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LETTERS TO THE EDITOR

COMMENTS ON "JOINT STIFFNESS CONTROL OF A ONE-LINK FLEXIBLE ARM"

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In a recent article [1], Pun and Semercigil successfully applied a joint variable stiffness control (VSC) scheme to a one-link flexible arm. A standard finite element method was used to discretize the continuous system and produce global matrices. The Newmark-β method was applied to integrate the global matrices. The numerical results of the joint angle and the tip deflection were presented [1] as surface plots for varying payload mass, link length, joint stiffness and damping. Pun and Semercigil also demonstrated the effectiveness of VSC in suppressing the transient vibration of the one-link flexible arm. In this note, aspects of the numerical integration and the boundary condition on the flexible arm will be discussed.

Frequency equation. The frequency equation of a flexible slewing link can be written in a dimensionless form [2, 3] (see reference [3] for symbols and illustration)

$$(\mathrm{csh}-\mathrm{sch})-2\alpha_1\lambda\,\mathrm{ssh}-\beta_0\lambda^3(1+\mathrm{cch})-2\beta_1\lambda^3\,\mathrm{cch}-\alpha_1\lambda^4(\beta_0+\beta_1)(\mathrm{csh}-\mathrm{sch}) + \beta_0\beta_1\lambda^6(\mathrm{csh}+\mathrm{sch})-\beta_0\beta_1\alpha_1\lambda^7(1-\mathrm{cch})=0, \quad (1)$$

in which $\mathrm{c}=\cos\lambda$, $\mathrm{s}=\sin\lambda$, $\mathrm{ch}=\cosh\lambda$ and $\mathrm{sh}=\sinh\lambda$, while $\beta_0=J_0/(\rho A l^3)$, $\beta_1=J_p/(\rho A l^3)$ and $\alpha_1=M_p/(\rho A l)$.

Equation (1) can be re-arranged as:

$$(\mathrm{csh}-\mathrm{sch}-2\alpha_1\lambda\,\mathrm{ssh})/\beta_0-\lambda^3(1+\mathrm{cch})-2\beta_1/\beta_0\lambda^3\,\mathrm{cch}-\alpha_1\lambda^4(1+\beta_1/\beta_0)(\mathrm{csh}-\mathrm{sch}) + \beta_1\lambda^6(\mathrm{csh}+\mathrm{sch})-\beta_1\alpha_1\lambda^7(1-\mathrm{cch})=0. \quad (2)$$

For cases associated with a cantilever base ($\beta_0\approx\infty$), equation (2) reduces to [see reference 4]

$$(1+\beta_1\alpha_1\lambda^4)+(1-\beta_1\alpha_1\lambda^4)\,\mathrm{cch}+\alpha_1\lambda(\mathrm{csh}-\mathrm{sch})-\beta_1\lambda^3(\mathrm{csh}+\mathrm{sch})=0. \quad (3)$$

If a lumped mass is attached to the flexible link ($\beta_1=0$), equation (3) can be further simplified to

$$(1+\cos\lambda_n\cosh\lambda_n)+\alpha_1\lambda_n(\cos\lambda_n\sinh\lambda_n-\sin\lambda_n\cosh\lambda_n)=0, \quad (4)$$

where λ_n is the eigenparameter corresponding to a tip-loaded uniform beam with a cantilever base [references 4–6], where n is the mode number. The mode frequency in Hertz is then $\omega_n=\beta_n^2/(2\pi)\sqrt{EI/(\rho A)}$ and $\lambda_n=\beta_n l$.

Numerical integration. Hamilton's principle has been utilized in the dynamic analysis of flexible manipulator arms [7–9]. Numerical integration methods are often used to obtain the system response. It is well-known that the accuracy of the solution through a numerical scheme depends upon the time step, Δt. A common rule of thumb: $\Delta t/\tau\leqslant 1/10$, which usually provides satisfactory results, was suggested in reference [10]. The condition that is required to yield the convergent solutions of vibratory systems by using different

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numerical integration methods was investigated in references [11, 12]. Three methods were each used in the response analysis of discrete and continuous systems [11]. It was found that the required coefficient $\Delta t/\tau$ for satisfactory results was higher than that suggested by Clough and Penzien [10]. The values were found to be respectively 0·318, 0·456 and 0·55 for linear systems, if the methods of central difference, fourth order Runge–Kutta, and Newmark-β were used. A smaller value was required for a non-linear vibratory systems [11]. Moreover, in a stability study of the rotating beam system considered, it was confirmed that the choice of the time step would depend essentially on the least period of vibration, τ_n. For example, the selected time step of the rotating beam system was based on the third mode frequency, $\omega_3 = 501{\cdot}3$ Hz [11].

Pun and Semercigil used the Newmark-β method to integrate the global matrices of the one-link flexible arm [1]. Each integration time step is 0·005 s, which corresponds to approximately 1/41 of the fundamental oscillation period of the system ($\omega = 4{\cdot}9$ Hz).

The frequency equation can also be used to determine the appropriate time step. The first two natural frequencies obtained by solving equation (4) are, respectively, 8·6 and 95·2 Hz, if the parameters given in Table 1 of reference [1] are used: $l = 0{\cdot}5$ m, $\rho A = 0{\cdot}85$ kg/m, $M = 0{\cdot}5$ kg and $EI = 73\ \mathrm{Nm}^2$. Note that the results were obtained without considering the elbow stiffness and damping.

Based on $\omega_2 = 95{\cdot}2$ Hz (thus $\tau_2 = 0{\cdot}0105$ s) and $\Delta t/\tau_2 = 0{\cdot}55$ for the Newmark-β method, the appropriate time step can be found: $\Delta t = 0{\cdot}0058$ s. The value is near to the value 0·005 s used in reference [1].

Boundary conditions. The effects of joint stiffness and payload's mass to the vibration of the flexible arm were studied and presented as surface plots [1]. As the stiffness increases, the flexibility decreases until the joint begins to lock up and produces a near-cantilever response. Also, the fundamental frequency decreases and the system becomes more flexible as the tip mass increases.

The rotating beam system is often modelled as an Euler–Bernoulli beam with clamped–free or pinned–free boundary conditions. Several works [2, 3, 6, 13–15] have addressed an important issue, that of selecting sets of modes for problems of elastic beams that undergo large rigid body displacements. Experimental results [13, 14] suggested that the exact natural frequencies are intermediate between the clamped and pinned cases. It was concluded in a recent work [3] that the parameters of hub and payload should be incorporated for an accurate model, regardless of the use of the pseudo-clamped or the pseudo-pinned case.

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