Journal of Sound and Vibration (1998) **212**(1), 37–59

TRANSIENT BEHAVIOUR OF ACOUSTIC GYROMETERS

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(*Received* 8 *October* 1997)

Acoustic gyrometry has been developed during the past 15 years as a new miniaturized or low-cost technology. The operation of acoustic gyrometers employs acoustic fields inside fluid-filled resonant cavities to determine angular velocities. Until now, research efforts and design methodology have concentrated both on trapezoidal miniaturized gyros (etched on silicon chips) as well as on cylindrical gyros designed by using classical techniques. These approaches are restricted to the frequency domain which involves only the steady Coriolis effect (constant rotation rate). Nowadays, the need for a time domain solution for inertial effects on acoustic fields clearly arises when dealing with applications involving strong variations of the rotation rates. This paper aims at providing such advances in "inertial–acoustic" theory and modelling. The discussion covers cylindrical gyros because the presence of an unsteady rotational velocity gradient (with respect to the radial co-ordinate) of a gas in a cylindrical cavity adds one of the most important features both in the basic physics underlying "inertial–acoustic" transient processes and in the behaviour of the gyros.

1. INTRODUCTION

In recent years, there has been a very strong motivation for designing new sensors involving smaller dimensions, lower manufacturing cost and lower power consumption, as well as higher reliability and improved lifetime. During the past decade, work has been carried out on a new kind of rate gyro, the acoustic gyro, which may be of practical use where simplicity, reliability, miniaturization or low cost is more important than the highest sensitivity. Attempts to develop practical devices have been concentrated both on miniaturized gyros etched on silicon chips as well as on gyros designed by using classical techniques.

The operation of acoustic gyrometers employs acoustic fields inside fluid-filled resonant cavities to determine angular velocities. Until now, research efforts have been concentrated both on the inertial effects on acoustic fields inside trapezoidal ultra-miniaturized cavities (on silicon chips) or inside rectangular and cylindrical cavities (typically less than 1 cm^3) [1–3], as well as on the behaviour of rectangular miniaturized microphones and loudspeakers [4]. But progress towards a theoretical description of the phenomena has been restricted to the frequency domain which involves only steady angular velocity (i.e., constant rotation rate). A deeper understanding requires investigation in the time domain. Apart from this, practical applications sought for this kind of gyro are additional reasons that serve as motivation to understand in some detail the complex phenomena that are involved in the transient processes (for strong variations of the rotation rates) and then

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to interpret experimental results. This paper aims at providing such advances in "inertial–acoustic" theory and modelling. The discussion covers cylindrical gyros because the presence of an unsteady velocity, which depends on the location (r, z) of the particle of the fluid considered in a cylindrical cavity and which is imposed by the transient variation of the rotation of the walls around the axis of the cavity, adds one of the most important features both in the basic physics underlying "inertial–acoustic" transient processes and in the behaviour of the gyros.

2. BRIEF REVIEW OF THE BEHAVIOUR OF THE STATIONARY ROTATING GYRO

A rate gyro provides signals that are measures of angular rates with respect to an inertial frame. The heart of the acoustic gyro under consideration in this paper (see Figure 1) is a thin cylindrical cavity, typically 1·5 cm in diameter with a height h much less than half the wavelength (0·4 cm for example), filled with a suitable working fluid [1] (SF6 under 2·5 bars for example). Using an acoustic driver coupled to the cavity through a hole roughly 0·05 cm in diameter set at the point $\varphi = 0$, the fluid within the cavity is excited to generate an acoustic standing wave, corresponding typically to the resonance of the first azimuthal mode labelled C_{010} and given by the eigenfunction $J_1(\gamma_{01} r/R_0)\cos\varphi$, where J_1 is the first order cylindrical Bessel function of the first kind, γ_{01} the first zero of the first derivative of $J_1(\gamma_{01} r/R_0)$ with respect to the radial co-ordinate r, R_0 the radius of the cavity and φ the azimuthal co-ordinate (a list of symbols is given in the Appendix). Actually, the viscous and thermal dissipation in the boundary layers are taken into account in the formalism. When the cavity rotates around the z-axis of the cylindrical cavity, the angular velocity $\mathbf{\Omega}$ of the non-inertial frame linked to the cavity, with respect to the inertial frame, being constant ($\mathrm{d}\mathbf{\Omega}/\mathrm{d}t = 0$), the Coriolis effect on the acoustic field is the only one to be considered, as the other terms vanish as far as the linear approximation is valid (the acceleration linked to the time rate of change of the angular velocity is equal to zero and the centripetal acceleration, proportional to $\mathbf{\Omega}^2$ times the particle displacement, is negligible in most applications). This Coriolis effect leads to energy transfer from the mode C_{010} mentioned above to the orthogonal mode labelled S_{010} and given by the eigenfunction $J_1(\gamma_{01} r/R_0)\sin\varphi$, which can be measured with a microphone set at the point $\varphi = \pi/2$, even if its amplitude is much lower than the amplitude of the primary mode C_{010} generated by the loudspeaker (which is null at $\varphi = \pi/2$).

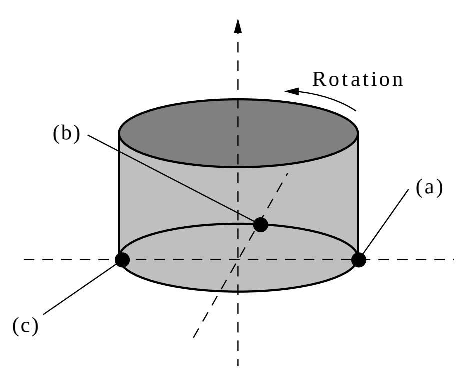

Figure 1. Schematic view of the cavity of the gyrometer: (a) the loudspeaker located at $\varphi = 0$; (b) the measurement microphone located at $\varphi = \pi/2$; (c) the reference microphone.

These two modes C_{010} and S_{010} are almost sufficient to describe a stationary solution for the steady state acoustic response of the rotating cavity, in the "ideal" case of a perfectly shaped cavity. Actually, in order to take into account unavoidable small perturbations, more terms must be included into the eigenfunctions expansion which is used to describe the acoustic field [2].

Moreover, the Coriolis force $\mathbf{f}_c$ can be interpreted in the wave equation as a source term given by $\operatorname{div}\mathbf{f}_c = 2\operatorname{div}(\mathbf{\Omega}\times\mathbf{v}) = -2\mathbf{\Omega}\,.\,\mathbf{curl}\,\mathbf{v} = -2\mathbf{\Omega}\,.\,\mathbf{curl}\,v_v$, emphasizing that the only vortical component v_v of the particle velocity v is involved in the Coriolis coupling (the acoustic and entropic components of the particle velocity do not play any direct role in the inertial coupling process). This vortical component is negligible except inside the viscous boundary layers, which therefore play an important role in the process. Hence, the energy transfer from the mode C_{010} to the mode S_{010} due to the Coriolis phenomena takes place only in the very thin boundary layers or, in other words, the equivalent "Coriolis sources" are distributed only at the boundaries in such a way that the mode S_{010} is generated. On the other hand, the strength of this "Coriolis resonant field" created in the cavity is proportional to both the amplitude of the primary resonant mode C_{010} generated by the loudspeaker and the rotation rate of the cavity [1, 2]. Then the transfer function between the output signal of the microphone set at $\varphi = \pi/2$, which measures the amplitude of the Coriolis mode S_{010}, and the output signal of another microphone set at $\varphi = \pi$, which measures the amplitude of the primary mode C_{010}, provides the value of the rotation rate $\mathbf{\Omega}$.

