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A NEW METHOD FOR DETERMINING COUPLING LOSS FACTORS FOR SEA

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1. INTRODUCTION

SEA (Statistical Energy Analysis) originated by R. H. Lyon [1] is a useful technique for solving vibration and noise problems in the frequency region of more than 500 Hz. SEA consists of breaking up a large system into a number of subsystems and writing down the power balance equation for each of them. Coupling loss factors which determine the power flow between subsystems are one of the most important parameters used in SEA.

Coupling loss factors and loss factors are required to predict vibration and noise levels of an existing product or a product at the drawing board stage. For a few simple cases involving conservative coupling, coupling loss factors can be theoretically calculated. However, for most practical situations, especially for those involving non-conservative coupling such as a bearing–shaft interface, they cannot be theoretically calculated and have to be obtained from experimental measurements. For products at the drawing board stage, coupling loss factors of a similar existing product can be measured and used as a first approximation. Loss factors which determine the power dissipation in each of the subsystems will have to be determined from experiments, but they are relatively easy to determine in comparison to the coupling loss factors. Reference [2] deals with measurement of coupling loss factors by matrix fitting, references [3–5] have presented the determination of coupling and loss factors from the power injection method.

Methods currently available in the literature for determining coupling loss factors have the following drawbacks.

(1) The matrix inversion involved in the computation of the coupling loss factors has some numerical problems due to the ill-conditioned matrix that is used for inversion.

(2) Sometimes the coupling loss factors turn out to be negative which is inconsistent.

(3) To avoid numerical problems during matrix inversion, it is necessary to assume that the subsystems are weakly coupled; this limits the possibility of using SEA for a general structural model which may have strong coupling.

(4) Some methods do not use information about the power injected into the system due to difficulties such as phase mismatch encountered in the past in computing the injected power. This has led to inaccurate determination of the coupling loss factors and also inconsistencies, like negative coupling factors.

The only advantage of the matrix fitting method presented in reference [2] is that it does not require information about the input power, which was difficult to determine in the past. The power injection method presented by reference [4] has succeeded to some extent in improving the accuracy of the coupling loss factors, but still some problems exist with the matrix inversion for strongly coupled systems. The method presented by reference [5] has been experimentally verified only for weakly coupled systems.

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The assumption of weak coupling in determining coupling loss factors limits the practical application of SEA. There are many structures of engineering interest which have strong coupling between their subsystems. Therefore the objective of this communication is to present a method of determining coupling loss factors by the power injection method (PIM) which is easy to compute, accurate and which is not affected by the strength of the coupling between subsystems. The method of determining coupling loss factors presented here assumes the possibility of accurately determining the power injected to subsystems.

2. THEORY AND FORMULATION

2.1. *Power flow between subsystems*

A typical model for SEA applications consists of many interconnected subsystems as shown in Figure 1. Consider a system of N interconnected subsystems for which $\pi_{i,in}$ is the power injected to system i, $\pi_{i,d}$ is the power lost due to dissipation in system i, π_{ij} is the net power flowing from system i to system j. The power balance for each of the systems results in a system of linear equations,

$$\{\mathbf{P}\} = [\mathbf{X}]\{\mathbf{E}\}, \tag{1}$$

where the matrix $[\mathbf{X}]$ consisting of coupling loss factors and damping factors is given by

$$[\mathbf{X}] = \begin{bmatrix} \eta_{1,t} & -\eta_{21} & -\eta_{31} & \cdots & -\eta_{n1} \\ -\eta_{12} & \eta_{2,t} & -\eta_{32} & \cdots & -\eta_{n2} \\ -\eta_{13} & -\eta_{23} & \eta_{3,t} & \cdots & -\eta_{n3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\eta_{1n} & -\eta_{2n} & -\eta_{3n} & \cdots & \eta_{n,t} \end{bmatrix}, \tag{2}$$

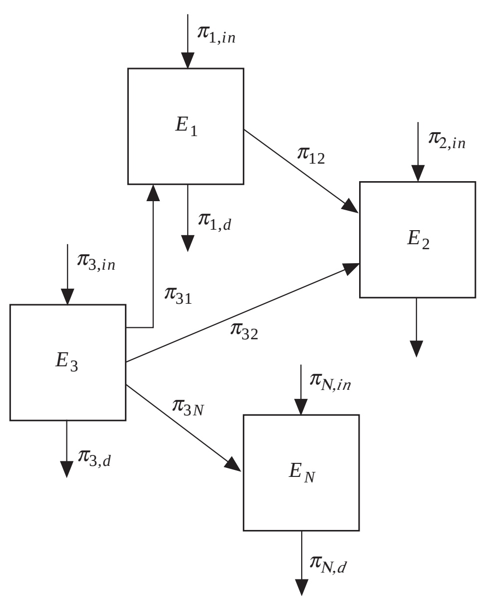

Figure 1. Power flow between connected systems.

$$\eta_{i,t} = \eta_i + \sum_{\substack{j=1 \\ j \neq i}}^{n} \eta_{ij}, \tag{3}$$

