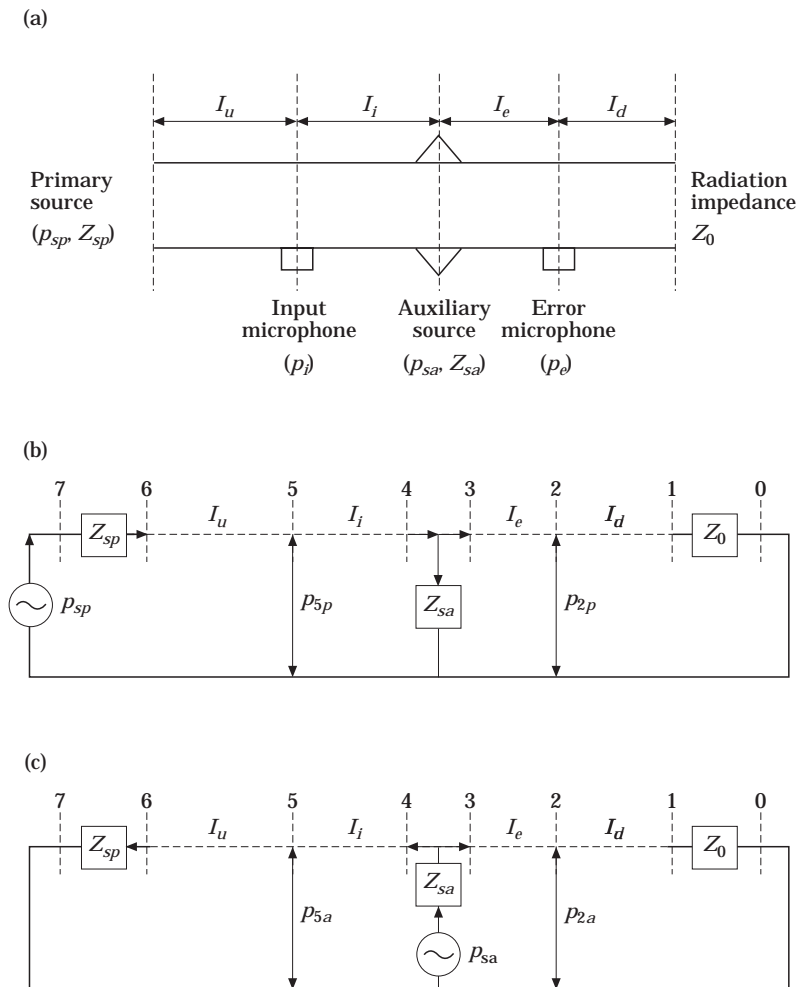


Figure 2. (a) Schematic diagram of an active noise control system in a duct, (b) electrical analogous circuit of the system with only the primary source active and (c) electrical analogous circuit of the system with only the auxiliary source active.

the auxiliary source by means of linear superposition. Analytical expressions have thus been derived for the ratio of the two source pressures and certain other ratios or relations of interest for complete or partial cancellation of noise downstream of the auxiliary source [41]. The same results have been obtained by means of block diagrams and transfer functions [42].

An active noise control system in a duct, until 1990 worked for plane waves only. For a relatively high frequency source and/or for ducts with large transverse dimensions, the duct cross-section had to be divided into two or more sections, which permitted plane waves only, up to the largest frequency of interest. For this purpose, the cut-on frequencies of a large round duct with azimuthal as well as radial partitions were calculated by the author [43].