The theory developed in the following sections, devoted to the description of the inertial–acoustic coupling, when strong unsteady and r-dependent variations of the rotation rates (which depend both on the time and the location in the cavity) occur, can be considered in some respect as an extension beyond the steady state "theory" briefly mentioned above (which assumed a constant rotation rate). Therefore, details needed to understand more deeply the preceding summary, describing stationary phenomena, are included in the remainder of the paper.

3. THE GENERAL PROBLEM

3.1. THE BASIC PROBLEM

The cavity studied here is a cylindrical one (height h, radius R_0), in which transducers are flush-mounted to the base ($z = 0$) near the circular lateral wall ($r \cong R_0$). We shall use an inertial reference frame whose origin is located at the centre of the base of the cylinder. The natural co-ordinates chosen are Cartesian (X, Y, Z) or cylindrical (r, θ, z) and the (Oz/OZ) axis is aligned along the geometric axis of the cylinder, which itself is coincident with the rotation vector of the moving cavity $\mathbf{\Omega}$ (see Figure 2).

The cavity is excited by the loudspeaker on its first acoustic azimuthal mode (0, 1, 0), in such a way that the measurement microphone, which is located at a right angle from the loudspeaker, sets on a node of pressure of that mode when the cavity is at rest.

At time $t = 0$, the cavity is set impulsively in rotation, with its angular speed being brought from zero to $\mathbf{\Omega}_0$ (see Figure 3). Then, the fluid motion goes through a transient stage where particles are gradually driven by the walls, acquiring the angular velocity $\mathbf{\Omega}(r, z, t)$, up to reaching the ultimate state of rigid body rotation (when the cylinder and the fluid rotate together with the uniform angular velocity $\mathbf{\Omega}_0$). As the system has an axial symmetry, this angular velocity is assumed to be independent of the θ-co-ordinate.

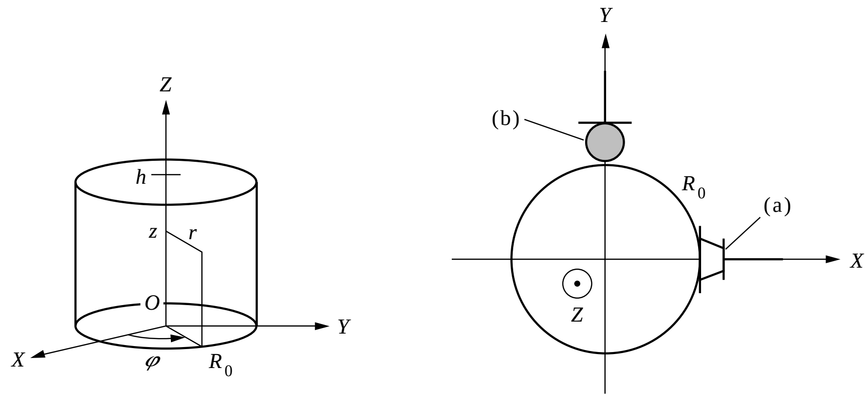

Figure 2. Schematic views of the cavity showing the systems of Cartesian and cylindrical co-ordinates, the loudspeaker (a) and the measurement microphone (b).

3.2. GENERAL EQUATIONS OF MOTION

The motion of the fluid can be described as the superposition of a circular flow (here linked to the boundary conditions expressing the rotation of the cavity (see equation (14))) and an acoustic perturbation generated by the acoustic source which is the rate of mass density creation $\rho_T q_s$. It is governed by the following set of fundamental equations: the Navier-Stokes equations,

$$\rho_T \, \mathrm{d}_t \, \boldsymbol{V}_T = \rho_T [\partial_t \, \boldsymbol{V}_T + (\boldsymbol{V}_T \cdot \mathbf{grad}) \boldsymbol{V}_T] = -\mathbf{grad} \, P_T + \mu \boldsymbol{\Delta} \boldsymbol{V}_T + \left(\eta + \frac{\mu}{3} \right) \mathbf{grad} \, (\mathrm{div} \, \boldsymbol{V}_T); \quad (1)$$

the conservation of mass equation,

$$\mathrm{d}_t \, \rho_T + \rho_T \, \mathrm{div} \, (\boldsymbol{V}_T) = \rho_T \, q_s \, ; \quad (2)$$

the heat conduction equation,

$$\rho_T \, T \, \mathrm{d}_t \, S_T = \lambda \boldsymbol{\Delta} T. \quad (3)$$

Here $\boldsymbol{V}_T$, ρ_T, and P_T are respectively the particle velocity, the density and the pressure associated to the fluid motion, λ, μ and η are respectively the coefficients of thermal conductivity, shear viscosity and bulk viscosity of the fluid, q_s being the rate of creation of fluid per unit volume of the loudspeaker, and S_T and T being respectively the entropy per unit volume and the temperature.

The study presented in the next section (3.3) concerns only the circular flow: that is, the motion of the fluid driven by the walls of the cavity in the absence of any acoustic source.

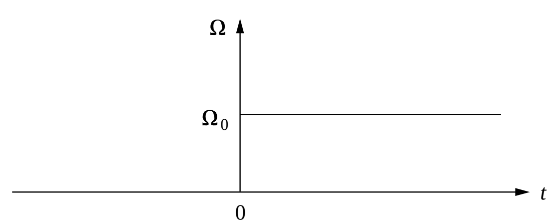

Figure 3. Angular velocity of the cavity walls as a function of time.

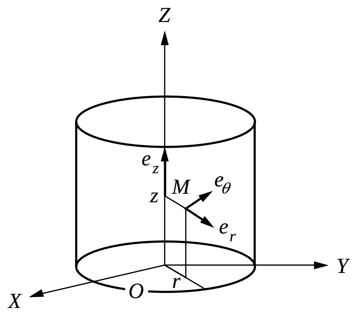

Figure 4. System of cylindrical co-ordinates.

This study enables one to determine the expression of the angular velocity $\Omega(r, z, t)$ of the fluid as function of position in space (r, z) and time (t) during the transient period.

3.3. THE UNSTEADY CIRCULAR FLOW

In this section, the fluid motion is studied in the absence of any acoustic source. A particle of fluid at the point **M** is located by its position vector

$$\mathbf{OM} = r\mathbf{e}_r + z\mathbf{e}_z, \tag{4}$$

where $(\mathbf{e}_r, \mathbf{e}_\theta, \mathbf{e}_z)$ are the three unit vectors of the system of cylindrical co-ordinates chosen (see Figure 4).

