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INFLUENCE OF AN ELECTRIC FIELD ON DIAPHRAGM STABILITY AND VIBRATION IN A CONDENSER MICROPHONE

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The electric field due to the constant bias voltage between the diaphragm and the base plate of a condenser microphone introduces a deflection dependent load intensity on the diaphragm. Since this load intensity increases with deflection, the electric field has a destabilizing influence on the plate vibration. Considering the diaphragm as a clamped circular plate, the dependence of the steady deflection and the natural vibration of the plate on the strength of the electric field is investigated. The analysis shows that as the field strength increases, the steady deflection also increases and reaches a neutral equilibrium state corresponding to a critical value of the field strength. Any further increase in the field strength buckles the diaphragm on to the base plate. The electric field decreases the fundamental natural frequency of vibration which approaches zero as the field strength reaches its critical value.

1. INTRODUCTION

In condenser microphones, the sound pressure impinging on its flexible diaphragm causes diaphragm deflection which changes the capacitance between the diaphragm and the rigid base plate. The changes in the capacitance are then converted to a voltage signal in an electrical circuit connected to the diaphragm and the base plate. Since the electric force field in the capacitor is always attractive, a bias voltage source is used in this circuit to maintain the potential difference between the diaphragm and the base plate to have the same sign.

The natural vibration of the diaphragm within the operating frequency range of the microphone can cause distortion in the output signal and hence, a vibration analysis of the diaphragm in the presence of the electric field is essential in the microphone design. The bias voltage between the diaphragm and the base plate can be considered as a constant for such natural vibration analysis.

In the absence of the electric field, exact solutions for the natural frequencies and modes of the clamped circular plate can be obtained in terms of Bessel functions [1, 2]. The electric field due to the constant bias voltage introduces a load intensity which increases with the plate deflection. The influence of electrostatic field on the static and dynamic deflections of circular membrane was studied by Warren *et al*. [3, 4] using numerical techniques. In their dynamic deflection analysis [4], they used an approximate thin air film model, developed by Hayasaka [5], to relate the reaction pressure of air in the gap to the dynamic deflection. A central difference numerical approach was then used to solve the coupled differential equations for the membrane and the air in the gap.

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An energy approach was used to study the influence of electrostatic field on the static deflection of rectangular plate type microphone diaphragms in references [6, 7], which showed that the static deflection of the plate increased with increasing electrostatic field. The vibration of a rectangular microphone diaphragm in the presence of electrostatic field was studied in reference [8] using the Rayleigh–Ritz method after linearizing the non-linear effects of the electrostatic field. While the assumed deflection shape accounted for the dynamic deflection only, the non-homogeneous terms resulting from following the Rayleigh–Ritz procedure on the linearized problem was also disregarded. As a result, it was wrongly concluded that the diaphragm stiffens with increasing strength of the electrostatic field.

Since the circular plate is a more realistic representation of the microphone diaphragm, the influence of the electric field on the circular diaphragm vibration is considered in the present study. The diaphragm is considered as a thin circular plate clamped at its periphery. Further, the differential equation of the plate is solved directly in order to examine the non-linear behavior and stability of the system. The vibration of the diaphragm is considered as a small perturbation from the static deflection due to the electrostatic field.

2. THEORY

The diaphragm of the microphone is modeled as a thin uniform circular plate which is clamped at its periphery as shown in Figure 1, for theoretical analysis. The electric field in the space between the diaphragm and the stationary plate introduces a deflection dependent load intensity $\varepsilon V^2/(d-w)^2$ on the diaphragm. The equation of motion of the diaphragm can be expressed as

$$D\nabla^4 w + m\partial^2 w/\partial t^2 = \varepsilon V^2/(d-w)^2. \tag{1}$$

For the present analysis, the applied voltage V between the diaphragm and the base plate is assumed to be constant. Using the non-dimensional polar co-ordinate (r, θ), which is related to the Cartesian co-ordinates (x, y) by the relations $x = ar\cos\theta$ and $y = ar\sin\theta$, equation (1) can be rewritten in non-dimensional form as

$$\bar{\nabla}^4\bar{w} + (ma^4\omega^2/D)\partial^2\bar{w}/\partial\tau^2 = \varepsilon a^4V^2/Dd^3(1-\bar{w})^2, \tag{2}$$

where

$$\bar{\nabla}^2 = \partial^2/\partial r^2 + (1/r)\partial/\partial r + (1/r^2)\partial^2/\partial\theta^2. \tag{3}$$