For measurement of acoustic pressures in ducts lined with turbulent mean flow for use in active noise control systems for low frequency noise flow machinery, a typical tube would have a thin longitudinal slit covered with a number of layers of cloth, with a half-inch microphone at the downstream end. This antiturbulence probe tube has been analysed for its acoustic sensitivity or frequency response by means of the distributed parameter approach wherein mutual interaction of the acoustic pressure fields in the probe tube and the annular duct is considered [44]. In a typical one-dimensional active noise control system, the auxiliary source (generally a loudspeaker) must produce an acoustic pressure equal (and opposite in phase) to that produced by the primary source at the loudspeaker junction. If the ratings of the power amplifier and the loudspeaker are not adequate, there will be little reduction of noise at the radiation end. One way out is to replace the duct length  $l_u$  by a passive muffler (reflective or dissipative) upstream of the input microphone. Such a hybrid system has been analysed by making use of electroacoustic analogies and the transfer matrix method [45].

## 9. CONCLUDING REMARKS

In keeping with the requirements of the INDO-US Symposium, this review article has concentrated on work done by the author and his students. The convective effect of meanflow for waves in variable area ducts [46, 47] and the effect of meanflow shear [48, 49] have not been discussed because they have been found to be of negligible practical importance. It is obvious, however, that over the last two decades, there have been considerable advances. It is now possible to design a preliminary configuration on a personal computer [1, 16, 37, 50]. However, considerable research inputs are still needed in (a) time domain analysis of the extended tube chamber and the perforated element muffler; (b) FEM analysis of complex geometries like perforated element mufflers in order to incorporate 3-D effects; (c) acoustic analysis of bellows, that are used primarily for vibration isolation; (d) development of the boundary element methods for 3-D analysis of mufflers; (e) aeroacoustic modelling of orifice plates and control valves; (f) frequency domain characterization of the engine exhaust source; (g) development of cost effective active noise control systems for the HVAC and engine exhaust systems, and (h) prediction of shell noise for typical commercial mufflers.

Work on some of these problems is going on in this Institute as also elsewhere in the world.

## REFERENCES

1. M. L. MUNJAL 1987 *Acoustics of ducts and mufflers*. New York: Wiley-Interscience.
2. H. F. OLSON 1966 *Dynamical analogies*. Princeton, NJ: Van Nostrand; third edition.