When the cavity is suddenly set in rotation around the (Oz) axis, the fluid element acquires a velocity $\boldsymbol{V}(r, z, t) = \mathrm{d}_t\,\mathbf{OM}$ and an acceleration which can be written (with the Eulerian description) as

$$\mathrm{d}_t\,\boldsymbol{V}(r, z, t) = \mathrm{d}^2_{tt}\,\mathbf{OM} = \partial_t\,\boldsymbol{V} + (\boldsymbol{V} \cdot \mathbf{grad})\boldsymbol{V}. \tag{5}$$

The set of fundamental equations governing the fluid motion (which are just particular cases of equations (1) and (2)) is the following:
the Navier-Stokes equation

$$\mathrm{d}_t\,\boldsymbol{V} = -\frac{1}{\rho_E}\,\mathbf{grad}\,P + v\boldsymbol{\Delta}\boldsymbol{V} + \left(\frac{\eta}{\rho_E} + \frac{v}{3}\right)\mathbf{grad}\,(\mathrm{div}\,\boldsymbol{V}); \tag{6}$$

the conservation of mass equation,

$$\mathrm{d}_t\,\rho_E + \rho_E\,\mathrm{div}\,(\boldsymbol{V}) = 0. \tag{7}$$

Here P is the pressure of the fluid, ρ_E its density and $v = \mu/\rho_E$ its coefficient of kinematic viscosity.

By neglecting the fluid compressibility (density is supposed to be independent of spatial co-ordinates and time: $\rho_E \cong \rho_0$) and by assuming consequently that the radial component V_r of the particle velocity $\boldsymbol{V}$ vanishes, this last equation (7) simplifies to

$$\mathrm{div}\,\boldsymbol{V} = 0. \tag{8}$$

This neglect of compressibility and radial velocity here is reasonable because to a first order the fluid motion caused by the cavity's rotation will be a pure shear flow.

Equation (8) can be written with the cylindrical co-ordinates (r, θ, z) as

$$\frac{1}{r}\,\partial_r\,(rV_r) + \frac{1}{r}\,\partial_\theta\,V_\theta + \partial_z\,V_z = 0, \quad \text{that is} \quad \frac{1}{r}\,\partial_r\,(rV_r) = 0, \tag{9}$$

because of the cylindrical symmetry ($\boldsymbol{V}$ is independent of θ and $V_z = 0$). The solution of this equation can be formulated as $V_r = C/r$, where the constant C must be equal to zero in order to satisfy the condition $V_r = 0$ on the walls (at $r = R_0$).

Therefore, the velocity $\boldsymbol{V}$ has only one non-vanishing component $V_\theta\,(r, z, t)$, which denotes a pure shear motion in conformity with the hypothesis chosen, and which leads to the expression

$$\boldsymbol{V}(r, z, t) = \mathrm{d}_t\,\mathbf{OM} = \boldsymbol{\Omega}(r, z, t) \times \mathbf{OM} = \Omega(r, z, t) r \mathbf{e}_\theta. \tag{10}$$

Note that this relationship satisfies equation (8):

$$\operatorname{div} \boldsymbol{V} = \operatorname{div}\,(\boldsymbol{\Omega} \times \mathbf{OM}) = \frac{1}{r}\,\partial_\theta\,(r\boldsymbol{\Omega}(r, z, t)) = 0. \tag{11}$$

Then, the acceleration of the fluid element can be expressed as

$$\mathrm{d}_t\,\boldsymbol{V} = \mathrm{d}_{tt}^2\,\mathbf{OM} = \mathrm{d}_t\,\boldsymbol{\Omega} \times \mathbf{OM} + \boldsymbol{\Omega} \times \mathrm{d}_t\,\mathbf{OM} = \mathrm{d}_t\,\boldsymbol{\Omega} \times \mathbf{OM} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{OM}), \tag{12}$$

where the first expression ($\mathrm{d}_t\,\boldsymbol{\Omega} \times \mathbf{OM}$) represents the angular acceleration of the particle and the second one ($\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{OM})$) the centripetal acceleration. One can easily verify that this last expression is equivalent to equation (5).

Finally, the radial and azimuthal components of the Navier-Stokes equations can be written respectively as

$$\frac{V_\theta^2}{r} = \frac{1}{\rho_0}\,\partial_r\,P, \qquad \partial_t\,V_\theta = \nu\left[\partial_{rr}^2 + \frac{1}{r}\,\partial_r - \frac{1}{r^2} + \partial_{zz}^2\right] V_\theta. \tag{13}$$

The first relationship shows that the centripetal acceleration creates a radial pressure gradient (with which no density gradient is associated as the hypothesis of incompressible fluids has been assumed). The solution of the second equation gives a general expression for the flow velocity V_θ.

The boundary and initial conditions to be satisfied by the angular velocity of the fluid and the walls are as follows, the rotation rate of the walls of the cavity involving the Heaviside step function at $t = 0$:

the velocity of the fluid

$$V_\theta\,(r, z, t \leqslant 0) = 0, \qquad V_\theta\,(r, z, t > 0) = \boldsymbol{\Omega}(r, z, t) r;$$

the velocity of the walls

$$V_\theta\,(r, z, t < 0) = 0, \qquad V_\theta\,(r, z = 0, t \geqslant 0) = \Omega_0\,r,$$

$$V_\theta\,(r, z = h, t \geqslant 0) = \Omega_0\,r, \qquad V_\theta\,(r = R_0, z, t \geqslant 0) = \Omega_0\,R_0. \tag{14}$$

The solution V_θ of the set of equations (13) which satisfies these boundary conditions (14) can be expressed as a double Fourier-Bessel expansion [5]:

$$V_\theta\,(r, z, t) = \Omega_0\,r\left(1 + \frac{R_0}{r} \sum_{m = 1,3\ldots} \sum_{n = 1,2\ldots} \frac{8}{m\pi} \sin\left(\frac{m\pi}{h}\,z\right) \frac{\mathrm{J}_1\,(\lambda_n\,r/R_0)}{\lambda_n\,\mathrm{J}_0\,(\lambda_n)}\,\mathrm{e}^{-k_{mn}^2\,\nu t}\right) U(t), \tag{15}$$

with $k_{mn}^2 = (m\pi/h)^2 + (\lambda_n\,/R_0)^2$ and where the coefficients λ_n are the zeros of the Bessel function of first order ($\mathrm{J}_1\,(\lambda_n) = 0$). The Heaviside step function $U(t)$ ensures the causality of this solution. Then, the expression for the angular velocity $\boldsymbol{\Omega}(r, z, t)$ takes the form

$$\mathbf{\Omega}(r, z, t) = \Omega_0\left(1 + \frac{R_0}{r}\sum_{m=1,3\ldots}\sum_{n=1,2\ldots}\frac{8}{m\pi}\sin\left(\frac{m\pi}{h}z\right)\frac{\mathrm{J}_1(\lambda_n r/R_0)}{\lambda_n \mathrm{J}_0(\lambda_n)}\mathrm{e}^{-k_{mn}^2 \nu t}\right)U(t). \tag{16}$$

One can easily verify that at time zero, this solution $\mathbf{\Omega}$ is equal to zero everywhere except on the walls, because

$$\sum_{m=1,3\ldots}\frac{4}{m\pi}\sin\left(\frac{m\pi}{h}z\right) = 1 \qquad \text{and} \qquad \frac{R_0}{r}\sum_{n=1,2\ldots}\frac{2\mathrm{J}_1(\lambda_n r/R_0)}{\lambda_n \mathrm{J}_0(\lambda_n)} = -1.$$

3.4. THE TOTAL PARTICLE MOTION

3.4.1. *Circular flow and "acoustic" motion*

In this section, the whole motion of the fluid is studied. It includes the unsteady circular flow presented in the previous section and an "acoustic" motion (the word "acoustic" is taken here globally and includes the thermal and vorticity motions which accompany the acoustic movement itself). The instantaneous position of the particle is described by means of the vector $\mathbf{OP} = \mathbf{OM} + \mathbf{MP}$, where $\mathbf{OM}$ is the position vector of the particle driven along the circular streamlines of the flow without acoustic displacement, and where $\mathbf{MP}$ represents the displacement of the element of fluid due to the acoustic motion.