It is desirable to subdivide the investigation into three cases, namely (1) vibration in the absence of electric field, $V = 0$, (2) time independent deflection due to electric field and (3) vibration in the presence of electric field. The time independent deflection of the diaphragm in the case (2) due to the constant bias voltage is axisymmetric. Since the aim of the present analysis is the determination of the fundamental frequency of diaphragm

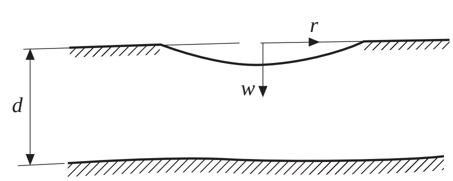

Figure 1. Diaphragm and base plate in a condenser microphone.

vibration, the circular plate deflections for the cases (1) and (3) are also considered to be axisymmetric.

2.1. CASE (1): AXISYMMETRICAL PLATE VIBRATION IN THE ABSENCE OF AN ELECTRIC FIELD

The equation of motion of the plate in the absence of an electric field is a special case of equation (1) corresponding to $V = 0$. Exact solutions for the pth axisymmetrical vibration mode $\bar{w} = \phi_{0p}(r)$ of a clamped circular plate can be expressed in terms of Bessel functions as [1, 2]

$$\phi_{0p}(r) = \mathrm{I}_0(\mu_p r)/\mathrm{I}_0(\mu_p) - \mathrm{J}_0(\mu_p r)/\mathrm{J}_0(\mu_p), \tag{4}$$

where

$$\mu_p = (ma^4\omega_p^{(0)2}/D)^{1/4}. \tag{5}$$

Here, μ_p is a non-dimensional parameter representing the square root of the natural frequency of the plate. The superscript $^{(0)}$ in the frequency symbol indicates the plate vibration condition in the absence of an electric field. The modal function set $\{\phi_{0p}(r)\}$ is orthogonal in the interval [0, 1] with respect to the weight function r.

Imposing the slope boundary condition at $r = 1$ on equation (4), the corresponding frequency equation can be simplified to

$$\mathrm{J}_0(\mu_p)\mathrm{I}_1(\mu_p) + \mathrm{J}_1(\mu_p)\mathrm{I}_0(\mu_p) = 0. \tag{6}$$

This frequency equation can be solved using the Newton–Raphson method [2] and the first positive root of the equation converges to $\mu_1 = 3{\cdot}19622$. Thus, the fundamental natural frequency of vibration of the plate in the absence of an electric field becomes, $\omega_1^{(0)} = 10{\cdot}21582(D/ma^4)^{1/2}$. This vibration frequency and mode can be used as a first approximation on the determination of the first natural frequency and mode of small vibration of the plate in the presence of an electric field.

2.2. CASE (2): TIME INDEPENDENT DEFLECTION OF PLATE DUE TO ELECTRIC FIELD

The time independent deflection of the plate is the special solution of equation (1), when the second term on the left side is not considered. Representing the time independent solution as $\bar{w}_s$, the governing equation becomes

$$\bar{\nabla}^4\bar{w}_s = \lambda/(1 - \bar{w}_s)^2, \tag{7}$$

where

$$\lambda = \varepsilon a^4 V^2/Dd^3. \tag{8}$$

The presence of the term on the right side of equation (7) renders it non-linear. The linearized form of this governing (7) for small $\bar{w}_s$ becomes