$\{\mathbf{P}\}$ is a column vector given by

$$\{\mathbf{P}\} = \begin{bmatrix} \pi_{1,in}/\omega_c \\ \pi_{2,in}/\omega_c \\ \vdots \\ \pi_{N,in}/\omega_c \end{bmatrix}, \tag{4}$$

and $\{\mathbf{E}\}$ is a column vector containing the total energy of each of the subsystems. $\eta_1, \eta_2 \ldots \eta_N$ are respectively loss factors of the N subsystems. η_{ij} is the coupling loss factor representing power flow from system i to system j and η_{ji} is the coupling loss factor representing power flow from system j to system i. From equation (1), the energy in each of the systems is given by

$$\{\mathbf{E}\} = [\mathbf{A}]\{\mathbf{P}\}, \tag{5}$$

where

$$[\mathbf{A}] = [\mathbf{X}]^{-1}. \tag{6}$$

The energy of each system is represented by $E_i = M_i\langle v_i^2\rangle$ for a vibrating system and $E_i = V_i\langle p_i^2\rangle/\rho c^2$ for an acoustic system; E_i is the energy, M_i is the mass, $\langle v_i^2\rangle$ is the space averaged mean square velocity in the frequency band of center frequency ω_c, V_i is the volume of the acoustic space, ρ is the density of the fluid medium filled in the acoustic space (generally air), c is the speed of sound in the fluid medium and $\langle p_i^2\rangle$ is the space averaged mean square sound pressure of the ith system in the frequency band of center frequency ω_c, where the parameters are appropriately used for vibratory and acoustic systems. If all the entries of $[\mathbf{X}]$ consisting of loss factors and coupling loss factors are known, and the input power is known, the energy of each subsystem can be determined from equation (5) and in turn the average velocity or pressure may be determined. In case coupling loss factors and loss factors are not known, they can be estimated by computing the energy of each subsystem for a known input power, by measuring the average velocity or pressure of each subsystem. This is the principle of the power injection method that is presented here. In this method, each subsystem will be excited one at a time, thereby generating a system of $N^2 \times N^2$ linear equations by relating the energy of the subsystems to the input power corresponding to N sets of experiments. Since the coupling loss factors and loss factors are frequency dependent, the above procedure will have to be repeated for all frequency bands (octave or 1/3 octave) of interest.

An experimental procedure which is based on measured values of power injection and energy of the subsystems (determined by measuring velocity or sound pressure) is presented in what follows, for determining coupling loss factors. A three subsystem model is chosen as an example and the procedure can be extended to any number of subsystems.

2.2. *Application to a three subsystems model*

Consider a three subsystems model which is subjected sequentially to power inputs $P_1^{(1)}$, $P_2^{(2)}$ and $P_3^{(3)}$ respectively. The loss factors of each of the subsystems are η_1, η_2 and η_3; coupling loss factors between subsystems are η_{12}, η_{21}, η_{13}, η_{31}, η_{23} and η_{32}. System 1 is first subjected to an input power $P_1^{(1)}$ and the energies of all the systems are measured and are, respectively, $E_1^{(1)}$, $E_2^{(1)}$ and $E_3^{(1)}$; the subscripts denote the subsystem number and the superscripts denote the experiment number. Similarly, the second experiment is conducted by injecting power $P_2^{(2)}$ to system 2 and measuring $E_1^{(2)}$, $E_2^{(2)}$ and $E_3^{(2)}$ and the third experiment is conducted by injecting power $P_3^{(3)}$ to system 3 and measuring $E_1^{(3)}$, $E_2^{(3)}$ and $E_3^{(3)}$. All the measurements are carried out in the frequency band of center frequency ω_c.

Equation (1) can be used to express the above measured parameters in terms of the loss and coupling loss factors as follows. Some row operations have been carried out to obtain a simplified matrix as:

$$\begin{bmatrix} E_1^{(1)} & E_2^{(1)} & E_3^{(1)} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & E_2^{(1)} & E_3^{(1)} & -E_1^{(1)} & E_2^{(1)} & -E_1^{(1)} & E_3^{(1)} & 0 & 0 \\ 0 & 0 & E_3^{(1)} & 0 & 0 & -E_1^{(1)} & E_3^{(1)} & -E_2^{(1)} & E_3^{(1)} \\ E_1^{(2)} & E_2^{(2)} & E_3^{(2)} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & E_2^{(2)} & E_3^{(2)} & -E_1^{(2)} & E_2^{(2)} & -E_1^{(2)} & E_3^{(2)} & 0 & 0 \\ 0 & 0 & E_3^{(2)} & 0 & 0 & -E_1^{(2)} & E_3^{(2)} & -E_2^{(2)} & E_3^{(2)} \\ E_1^{(3)} & E_2^{(3)} & E_3^{(3)} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & E_2^{(3)} & E_3^{(3)} & -E_1^{(3)} & E_2^{(3)} & -E_1^{(3)} & E_3^{(3)} & 0 & 0 \\ 0 & 0 & E_3^{(3)} & 0 & 0 & -E_1^{(3)} & E_3^{(3)} & -E_2^{(3)} & E_3^{(3)} \end{bmatrix} \begin{Bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_{12} \\ \eta_{21} \\ \eta_{13} \\ \eta_{31} \\ \eta_{23} \\ \eta_{32} \end{Bmatrix} = \begin{Bmatrix} P_1^{(1)}/\omega_c \\ 0 \\ 0 \\ P_2^{(2)}/\omega_c \\ P_2^{(2)}/\omega_c \\ 0 \\ P_3^{(3)}/\omega_c \\ P_3^{(3)}/\omega_c \\ P_3^{(3)}/\omega_c \end{Bmatrix} \tag{7}$$