As the fundamental Navier-Stokes equations (1) and conservation of mass equation (2), which describe the particle motion, are expressed in an inertial reference frame, the operator "d_t" giving the material derivative will now be denoted "$\mathrm{d}_t^{(i)}$", and the particle velocity and the acceleration associated to the global motion are written respectively as:

$$\boldsymbol{V}_T = \mathrm{d}_t^{(i)}\,\mathbf{OP}, \qquad \mathrm{d}_t^{(i)}\,\boldsymbol{V}_T = \mathrm{d}_{tt}^{2(i)}\,\mathbf{OP}. \tag{17, 18}$$

Then, the Navier-Stokes equations (1) and the conservation of mass equation (2) take the following forms:

$$\rho_T\,\mathrm{d}_{tt}^{2(i)}\,\mathbf{OP} = -\mathbf{grad}\,P_T + \mu\mathbf{\Delta}(\mathrm{d}_t^{(i)}\,\mathbf{OP}) + \left(\eta + \frac{\mu}{3}\right)\mathbf{grad}\,[\mathrm{div}\,(\mathrm{d}_t^{(i)}\,\mathbf{OP})], \tag{19}$$

$$\mathrm{d}_t^{(i)}\,\rho_T + \rho_T\,\mathrm{div}\,(\mathrm{d}_t^{(i)}\,\mathbf{OP}) = \rho_0\,q_s. \tag{20}$$

Furthermore, both quantities P_T, ρ_T can be separated into two parts:

$$P_T = P + p, \qquad \rho_T = \rho_0 + \rho, \tag{21, 22}$$

where P and ρ_0 are the pressure and density associated with the circular flow only and where p and ρ are the pressure and density variations due to the acoustic movement superposed to the flow.

In order to reveal the expected inertial factors explicitly, the quantities which occur in the fundamental equations above must be expressed as functions of the corresponding quantities relative to a moving reference frame.

3.4.2. *The inertial factors*

The equations of motion are expressed by using a moving reference frame (m), chosen in such a way that it is linked at each time t to the motion of a fluid element located at the same distance r and the same z-co-ordinate of the z-axis (i.e., situated on the same circular streamline). Therefore, the rotation velocity $\mathbf{\Omega}(r, z, t)$ of the moving frame is equal to that of the element of fluid at the time and the position considered. The origin O of this moving frame is the same as that of the fixed reference system (see Figure 5).

The particle velocity relative to the inertial frame can be expressed as a function of quantities relative to the moving frame [5] as

$$\mathrm{d}_t^{(i)}\, \mathbf{OP} = V(O) + \mathrm{d}_t^{(m)}\, \mathbf{OP} + \boldsymbol{\Omega} \times \mathbf{OP}, \tag{23}$$

where $\mathrm{d}_t^{(m)}$ is the operator giving the material derivative in the moving reference frame. In our particular case, the translational velocity $V(O)$ of the origin of the moving frame is equal to zero and as this frame is linked to the unsteady circular motion of the particle, the velocity corresponding to this circular motion in the moving frame is equal to zero: $\mathrm{d}_t^{(m)}\, \mathbf{OM} = 0$. Then, this last expression (23) simplifies to

$$\mathrm{d}_t^{(i)}\, \mathbf{OP} = (\mathrm{d}_t^{(m)} + \boldsymbol{\Omega} \times)\mathbf{OP} = \mathrm{d}_t^{(m)}\, \mathbf{MP} + \boldsymbol{\Omega} \times \mathbf{OP}. \tag{24}$$

The acceleration of the particle is determined by applying once again the operator $[\mathrm{d}_t^{(i)} = \mathrm{d}_t^{(m)} + \boldsymbol{\Omega} \times\,]$ to the last result (24):

$$\begin{aligned} \mathrm{d}_{tt}^{2(i)}\, \mathbf{OP} &= \mathrm{d}_t^{(i)}\, (\mathrm{d}_t^{(i)}\, \mathbf{OP}) = [\mathrm{d}_t^{(m)} + \boldsymbol{\Omega} \times\,]\, [\mathrm{d}_t^{(m)}\, \mathbf{MP} + \boldsymbol{\Omega} \times \mathbf{OP}] \\ &= \mathrm{d}_{tt}^{2(m)}\, \mathbf{MP} + 2\boldsymbol{\Omega} \times \mathrm{d}_t^{(m)}\, \mathbf{MP} + \mathrm{d}_t^{(m)}\, \boldsymbol{\Omega} \times \mathbf{OP} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{OP}). \end{aligned} \tag{25}$$

In this last expression, one can recognize the usual inertial factors: the Coriolis acceleration ($2\boldsymbol{\Omega} \times \mathrm{d}_t^{(m)}\, \mathbf{MP}$), the centripetal acceleration ($\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{OP})$), and the transient factor ($\mathrm{d}_t^{(m)}\, \boldsymbol{\Omega} \times \mathbf{OP}$) linked to the angular acceleration of the particle.

3.4.3. *Linearization of the fundamental equations*

The expressions of velocities and accelerations (24) and (25) are introduced in the Navier-Stokes (19) and conservation of mass (20) equations, which then may be respectively written as

$$\begin{aligned} &\rho_T\, \mathrm{d}_{tt}^{2(m)}\, \mathbf{MP} + 2\rho_T\, \boldsymbol{\Omega} \times \mathrm{d}_t^{(m)}\, \mathbf{MP} + \rho_T\, \mathrm{d}_t^{(m)}\, \boldsymbol{\Omega} \times \mathbf{OP} + \rho_T\, \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{OP}) \\ &\quad = -\mathbf{grad}\,(P + p) + \mu\boldsymbol{\Delta}\, \mathrm{d}_t^{(m)}\, \mathbf{MP} + \mu\boldsymbol{\Delta}(\boldsymbol{\Omega} \times \mathbf{OP}) \\ &\qquad + \left(\eta + \frac{\mu}{3}\right) \mathbf{grad}\,(\mathrm{div}\,(\mathrm{d}_t^{(m)}\, \mathbf{MP})) + \left(\eta + \frac{\mu}{3}\right) \mathbf{grad}\,(\mathrm{div}\,(\boldsymbol{\Omega} \times \mathbf{OP})), \end{aligned} \tag{26}$$

$$\mathrm{d}_t^{(m)}\, \rho_T + \rho_T\, \mathrm{div}\,(\mathrm{d}_t^{(m)}\, \mathbf{MP}) + \rho_T\, \mathrm{div}\,(\boldsymbol{\Omega} \times \mathbf{OP}) = \rho_0\, q_s, \tag{27}$$

where $\mathrm{d}_t^{(i)}\, \rho_T = \mathrm{d}_t^{(m)}\, \rho_T$ because the time variation of ρ_T for a given particle is the same in the inertial frame and in the moving one.