$$\bar{\nabla}^4\bar{w}_s - 2\lambda\bar{w}_s \approx \lambda, \tag{9}$$

which can be solved exactly in terms of Bessel functions as

$$\bar{w}_s \approx \tfrac{1}{2}\left\{\frac{\mathrm{I}_1(\lambda_c)\mathrm{J}_0(\lambda_c r) + \mathrm{J}_1(\lambda_c)\mathrm{I}_0(\lambda_c r)}{\mathrm{I}_1(\lambda_c)\mathrm{J}_0(\lambda_c) + \mathrm{J}_1(\lambda_c)\mathrm{I}_0(\lambda_c)} - 1\right\}, \tag{10}$$

where $\lambda_c = (2\lambda)^{1/4}$. When λ is small, the series expressions for Bessel functions give the first order approximation of equation (10) as $\bar{w}_s = \lambda(1 - r^2)^2/64 + \mathcal{O}(\lambda^2)$. This approximation indicates that, when λ is small, the time independent deflection $\bar{w}_s$ is of order λ and consequently, the solution (10) for $\bar{w}_s$ is reasonably accurate for small λ.

When λ is small, the solution (10) of the linearized equation (9) can be used as a first aproximation in an iterative scheme for the solution of the non-linear equation (7). In this iterative scheme, the improved $(i+1)$th solution is calculated from the assumed ith solution using the relation

$$\bar{\nabla}^4 \bar{w}_s^{(i+1)} = \lambda/(1-\bar{w}_s^{(i)})^2. \tag{11}$$

For the numerical solution of equation (11), the independent variable r in the range [0, 1] is divided into equal intervals and the $\bar{w}_s^{(i)}$ values at these points are used to compute the corresponding values of $\bar{w}_s^{(i+1)}$. A cubic spline approximation of the integrand is used to evaluate each of the four successive numerical integrations with respect to r associated with the operator $\bar{\nabla}^4$.

The approximate solution (10) is assumed as the first approximation for $\lambda = 0{\cdot}5$ to evaluate the solution of the non-linear equation (7). The λ values are progressively increased in steps of 0·1 and the non-linear solution corresponding to the previous λ, instead of the linear solution, is subsequently used as first approximation to improve the convergence rate. The iteration was continued until the relative error is less than 10^{-16}. However, the scheme fails to converge when λ exceeds a certain critical value.

Refining the λ intervals to 0·01 and 0·001, the largest value of λ for which the iteration scheme (11) converges is determined accurately to five significant places as 13·887 and the non-dimensional central deflection for this value of λ is found to be $\bar{w}_s = 0{\cdot}459205$. The computations are carried out on a VAX digital computer in quadruple precision and the convergence of the numerical results are confirmed by increasing the number of intervals in the radial direction. Convergence was obtained at 500 intervals; however, the results are presented using 1000 intervals.

Since the electrical force on the plate increases with the deflection, it can become strong enough to destabilize the plate when λ is large. When $\lambda = 13{\cdot}888$, the iterative scheme yields progressively increasing values of $\bar{w}_s$ which ultimately exceeds unity. Thus, when λ exceeds its critical value, it is reasonable to consider the diaphragm to buckle under the dominating influence of the destabilizing electric force.

The electric field pulls the diaphragm away from the equilibrium position and this destabilizing pull increases with the deflection. For small vibration about the steady deflection, the electric force field can be represented by a negative stiffness intensity given by $2\varepsilon V^2/(d-w_s)^3$. An increase in the steady deflection strengthens this negative stiffness which ultimately neutralizes the diaphragm stiffness at the critical field strength and buckles the diaphragm.