3. D. D. DAVIS 1957 *Handbook of Noise Control* (editor C. M. Harris), Chapter 21. Acoustical Filters and Mufflers.
4. A. V. SREENATH and M. L. MUNJAL 1970 *Journal of Sound and Vibration* **12**, 1–19. Evaluation of noise attenuation due to exhaust mufflers.
5. M. L. MUNJAL 1975 *Journal of Sound and Vibration* **39**, 105–119. Velocity ratio cum transfer matrix method for evaluation of a muffler.
6. R. J. ALFREDSON and P. O. A. L. DAVIES 1971 *Journal of Sound and Vibration* **15**, 175–196. Performance of exhaust silencer components.
7. M. L. MUNJAL 1980 *Proceedings of DAGA '80* (Munich:VDE- Verlag), 281–284. A new look at the performance of reflective exhaust mufflers.
8. V. B. PANICKER and M. L. MUNJAL 1981 *Journal of the Indian Institute of Science* **A63**, 1–19. Aeroacoustic analysis of straight-through mufflers with simple and extended-tube expansion chambers.
9. V. B. PANICKER and M. L. MUNJAL 1981 *Journal of the Indian Institute of Science* **A63**, 21–38. Aeroacoustics of mufflers with flow reversals.
10. V. B. PANICKER and M. L. MUNJAL 1981 *Journal of the Acoustical Society of India* **91**, 95–101. Acoustic dissipation in a uniform tube with moving medium.
11. J. W. SULLIVAN 1979 *Journal of the Acoustical Society of America* **66**, 772–778. A method of modeling perforated tube muffler components. I. Theory.
12. J. W. SULLIVAN 1979 *Journal of the Acoustical Society of America* **66**, 779–788. A method of modelling perforated tube muffler components. II. Applications.
13. K. N. RAO and M. L. MUNJAL 1986 *Sadhana (Academy Proceedings in Engineering Sciences)* **9**, 255–269. Noise reduction with perforated three-duct muffler components.
14. M. L. MUNJAL, K. N. RAO and A. D. SAHASRABUDHE 1987 *Journal of Sound and Vibration* **114**, 173–188. Aeroacoustic analysis of perforated muffler components.
15. K. S. PEAT 1988 *Journal of Sound and Vibration* **123**, 199–212. A numerical decoupling analysis of perforated pipe silencer elements.
16. G. R. GOGATE and M. L. MUNJAL 1995 *Journal of the Acoustical Society of America* **97**, 2919–2927. Analytical and experimental aeroacoustic studies of open-ended three-duct perforated elements used in mufflers.
17. M. L. MUNJAL and M. G. PRASAD 1986 *Journal of the Acoustical Society of America* **80**, 1501–1506. On plane wave propagation in a uniform pipe in the presence of a mean flow and a temperature gradient.
18. K. S. PEAT 1988 *Journal of Sound and Vibration* **123**, 43–53. The transfer matrix of a uniform duct with a linear temperature gradient.
19. R. S. BENSON, R. S. GARG and D. WOOLLATT 1964 *International Journal of Mechanical Science* **6**, 117–144. A universal study of unsteady flow problems.
20. V. H. GUPTA and M. L. MUNJAL 1993 *Sadhana (Academy Proceedings in Engineering Sciences)* **18**, 911–925. Time-domain finite-wave analysis of the engine exhaust system by means of the stationary-frame method of characteristics. Part I. Theory.
21. V. H. GUPTA and M. L. MUNJAL 1993 *Sadhana (Academy Proceedings in Engineering Sciences)* **18**, 927–941. Time-domain finite-wave analysis of the engine exhaust system by means of the stationary-frame method of characteristics. Part II. Computed results and experimental corroboration thereof.
22. V. H. GUPTA and M. L. MUNJAL 1992 *Journal of the Acoustical Society of America* **92**, 2716–2725. On numerical prediction of the acoustic source characteristics of an engine exhaust system.
23. A. D. JONES 1984 *Noise Control Engineering Journal* **23**, 12–31. Modelling the exhaust noise radiated from reciprocating internal combustion engines.
24. ASTM C 1972 *American Society of Testing and Materials, Philadelphia, USA*. Impedance and absorption of acoustical material by the Tube method—reapproved.
25. K. N. RAO and M. L. MUNJAL 1986 *Journal of Sound and Vibration* **108**, 283–295. Experimental evaluation of impedance of perforates with grazing flow.
26. J. Y. CHUNG and D. A. BLASER 1980 *Journal of the Acoustical Society of America* **68**, 907–913. Transfer function method of measuring in-duct acoustic properties I. Theory.
27. J. Y. CHUNG and D. A. BLASER 1980 *Journal of the Acoustical Society of America* **68**, 914–921. Transfer function method of measuring in-duct acoustic properties II. Experiment.
28. M. L. MUNJAL and A. G. DOIGE 1990 *Journal of Sound and Vibration* **137**, 135–138. The two microphone method incorporating the effects of mean flow and acoustic damping.