In the following, the "acoustic" particle displacement is denoted $\boldsymbol{\xi} = \mathbf{MP}$ and the "acoustic" particle velocity relative to the moving frame is denoted $\mathbf{v} = \mathrm{d}_t^{(m)}\, \mathbf{MP}$, the word

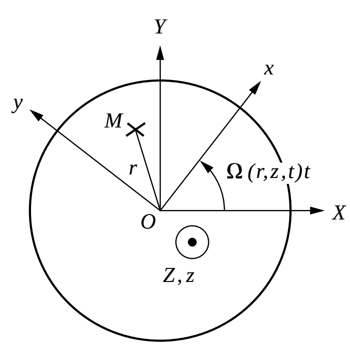

Figure 5. Reference frames: fixed frame (O, X, Y, Z) and moving frame $(0, x, y, z)$.

"acoustic" being still taken globally: i.e., including the thermal and vorticity motions which accompany the acoustic motion itself. Nevertheless, one can note that $d_t^{(i)}\,\boldsymbol{\Omega} = d_t^{(m)}\,\boldsymbol{\Omega} = \partial_t^{(m)}\,\boldsymbol{\Omega}$ (which can be written either $d_t\,\boldsymbol{\Omega}$ or $\partial_t\,\boldsymbol{\Omega}$).

Therefore, linear Navier-Stokes equation (26) and linear conservation of mass equation (27) are written, taking into account the equation $\mathrm{div}\,(\boldsymbol{\Omega} \times \mathbf{OP}) = 0$ (see equation (11)), as

$$\partial_t^{(m)}\,\mathbf{v} + 2\boldsymbol{\Omega} \times \mathbf{v} + d_t\,\boldsymbol{\Omega} \times \mathbf{OP} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{OP})$$

$$= -\frac{1}{\rho_0}\,\mathbf{grad}\,(P + p) + \nu\boldsymbol{\Delta}\mathbf{v} + \nu\boldsymbol{\Delta}(\boldsymbol{\Omega} \times \mathbf{OP}) + \left(\frac{\eta}{\rho_0} + \frac{\nu}{3}\right)\mathbf{grad}\,(\mathrm{div}\,(\mathbf{v})), \tag{28}$$

and

$$\partial_t^{(m)}\,\rho + \rho_0\,\mathrm{div}\,(\mathbf{v}) = \rho_0\,q_s. \tag{29}$$

3.4.4. *Fundamental equations governing the acoustic motion*

Actually, the complete motion (circular flow and "acoustic" motion) of the element of fluid located at the position P in the cavity satisfies equations (28) and (29). But upon taking into account equation (8) $\mathrm{div}\,(\mathbf{V}) = 0$ and expression (12) of the acceleration of the circular flow, the Navier-Stokes equation governing the motion of the circular flow without acoustic perturbation (6) can be written as

$$d_t\,\boldsymbol{\Omega} \times \mathbf{OM} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{OM}) = -\frac{1}{\rho_0}\,\mathbf{grad}\,P + \nu\boldsymbol{\Delta}(\boldsymbol{\Omega} \times \mathbf{OM}). \tag{30}$$

Combining this last equation (30) and equation (28), and taking into account that $\mathbf{OP} = \mathbf{OM} + \boldsymbol{\xi}$ leads to the Navier-Stokes equation for the "acoustic" motion, namely

$$\partial_t^{(m)}\,\mathbf{v} + 2\boldsymbol{\Omega} \times \mathbf{v} + d_t\,\boldsymbol{\Omega} \times \boldsymbol{\xi} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{\xi}) = -\frac{1}{\rho_0}\,\mathbf{grad}\,p + \nu\Delta(\mathbf{v} + \boldsymbol{\Omega} \times \boldsymbol{\xi})$$

$$+ \left(\frac{\eta}{\rho_0} + \frac{\nu}{3}\right)\mathbf{grad}\,(\mathrm{div}\,(\mathbf{v})). \tag{31}$$

The conservation of mass equation (29) for the "acoustic" motion remains unchanged.

Finally, the viscous and thermal dissipative processes inside the cavity are always negligible in comparison with those which take place in the boundary layers near the walls. Therefore, the factors which denote these dissipative processes in the fundamental equation (31) are neglected in the following. This assumption enables one to simplify greatly the expression for the problem under consideration. Actually, the Navier-Stokes equation for the acoustic motion (and only it) reduces to the Euler equation and the hypothesis of adiabatic compressibility can then be assumed in the continuity equation for the entropy (3). In the following sections, the Green function is chosen in such a way that it includes the dissipative processes in the boundary layers.

Hence, by reducing the notation $\partial_t^{(m)}$ to the more simple notation ∂_t in the following, the set of fundamental equations which remains finally is the following:
the Euler equation

$$\partial_t\,\mathbf{v} + 2\boldsymbol{\Omega} \times \mathbf{v} + d_t\,\boldsymbol{\Omega} \times \boldsymbol{\xi} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{\xi}) = -\frac{1}{\rho_0}\,\mathbf{grad}\,p; \tag{32}$$

the conservation of mass equation,

$$\partial_t \rho + \rho_0 \operatorname{div}(\mathbf{v}) = \rho_0 q_s ; \tag{33}$$

the equation characterizing adiabatic compressibility (entropy fluctuation $dS_T = 0$),

$$p = \rho c^2, \tag{34}$$

where $c = \sqrt{\gamma/\rho_0 \chi_T}$ is the adiabatic speed of sound.

As has been noted previously, the predominant dissipative processes linked to the viscosity and the heat conduction of the fluid are those which occur in boundary layers near the walls of the cavity (the dissipative processes in the bulk of the cavity are then negligible). They are taken into account in the appropriate impedance-like boundary condition [6]. But because the inertial factors, which represent the inertial acoustic mode coupling, depend strongly on not only acoustic movement (subscript "a"), but also the vorticity (subscript "v") and the thermal (subscript "h") movements, these components of the particle motion in the boundary layers must be explicitly taken into account when one calculates the effect of the fluid rotation on the acoustic field.

4. THE ACOUSTIC WAVE MOTION

4.1. GENERAL SOLUTION

4.1.1. *The equation of propagation*

The propagation equation for the pressure variation is readily obtained from the three equations (32), (33) and (34). By regrouping the factors linked to rotation and the source factor in the second member, it takes the form

$$\left(\mathbf{\Delta} - \frac{1}{c^2}\,\partial^2_{tt}\right) p(\mathbf{r}, t) = -\rho_0\, \partial_t\, q_s - \rho_0 \operatorname{div} \boldsymbol{\gamma} \tag{35}$$

in the domain D (the bulk of the cavity), where

$$\boldsymbol{\gamma} = \boldsymbol{\gamma}_{co} + \boldsymbol{\gamma}_a + \boldsymbol{\gamma}_{ce}, \tag{36}$$

with for $t > 0$ ($\gamma = 0$ for $t < 0$)

$$\boldsymbol{\gamma}_{co} = 2\mathbf{\Omega} \times \mathbf{v}, \qquad \boldsymbol{\gamma}_a = d_t\, \mathbf{\Omega} \times \boldsymbol{\xi}, \qquad \boldsymbol{\gamma}_{ce} = \mathbf{\Omega} \times (\mathbf{\Omega} \times \boldsymbol{\xi}). \tag{37–39}$$

The factor $\boldsymbol{\gamma}_{co}$ represents the Coriolis acceleration, $\boldsymbol{\gamma}_{ce}$ the centripetal acceleration and $\boldsymbol{\gamma}_a$ the transient factor linked to the angular acceleration.

The problem must be solved in the time domain by using real functions and not the associated analytic functions because of the complexity of the Hilbert transformation of the right side of the propagation equation. Even if this method leads to heavier expressions to write down, it greatly simplifies all the calculations.