2.3. CASE (3): SMALL VIBRATION IN THE PRESENCE OF AN ELECTRIC FIELD

The time independent deflection of the plate due to electric field satisfies equation (7). Substituting the solution $\bar{w} = \bar{w}_s + \Delta\bar{w}\cos\tau$ for small vibration of the plate about this steady deflection into equation (2) and approximating the resulting equation to first order gives

$$\bar{\nabla}^4 \Delta\bar{w} - \Omega^2 \Delta\bar{w} = \Lambda_s \Delta\bar{w}, \tag{12}$$

where

$$\Omega = \omega a^2\sqrt{m/D}, \qquad \Lambda_s = 2\lambda/(1-\bar{w}_s)^3. \tag{13, 14}$$

Even though equation (12) governing small vibration is linear, the implicit dependence of Λ_s on r renders its closed form solution difficult. It can be seen from equation (14) that for sufficiently small $\bar{w}_s$ the parameter Λ_s on the right-side of equation (12) approximates to 2λ. Consequently, an approximate closed form expression for the plate frequency in the presence of weak electric field can be deduced from equation (12) as

$$\omega_p/\omega_p^{(0)} \approx \sqrt{\{1 - 2\lambda/\mu_p^4\}}. \tag{15}$$

The corresponding vibration modes become identical to that of the plate in the absence of the electric field given in equation (4).

The approximate natural frequency given in equation (15) shows that the electric field can destabilize the system when $\lambda \approx \frac{1}{2}\mu_1^4 \approx 52{\cdot}18$. However, the accurate solution of equation (7) for the steady deflection indicated that the destabilization occurs when λ exceeds 13·887. Thus, the approximate frequency, expressed in closed form in equation (15), is not accurate for those values of λ near this critical value.

When $\bar{w}_s$ is sufficiently large compared to the amplitude of vibration $\Delta\bar{w}$, equation (12) can be solved for the fundamental natural frequency and mode using the standard iterative method based on the successive determination of the deflection due to system flexibility corresponding to the inertia forces associated with an assumed deflection mode. Here, the assumed ith approximations $\Delta\bar{w}^{(i)}$ and $\Omega^{(i)}$, are used to calculate the intermediate $\Delta\bar{w}^{(c)}$ given by

$$\bar{\nabla}^4\Delta\bar{w}^{(c)} = \Lambda_s\Delta\bar{w}^{(c)} + \Omega^{(i)2}\Delta\bar{w}^{(i)} \tag{16}$$

and satisfy the clamped plate boundary conditions $\Delta\bar{w}^{(c)}(1) = 0$ and $\nabla\Delta\bar{w}^{(c)}(1) = 0$. Multiplication of equation (16) by the scale factor $\{\Delta\bar{w}^{(i)}(0)/\Delta\bar{w}^{(c)}(0)\}$ expresses the iterative scheme in the standard form of as

$$\bar{\nabla}^4\Delta\bar{w}^{(i+1)} - \Lambda_s\Delta\bar{w}^{(i+1)} = \Omega^{(i+1)2}\Delta\bar{w}^{(i)}, \tag{17}$$

where

$$\Omega^{(i+1)2} = \{\Delta\bar{w}^{(i)}(0)/\Delta\bar{w}^{(c)}(0)\}\Omega^{(i)2}, \qquad \Delta\bar{w}^{(i+1)} = \{\Delta\bar{w}^{(i)}(0)/\Delta\bar{w}^{(c)}(0)\}\Delta\bar{w}^{(c)}. \quad (18, 19)$$

In this scheme the $(i+1)$th approximation is the deflection due to the combined influence of the plate elasticity and the electric field for the inertia force associated with the ith approximation. It can be noticed that $\Delta\bar{w}^{(i+1)}$ satisfies the clamped plate boundary conditions and the requirement $\Delta\bar{w}^{(i+1)}(0) = \Delta\bar{w}^{(i)}(0)$. Thus, the actual iterative scheme used in the determination of the fundamental frequency and mode is given by equation (17) and the additional equations (16, 18, 19) are included to clarify the numerical method used in the implementation of this scheme.