In the above experiments it is presumed that the power input was calculated on the basis of an average input power over the extent of the subsystems so the local modal characteristics do not influence the amount of power input. Similarly, the energy of each system is the spatial average within the frequency band of center frequency ω_c. By solving the system of linear equations represented by equation (7), coupling loss factors and loss factors can be obtained. The matrix on the left side of equation (7) which contains energy terms will be referred to as the energy matrix in this paper. The above procedure of determining coupling loss factors and loss factors can be extended to other frequency bands of interest. The entire procedure can be extended to a system of any number of subsystems.

3. NUMERICAL RESULTS AND DISCUSSION

In order to test the numerical accuracy of inverting the energy matrix of equation (7), the condition number of the energy matrix will have to be computed. A numerical experiment was conducted by assuming a fictitious set of three subsystems corresponding to both weak coupling and strong coupling. For this model it was assumed that each of them are driven one at a time. An energy ratio ε is defined which is the ratio of the energy of a subsystem to the driven system and this is assumed to be the same for all the non-driven systems, in all the three numerical experiments when each of the subsystems are assumed to be driven one at a time. The above numerical procedure can be summarized as follows for a specific value of energy ratio of 0·05 for the non-driven systems.

(1) System 1 is driven first, and its energy $E_1^{(1)}$ can be assumed to be 1. The energy of system 2 and system 3 are respectively, $E_2^{(1)} = 0{\cdot}05$, $E_3^{(1)} = 0{\cdot}05$.

TABLE 1

Condition number of the energy matrix

No.	Energy ratio (ε)	Condition number	Error (dB)
1	0·05	5·3826	0·73
2	0·10	5·8944	0·7704
3	0·30	9·5020	0·9778
4	0·50	16·5558	1·2190
5	0·80	54·4306	1·7358

(2) System 2 is driven and its energy $E_2^{(2)}$ can be assumed to be 1. The energy of system 1 and system 3 are respectively, $E_1^{(2)} = 0{\cdot}05$, $E_3^{(2)} = 0{\cdot}05$.

(3) System 3 is driven and its energy $E_3^{(3)}$ can be assumed to be 1. The energy of system 1 and system 2 are respectively, $E_1^{(3)} = 0{\cdot}05$, $E_2^{(3)} = 0{\cdot}05$.

The above energy ratio of 0·05 corresponds to weak coupling between systems. The assumption of the same energy ratio for all the non-driven subsystems in all the three experiments is only hypothetical. The same procedure is repeated for other values of energy ratios such as 0·1, 0·3, 0·5, 0·8. The energy matrix in equation (7) is formulated for each of the energy ratios and the condition number of the energy matrix is computed. The results are as shown in Table 1. A higher energy ratio indicates stronger coupling.

Although the condition number increased corresponding to strong coupling, the increase was not significant enough to anticipate any numerical difficulty or errors in matrix inversion. As long as the coupling loss factors and loss factors are positive, which can only be confirmed by actual measurements, this method can be a useful approach for estimating coupling loss factors and loss factors. Since the condition number is sufficiently low even for a stronger coupling, it can be concluded that this method is sufficiently accurate for determining coupling loss factors and loss factors.

4. CONCLUSIONS

The method presented in this paper to determine loss factors and coupling loss factors does not have any numerical difficulty and is therefore useful for both strong and weak coupling. The only apprehension many would have is the accuracy of determining the input power, which in turn affects the accuracy of computing coupling loss factors and loss factors. By using an impedance head and shaker or instrumented hammer and accelerometer for vibration measurement and sound intensity probe for sound power measurement, it would be possible in most cases to calculate the power injected to the subsystems.

Bies and Hamid [3], who used the power injection method in 1980, had suggested some changes in the transducer setup that would match the impedance of the exciter to that of the driving point of the structure, for accurately estimating the input power. The phase mismatch problem seem to have been almost resolved in the recent past. References [4] and [5] do not mention any such problem with power measurement. Since phase is used as a very important parameter in many noise and vibration studies in the recent past, better instrumentation is available for determining the phase component of signals. This will be very helpful in accurately determining the input power.

The author of this paper is at present working on the experimental verification of the method of determining coupling and loss factors presented here for weak and strong coupled systems. In addition to experimental verification of the procedure for determining coupling loss factors, a comparative study is being carried out to determine whether excitation through a shaker or instrumented hammer will yield more accurate estimation of the input power. Both the results will be published in the near future.

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