4.1.2. *The Green function*

In the time domain, the Green function associated to the problem considered satisfies the wave equation

$$\left(\mathbf{\Delta} - \frac{1}{c^2}\,\partial^2_{tt}\right) G(\mathbf{r}, t; \mathbf{r}_0, t_0) = -\delta(\mathbf{r} - \mathbf{r}_0)\delta(t - t_0), \quad \text{in the domain } D. \tag{40}$$

In the frequency domain, this Green function is chosen to satisfy the boundary conditions of the problem. It is then the solution of the set of equations

$$(\mathbf{\Delta} + k^2)G(\mathbf{r}, \mathbf{r}_0) = -\delta(\mathbf{r} - \mathbf{r}_0), \quad \text{in the domain } D, \tag{41a}$$

$$(\partial_n + \mathrm{i}k\beta)G(\mathbf{r}, \mathbf{r}_0) = 0, \quad \text{on the walls } \partial D, \tag{41b}$$

where $k = \omega/c$.

In equation (41b), the term β denotes a boundary specific admittance, which takes into account the effects of the viscous and thermal boundary layers. For $\omega > 0$, it can be written as [6]

$$\beta = (1 + i)\sqrt{\omega}\frac{1}{\sqrt{2c}}\left[\left(1 - \frac{k_\perp^2}{k^2}\right)\sqrt{l'_v} + (\gamma - 1)\sqrt{l_h}\right], \tag{42}$$

where $k_\perp$ is the component normal to the walls of the wavenumber $\mathbf{k}$, and where l'_v and l_h are the characteristic lengths defined as $l'_v = \mu/\rho_0 c$ and $l_h = \lambda/\rho_0 cC_p$, μ being the shear viscosity coefficient, λ the heat conduction coefficient, and C_p the heat coefficient at constant pressure per unit of mass of the fluid.

The Green function is expressed as an eigenfunction expansion where the orthonormal eigenfunctions $\Psi_n(r, \varphi, z)$ are solutions to the Neumann's boundary value problem:

$$(\mathbf{\Delta} + k_n^{o2})\Psi_n(\mathbf{r}) = 0, \quad \text{in the domain } D,$$

$$\partial_\mathrm{n}\Psi_\mathrm{n}(\mathbf{r}) = 0, \quad \text{on the boundaries } \partial D. \tag{43}$$

The Green function in the time domain is given by integrating its expression in the frequency domain, by using the residue integration method, leading to

$$G(\mathbf{r}, t; \mathbf{r}_0, t_0) = c^2 U(t - t_0)\sum_n \frac{\sin(\omega_n(t - t_0))}{\omega_n}\mathrm{e}^{-\gamma_n(t-t_0)}\Psi_n(\mathbf{r}_0)\Psi_n(\mathbf{r}), \tag{44}$$

where $\omega_n = \omega_n^0 - \gamma_n$, with

$$\gamma_n = \sqrt{\omega_n^0}\frac{\sqrt{c}}{2\sqrt{2}}\iint_{\partial D}\left[\left(1 - \frac{k_\perp^2}{k^2}\right)\sqrt{l'_v} + (\gamma - 1)\sqrt{l_h}\right]\Psi_n^2(\mathbf{r})\,\mathrm{d}\mathbf{r}. \tag{45}$$

Note that as the derivative normal to the walls "∂_n" of the truncated Green function is equal to zero, it might happen that, in some calculations, this approximation would not be relevant, especially when some factors in the integrals of coupling (see further) become very important just near the walls. In these cases, the normal derivative "∂_n" must be replaced by its expression $(-\mathrm{i}k\beta)$ from the boundary conditions (41b) when the operator acts on analytical functions, or by $(k\,\mathrm{Im}(\beta))$ when it acts on real functions.

4.1.3. *The integral equation*

The solution of the non-homogeneous wave equation (35) which satisfies some boundary conditions specified further, can be obtained, for $t \geqslant 0$ (the time interval in which the problem is considered) from the integral equation [7]

$$p(\mathbf{r}, t) = \int_0^{+\infty}\mathrm{d}t_0\iiint_V W(\mathbf{r}_0, t_0)G(\mathbf{r}, t; \mathbf{r}_0, t_0)\,\mathrm{d}\mathbf{r}_0$$

$$+ \int_0^{+\infty}\mathrm{d}t_0\iint_S \mathrm{d}S_0\cdot(G(\mathbf{r}, t; \mathbf{r}_0, t_0)\nabla_0\,p(\mathbf{r}_0, t_0) - p(\mathbf{r}_0, t_0)\nabla_0\,G(\mathbf{r}, t; \mathbf{r}_0, t_0))$$

$$-\frac{1}{c^2}\iiint_V \mathrm{d}\mathbf{r}_0\,[p(\mathbf{r}_0, t_0)\partial_{t_0} G(\mathbf{r}, t; \mathbf{r}_0, t_0) - G(\mathbf{r}, t; \mathbf{r}_0, t_0)\partial_{t_0} p(\mathbf{r}_0, t_0)]_{t_0=0}, \tag{46a}$$

where

$$W(\mathbf{r}_0, t_0) = \rho_0\,\partial_{t_0} q_s + \rho_0 \operatorname{div}(\boldsymbol{\gamma}). \tag{46b}$$

The first integral can be interpreted as the effect of the real and image sources, i.e., the inertial sources ($\rho_0 \operatorname{div}(\boldsymbol{\gamma})$) which begin to occur from the time $t = 0$, and the loudspeaker for $t > 0$ only. The second integral contains the correction to take into account if the boundary conditions chosen for the Green function and the real boundary conditions are not the same. This second integral is here equal to zero as the impedance-like boundary condition chosen for the Green function rightly corresponds, in a first approximation, to the real boundary conditions. The third integral represents the effect of the initial conditions, due to the energy provided by the loudspeaker at $t \leqslant 0$.

The upper integration limit ($t_0 = \infty$) is in fact $t_0 = t$ because of the principle of causality (given by the Heaviside step function $U(t - t_0)$ in the expression of the Green function). The lower integration limit ($-\infty$) is replaced by $t_0 = 0$ (the origin of the time).

The solution for the pressure variation can be expressed as an eigenfunction expansion,

$$p(\mathbf{r}, t) = \sum_n \Phi_n(t)\Psi_n(\mathbf{r}), \tag{47}$$

where the eigenfunctions Ψ_n are those of the cavity (see section 4.1.2) and where the coefficients Φ_n depend on the time. Introducing this last expression (47) and that of the Green function (44) in equation (46a), and making use of the orthogonality property of the eigenfunctions, leads to the equation satisfied by the coefficients $\Phi_n(t)$ for $t \geqslant 0$:

$$\Phi_n(t) = \frac{c^2}{\omega_n}\int_0^t \mathrm{d}t_0 \sin(\omega_n(t - t_0))\, \mathrm{e}^{-\gamma_n(t-t_0)} \iiint_V W(\mathbf{r}_0, t_0)\Psi_n(\mathbf{r}_0)\,\mathrm{d}\mathbf{r}_0$$

$$+\,\mathrm{e}^{-\gamma_n t}\left\{\left(\cos(\omega_n t) - \frac{\gamma_n}{\omega_n}\sin(\omega_n t)\right)\Phi_n(0) + \frac{1}{\omega_n}\sin(\omega_n t)\partial_{t_0}\Phi_n(t_0)|_{t_0=0}\right\}. \tag{48}$$

One can note that this expression is indeed equal to $\Phi_n(0)$ for $t = 0$.