The dependence of Λ_s on r through $\bar{w}_s$ renders the determination of $\Delta\bar{w}^{(c)}$, which satisfies equation (16) and the clamped boundary condition, difficult. Using an assumed approximation for $\Delta\bar{w}^{(c)}$ in the right side of equation (16), an improved approximation for $\Delta\bar{w}^{(c)}$ is determined by four successive numerical integrations. Here, $\Delta\bar{w}^{(i)}$ is used as the first approximation, and the successive approximations for $\Delta\bar{w}^{(c)}$ are calculated until the relative error is less than 10^{-8}. The improved $\Omega^{(i+1)}$ and $\Delta w^{(i+1)}$ which satisfy equation (17) are then evaluated from equations (18) and (19). The iterative scheme (17) for the determination of the fundamental frequency is continued until the relative error is less than 10^{-8}, and iteration converges for all values of λ, including those which are slightly less than the critical value for snap instability. The computations are carried out on a VAX digital computer in quadruple precision and the convergence is validated using 1000 radial intervals for the numerical integration.

Using finite difference expression for $\bar{\nabla}^4\Delta\bar{w}$, equation (17) can be expressed as a matrix eigenvalue problem for determination of the natural frequencies and nodal deflection vectors. In such an expression, the second term on the left side due to the electric field introduces negative elements in the leading diagonal of the stiffness matrix. The numerical

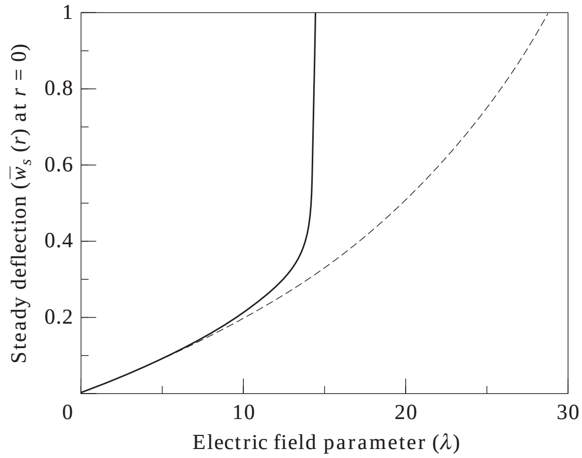

Figure 2. Variation of steady central deflection with electric field: ——, equation (7); - - - -, equation (10).

values of these negative elements increases with λ and thereby reduces the determinant of the stiffness matrix which ultimately vanishes as the parameter λ approaches its critical value. This formulation as a matrix eigenvalue problem involves very large order matrices and is not suitable for numerical computation. However, it emphasizes the fact that the iterative scheme (17) must lead to convergent solutions for the fundamental frequency and mode.

3. RESULTS AND DISCUSSION

The diaphragm of a microphone can be represented as a thin clamped circular plate. The exact solution for vibration frequencies and modes of clamped circular plate in terms of Bessel functions is well known. However, the presence of an electric field between the diaphragm and the rigid base plate renders the vibration analysis of the diaphragm difficult. The electric field induces a deflection dependent load intensity on the diaphragm, which introduces non-linearity in the governing equation (1). The diaphragm deflection has a time independent steady state solution represented by equation (7) and the small vibrations of the diaphragm about this steady state is studied using equation (12).

The load intensity due to the constant applied voltage between the plates causes a steady deflection of the diaphragm. The parameter λ, defined by $\varepsilon a^4 V^2/Dd^3$, represents the square of this applied voltage in non-dimensional form. This parameter is assumed to be constant for the present analysis. The dependence of the axisymmetric steady deflection of the plate on the parameter λ is expressed in equation (7), which is solved using an iterative procedure. For small λ, the approximate solution given in equation (10) is used as a first approximation to start the iterations.