29. T. Y. LUNG and A. G. DOIGE 1983 *Journal of the Acoustical Society of America* **73**, 867–876. A time-averaging transient testing method for acoustic properties of piping systems and mufflers with flow.
30. M. L. MUNJAL and A. G. DOIGE 1990 *Journal of Sound and Vibration* **141**, 323–333. Theory of a two-source–location method for direct experimental evaluation of the four-pole parameters of an aeroacoustic element.
31. W. S. KIM and M. G. PRASAD 1992 *Journal of the Acoustical Society of America* **91**, 2716–2725. Acoustic impedance of a finite length duct with dynamic termination.
32. W. S. KIM and M. G. PRASAD 1993 The ASME Winter Annual Meeting, New Orleans, Louisiana, 93-WA-NCA-5. Modified transfer function method for acoustical source characterization in duct systems.
33. M. L. MUNJAL and U. S. SHIRAHATTI 1987 *Transactions of ASME, Journal of Vibration, Acoustics, Stress and Reliability* **109**, 366–371. Analysis of lined ducts with mean flow, with application to dissipative mufflers.
34. M. L. MUNJAL and P. T. THAWANI 1997 *Noise Control Engineering Journal* **45**, 14–18. Effect of protective layer on the performance of absorptive ducts.
35. M. L. MUNJAL 1987 *Journal of Sound and Vibration* **116**, 71–88. A simple numerical method for a 3-dimensional analysis of simple expansion chamber mufflers of rectangular as well as circular cross-section with stationary medium.
36. A. D. SAHASRABUDHE, S. ANANTHA RAMU and M. L. MUNJAL 1991 *Journal of Sound and Vibration* **147**, 371–394. Matrix condensation and transfer matrix techniques in the 3-D analysis of expansion chamber mufflers.
37. A. D. SAHASRABUDHE, M. L. MUNJAL and S. ANANTHA RAMU 1992 *Noise Control Engineering Journal* **38**, 27–38. Design of expansion chamber mufflers incorporating 3-D effects.
38. A. D. SAHASRABUDHE, M. L. MUNJAL and S. ANANTHA RAMU 1995 *Journal of Sound and Vibration* **185**, 515–529. Analysis of inertance due to the higher order mode effects in a sudden area discontinuity.
39. V. EASWARAN and M. L. MUNJAL 1995 *Journal of the Acoustical Society of America* **97**, 3494–3501. A note on the effect of wall compliance on lowest-order mode propagation in fluid-filled/submerged impedance tubes.
40. M. L. MUNJAL and P. T. THAWANI 1995 *Proceedings of Inter-noise 95*, Newport Beach, USA, 361–366. A simple model for wave propagation along and across a hose.
41. M. L. MUNJAL and L. J. ERIKSSON 1988 *Journal of the Acoustical Society of America* **84**, 1086–1093. An analytical, one-dimensional standing wave model of a linear active noise control system in a duct.
42. M. L. MUNJAL and L. J. ERIKSSON 1989 *Journal of Sound and Vibration* **129**, 443–455. Analysis of a linear one-dimensional noise control system by means of block diagrams and transfer function.
43. M. L. MUNJAL 1988 *Journal of the Acoustical Society of America* **84**, 1936–1939. On the cut-on frequencies of a large round duct with azimuthal as well as radial partitions.
44. M. L. MUNJAL and L. J. ERIKSSON 1989 *Journal of the Acoustical Society of America* **85**, 582–587. An exact one-dimensional analysis of the acoustical sensitivity of the anti-turbulence probe-tube in a duct.
45. M. L. MUNJAL and L. J. ERIKSSON 1989 *Journal of the Acoustical Society of America* **86**, 832–834. Analysis of a hybrid noise control system for a duct.
46. V. EASWARAN and M. L. MUNJAL 1992 *Journal of Sound and Vibration* **152**, 73–93. Plane wave analysis of conical and exponential pipes with incompressible mean flow.
47. V. EASWARAN and M. L. MUNJAL 1991 *Journal of the Acoustical Society of America* **90**, 2163–2172. Transfer matrix modelling of hyperbolic and parabolic ducts with incompressible mean flow.
48. G. R. GOGATE and M. L. MUNJAL 1993 *Journal of Sound and Vibration* **160**, 465–484. Analytical solution of sound propagation in lined or unlined circular ducts with laminar mean flow.
49. G. R. GOGATE and M. L. MUNJAL 1992 *Journal of the Acoustical Society of America* **92**, 2915–2923. Analytical solution of the laminar mean flow wave equation in a lined or unlined two-dimensional rectangular duct.
50. M. L. MUNJAL, S. KRISHNAN and M. M. REDDY 1993 *Noise Control Engineering Journal* **40**, 159–166. Flow-acoustic performance of perforated element mufflers with application to design.