4.2. CYLINDRICAL SOLUTION

4.2.1. *Eigenfunctions*

Upon using the cylindrical co-ordinates, the eigenfunctions Ψ_n, which are the solutions of the Neumann's boundary problem, take the forms Ψ_n^C and Ψ_n^S, as follows:

$$\Psi_n\begin{bmatrix} C \\ S \end{bmatrix}(r, \varphi, z) = \frac{1}{\alpha_{n_r}\alpha_{n_\varphi}\alpha_{n_z}} \mathrm{J}_{n_\varphi}\left(\frac{\gamma_{n_r n_\varphi}}{R_0} r\right)\begin{bmatrix} \cos \\ \sin \end{bmatrix}(n_\varphi \varphi)\cos\left(\frac{n_z \pi}{h} z\right), \tag{49}$$

with

$$\alpha_{n_r}^2 = \left\{\begin{matrix} \dfrac{R_0^2}{2}\left[1 - \left(\dfrac{n_\varphi}{\gamma_{n_r n_\varphi}}\right)^2\right] J_{n_\varphi}^2(\gamma_{n_r n_\varphi}), & \text{if } \gamma_{n_r n_\varphi} \neq 0, \\ \dfrac{R_0^2}{2}, & \text{if } \gamma_{n_r n_\varphi} = 0 \end{matrix}\right\}, \tag{50}$$

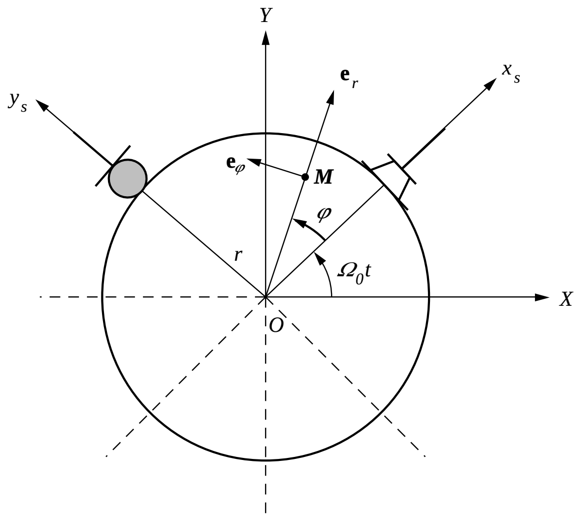

Figure 6. The angular co-ordinate φ of a point M of the cavity.

and $\alpha^2_{n_\varphi} = (1 + \delta_{n_\varphi 0})\pi$, $\alpha^2_{n_z} = h/(2 - \delta_{n_z 0})$. The factor $\gamma_{n_r n_\varphi}$ is the $(n+1)$th zero of the spatial derivative of the cylindrical Bessel function J_{n_φ}. The integers n_r, n_φ, n_z are associated respectively with the co-ordinates r, φ, z and to the eigenvalue $k_n^{\circ 2}$:

$$k_n^{\circ 2} = (\gamma_{n_r n_\varphi}/R_0)^2 + (n_z \pi/h)^2. \tag{51}$$

Then, the pressure expansion can be written as

$$p(\mathbf{r}, t) = \sum_n [\Phi^c_n(t)\Psi^c_n(\mathbf{r}) + \Phi^s_n(t)\Psi^s_n(\mathbf{r})]. \tag{52}$$

4.2.2. *Restriction to the two first azimuthal modes*

In the following, the pressure expansion is truncated to include only the first two resonant azimuthal modes $n = 1$, "C" and "S", which correspond to the quantum numbers $n_r = 0$, $n_\varphi = 1$, $n_z = 0$ and the same eigenvalue $\gamma_{01} = 1{\cdot}84$. The other modes, including the mode (0, 0, 0), are assumed to be negligible. This assumption can be made only if the azimuthal angular co-ordinate φ has its origin on the (Ox_s) axis which is linked to the loudspeaker. Then, the source and the microphone are located respectively at $\varphi_{\text{Loudspeaker}} = 0$ and $\varphi_{\text{micro}} = \pi/2$ (see Figure 6).

Furthermore, the height of the cavity is assumed to be much smaller than its diameter, enabling one to neglect the influence of the acoustic modes $n_z \neq 0$ at the operating frequency of the gyrometer (that is, the frequency of the first resonant mode (0, 1, 0) for the cavity under consideration).

Then, upon assuming these approximations, the eigenfunction expansion of the acoustic pressure may be written as

$$p(\mathbf{r}, t) = \Phi^c_1(t)\Psi^c_1(\mathbf{r}) + \Phi^s_1(t)\Psi^s_1(\mathbf{r}), \tag{53}$$

where

$$\Psi_1 \begin{bmatrix} C \\ S \end{bmatrix} (r, \varphi, z) = \frac{1}{\alpha_{0r}\sqrt{h\pi}} \mathrm{J}_1\left(\frac{\gamma_{01}}{R_0} r\right) \begin{bmatrix} \cos\varphi \\ \sin\varphi \end{bmatrix}, \tag{54a}$$

with

$$\alpha_{0r} = R_0 \, \mathrm{J}_1(\gamma_{01}) \sqrt{\gamma_{01}^2 - 1}/\sqrt{2}\gamma_{01}. \tag{54b}$$

The mode "C" ($\cos\varphi$) represents the acoustic field provided by the loudspeaker (located at $\varphi = 0$) and the mode "S" the field due to the inertial effects (measured by the microphone situated at $\varphi = \pi/2$).

4.2.3. *The integral equation*

For the $n = 1$ (mode (0, 1, 0)), the pressure amplitude of the main mode ($\cos\varphi$) generated by the loudspeaker is given by the coefficient $\Phi_1^c(t)$, and the amplitude of the other mode ($\sin\varphi$) corresponding to the signal measured by the microphone is given by the coefficient $\Phi_1^s(t)$. This last function $\Phi_1^s(t)$ satisfies the integral equation (48), written for this coefficient by restricting the expansion of p to the first azimuthal modes $n = 1$ in the factor $W(\mathbf{r}_0, t_0)$: i.e.,

$$\Phi_1^s(t) = \frac{c^2}{\omega_1} \int_0^t \sin(\omega_1(t - t_0)) \, \mathrm{e}^{-\gamma_1(t-t_0)} \iiint_D W(\mathbf{r}_0, t_0) \Psi_1^s(\mathbf{r}_0) \, \mathrm{d}\mathbf{r}_0 \, \mathrm{d}t_0$$

$$+ \mathrm{e}^{-\gamma_1 t} \left\{ \left(\cos(\omega_1 t) - \frac{\gamma_1}{\omega_1} \sin(\omega_1 t) \right) \Phi_1^s(0) + \frac{1}{\omega_1} \sin(\omega_1 t) \left[\partial_{t_0} \Phi_1^s(t_0) \right]_{t_0 = 0} \right\}, \tag{55}$$

with

$$\gamma_1 = \sqrt{\omega_1^\circ} \frac{\sqrt{c}}{\sqrt{2}} \left[\left(\frac{1}{R_0(\gamma_{01}^2 - 1)} + \frac{1}{h} \right) \sqrt{l_v'} + \left(\frac{\gamma_{01}^2}{R_0(\gamma_{01}^2 - 1)} + \frac{1}{h} \right) (\gamma - 1) \sqrt{l_h} \right]. \tag{56}$$

The initial conditions vanish because the function $\Phi_1^s(0)$ and its derivative are equal to zero. Actually, at time $t = 0$, the microphone, which is located on a node of pressure of the main mode, does not detect anything.