The variation of non-dimensional deflection $\bar{w}_s$ at the center of the plate against the parameter λ is shown in Figure 2. The approximate solution (10), based on the assumption that λ is small, is also included for the purpose of comparison. The solution of equation (7) shows that the central deflection increases with λ until it reaches about 45·92% of the gap between the plates. Any further increase in λ beyond its critical value of 13·887 causes the plate to buckle. However, the approximate solution (10), which is valid for small λ, increases with λ, and reaches unity when $\lambda = 28{\cdot}44$. Thus, this solution of the linearized equation (7) does not indicate the instability at the critical strength of the electric field.

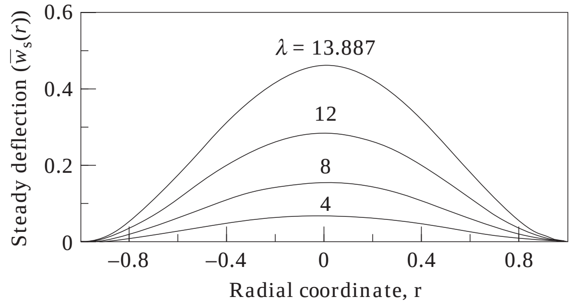

Figure 3. Variation of steady deflection shape with the electric field along a diameter.

A comparison with the solution of the non-linear equation (7) shows that this approximate solution underestimates the plate deflection and the error reaches 10% when $\lambda \approx 11{\cdot}036$. However, the approximate solution is in 34·87% error at the critical value of λ. Thus, the approximate closed form solution is reasonably accurate for microphone design purposes. The exact steady deflection shapes of the plate corresponding to λ values of 4, 8, 12 and 13·887 are shown in Figure 3.

The small vibration of the plate about its steady deflected position is governed by equation (12), which is solved by iterative steps given in equations (16–19). The variation of the fundamental frequency of vibration with λ is shown in Figure 4. The approximate solution (15) for this frequency is also included in Figure 4 for comparison. The destabilizing electric field decreases the natural frequency of plate vibration. Further, as λ approaches its critical value, this natural frequency approaches zero, indicating the buckling instability condition. The approximate solution (15) for the frequency of vibration always overestimates the fundamental frequency and is at 10% error when $\lambda = 10{\cdot}27$. This closed form solution is also reasonably accurate for design calculations. However, this solution becomes inaccurate near the critical value of λ associated with the buckling instability.

Within the accuracy of the numerical scheme (16–19), the small vibration mode shapes of the plate in the presence of the electric field always converges to the normalized vibration mode given by equation (4). Thus, the electric field does not have an appreciable influence

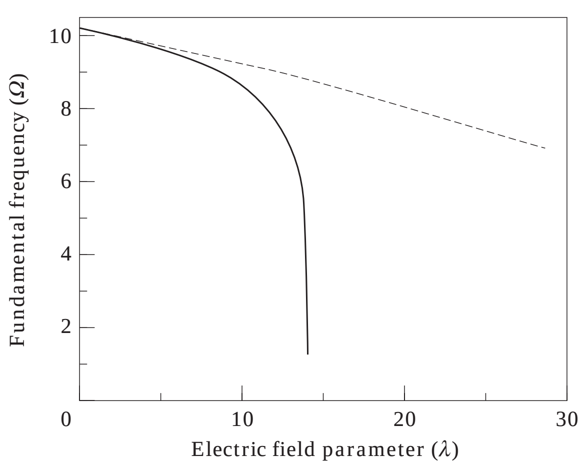

Figure 4. Variation of fundamental frequency with the electric field: ——, equation (12); - - - -, equation (15).

on the fundamental mode of small vibration. Since $\bar{\nabla}^4\phi_{01}(r) = \mu_{01}^4\phi_{01}(r)$, a careful re-examination of equation (12) reveals that the solution for $\Delta\bar{w}$ cannot be completely independent of the electric field. Substitution of the series solution $\Delta\bar{w} = \Sigma c_i\phi_{0i}(r)$ into equation (12) simplifies it to the matrix eigenvalue problem

$$\sum_k (\mu_{0i}^4\delta_{ik} - \Lambda_{ik}^{(eq)})c_k = \Omega^2 c_i, \tag{20}$$

where

$$\Lambda_{ik}^{(eq)} = \int_0^1 \Lambda_s\phi_{0i}(r)\phi_{0k}(r)r\,\mathrm{d}r \tag{21}$$

and δ_{ik} is the Kronecker delta. In this formulation, the negatives of $\{\Lambda_{ik}^{(eq)}\}$ represents the elements of the ''stiffness matrix'' due to the electric field. The numerical result for the fundamental mode indicates that c_2, c_3, etc., are small in comparison to c_1, and consequently equation (20) can be used to obtain an approximate expression for the fundamental frequency, $\Omega \approx \sqrt{\mu_{01}^4 - \Lambda_{11}^{(eq)}}$, and further discussion on the numerical accuracy of this expression falls outside the scope of the present investigation.

The steady deflection of the plate in the presence of the electric field has a unique stable solution which is axisymmetric. However, the small vibration modes governed by equation (12) need not necessarily be axisymmetric. Equation (12) can still be used to determine the other modes and frequencies of small vibration of the plate in the presence of the electric field. Since the derivative of the vibration amplitude with respect to θ appears only in the operator $\bar{\nabla}^4$ in equation (12), the dependence of the vibration mode on θ can be handled by assuming the modes with n diagonals as $\Delta\bar{w} = \Delta\bar{w}_n \cos n\vartheta$ and the lowest natural frequencies and modes in each of these mode categories can be similarly determined. However, the approximate numerical methods such as the Rayleigh–Ritz, are more appropriate for the determination of a large number of these other natural frequencies and modes of the plate in the presence of the electric field.

4. CONCLUSION

Vibration of the microphone diaphragm subjected to an electric force field, which induces a deflection dependent load intensity on the plate has been analyzed in two steps.

The steady deflection due to the non-linear load intensity is considered in the first step. The electric field causes a steady plate deflection which increases with the field strength. However, when the field strength reaches a critical value, it destabilizes this rest position associated with the steady deflection and thereby causes the plate to buckle.

When the system has a stable rest position, the dependence of its small vibration about this position on the electric field is studied in the next step. The fundamental frequency and the corresponding vibration mode are computed. The results show that, as the field strength increases, the frequency decreases and approaches zero as the field approaches its critical value corresponding to the buckling instability.

The negative stiffness intensity due to the electric field has a destabilizing influence which is indicated by the reduction in natural frequency. At the critical field strength, the negative stiffness due to the field completely neutralizes the plate stiffness and brings the system to a neutral equilibrium state. When the plate is displaced from the neutral equilibrium state towards the other plate, the electric field pulls it to buckle.

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NOMENCLATURE

a	plate radius
d	gap between diaphragm and base plate when $w = 0$
D	flexural rigidity of plate, $D = Eh^3/12(1 - v^2)$
E	Young's Modulus
h	plate thickness
m	mass per unit area of plate
r	non-dimensional radial co-ordinate
t	time
V	voltage between the plates
w	plate deflection
$\bar{w}$	non-dimensional deflection, $\bar{w} = w/d$
w_s	steady deflection of plate
$\bar{w}_s$	non-dimensional steady deflection, $\bar{w}_s = w_s/d$
Δw	amplitude of small vibration
$\Delta\bar{w}$	non-dimensional vibration amplitude, $\Delta\bar{w} = \Delta w/d$
ε	permittivity of air
λ	non-dimensional parameter representing the square of voltage expressed in equation (8)
Λ_s	defined in equation (14)
μ_p	defined in equation (5)
v	Poisson ratio
τ	non-dimensional time, $\tau = \omega t$
ω	frequency of vibration
$\omega^{(0)}$	frequency of vibration when $\lambda = 0$
Ω	non-dimensional frequency defined in equation (13)