Therefore, by using the notation

$$S_1^s(t_0) = \iiint_D W(\mathbf{r}_0, t_0) \Psi_1^s(\mathbf{r}_0) \, \mathrm{d}\mathbf{r}_0, \tag{57a}$$

the integral equation (55) for the coefficient $\Phi_1^s(t)$ becomes

$$\Phi_1^s(t) = \frac{c^2}{\omega_1} \int_0^t \sin(\omega_1(t - t_0)) \, \mathrm{e}^{-\gamma_1(t-t_0)} S_1^s(t_0) \, \mathrm{d}t_0. \tag{57b}$$

4.3. ANALYTICAL SOLUTION FOR $\Phi_1^s(t)$

The "source" factor denoted $S_1^s(t)$ (equation (57a)) depends on the function $W(\mathbf{r}_0, t)$, which involves the effects of the loudspeaker and the inertial effects. Upon denoting $\boldsymbol{\xi} = \partial_t^{-1} \mathbf{v}$ the acoustic particle displacement, its expression (46b) is given by

$$W(\mathbf{r}_0, t) = \rho_0 \, \partial_t \, q_s + 2\rho_0 \, \mathrm{div}(\boldsymbol{\Omega} \times \mathbf{v}) + \rho_0 \, \mathrm{div}(\partial_t \boldsymbol{\Omega} \times \partial_t^{-1} \mathbf{v}) + \rho_0 \, \mathrm{div}(\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \partial_t^{-1} \mathbf{v})).$$

The source function $S_1^s(t)$ can be written as the sum of the factor $Q_1^s(t)$, which denotes the energy transfer from the loudspeaker to the mode $n = 1$ "S", and of three other terms $Co_1^s(t)$, $Aa_1^s(t)$ and Ce_1^s (t) which represent the coupling associated to the effects of the rotation of the fluid:

$$S_1^s(t) = Q_1^s(t) + Co_1^s(t) + Aa_1^s(t) + Ce_1^s(t), \tag{58a}$$

with

$$Q_1^s(t) = \rho_0 \iiint_V \partial_t\, q_s(\mathbf{r}_0, t)\Psi_1^s(\mathbf{r}_0)\, \mathrm{d}\mathbf{r}_0, \qquad Co_1^s(t) = 2\rho_0 \iiint_V \operatorname{div}(\mathbf{\Omega} \times \mathbf{v})\Psi_1^s(\mathbf{r}_0)\, \mathrm{d}\mathbf{r}_0, \tag{58b, c}$$

$$Aa_1^s(t) = \rho_0 \iiint_V \operatorname{div}(\partial_t\, \mathbf{\Omega} \times \partial_t^{-1}\, \mathbf{v})\Psi_1^s(\mathbf{r}_0)\, \mathrm{d}\mathbf{r}_0, \tag{58d}$$

$$Ce_1^s(t) = \rho_0 \iiint_V \operatorname{div}(\mathbf{\Omega} \times (\mathbf{\Omega} \times \partial_t^{-1}\, \mathbf{v}))\Psi_1^s(\mathbf{r}_0)\, \mathrm{d}\mathbf{r}_0. \tag{58e}$$

Then, the solution $\Phi_1^s(t)$ is the sum

$$\Phi_1^s(t) = \Phi_{Q_1}^s(t) + \Phi_{C_{01}}^s(t) + \Phi_{Aa_1}^s(t) + \Phi_{Ce_1}^s(t), \tag{59a}$$

of the integrals

$$\Phi_{Q_1}^s(t) = \frac{c^2}{\omega_1} \int_0^t \sin(\omega_1(t - t_0))\, \mathrm{e}^{-\gamma_1(t - t_0)} Q_1^s(t_0)\, \mathrm{d}t_0, \tag{59b}$$

$$\Phi_{C_{01}}^s(t) = \frac{c^2}{\omega_1} \int_0^t \sin(\omega_1(t - t_0))\, \mathrm{e}^{-\gamma_1(t - t_0)} Co_1^s(t_0)\, \mathrm{d}t_0, \tag{59c}$$

$$\Phi Aa_1^s(t) = \frac{c^2}{\omega_1} \int_0^t \sin(\omega_1(t - t_0))\, \mathrm{e}^{-\gamma_1(t - t_0)} Aa_1^s(t_0)\, \mathrm{d}t_0, \tag{59d}$$

$$\Phi Ce_1^s(t) = \frac{c^2}{\omega_1} \int_0^t \sin(\omega_1(t - t_0))\, \mathrm{e}^{-\gamma_1(t - t_0)} Ce_1^s(t_0)\, \mathrm{d}t_0. \tag{59e}$$

In section 4.3, these integrals are calculated successively in order to provide an analytical form for the solution $\Phi_1^s(t)$, which permits one to obtain the pressure amplitude measured by the microphone as a function of the time.

4.3.1. *The loudspeaker contribution*

The rate of volume creation by the loudspeaker, which is considered as a point source at $\mathbf{r} = \mathbf{r}_s$, can be expressed as

$$q_s = Q_0 \cos(\omega_1 t)\delta(\mathbf{r} - \mathbf{r}_s), \tag{60}$$

where Q_0 represents the amplitude of the rate of volume creation provided by the acoustic source.

For $t \geqslant 0$, the acoustic source, which is flush-mounted to the base of the cavity ($z = 0$), near the lateral wall ($r_s \cong R_0$), rotates with the angular speed Ω_0. Its rate of volume creation takes the form

$$q_s = Q_0 \cos(\omega_1 t) \frac{1}{r} \delta(r - R_0)\delta(\varphi)\delta(z). \tag{61}$$

Therefore, the term source $Q_1^s(t)$ (equation 58b) becomes

$$Q_1^s(t) = \rho_0 Q_0 \partial_t \iiint_D \cos(\omega_1 t)\delta(\mathbf{r} - \mathbf{r}_s)\Psi_1^s(\mathbf{r})\, d\mathbf{r}, \tag{62}$$

which can be written, after integration over the volume D of the cavity, as

$$Q_1^s(t) = -\omega_1 \rho_0 Q_0 \sin(\omega_1 t)\Psi_1^s(R_0, \varphi = 0, z = 0). \tag{63}$$

As the eigenfunction $\Psi_1^s(R_0, \varphi = 0, z = 0)$ is equal to zero on the loudspeaker, the inertial source factor $Q_1^s(t)$, which expresses the energy transfer from the loudspeaker to the mode Ψ_1^s, vanishes (as assumed in section 4.2.3). Therefore, $Q_1^s(t) = 0$.

4.3.2. *Acoustic, entropic and vortical particle velocities*

The inertial factors (equations (58c), (58d) and (58e)) involve the expression of the whole "acoustic" particle velocity. This particle velocity is written as the sum of the laminar acoustic and laminar thermal velocities v_a and v_h and the vortical velocity v_v [8].

The real expressions of these three velocities are determined by assuming the Born approximation. The particle velocity $\mathbf{v}$ in equations (58c), (58d) and (58e) is assumed to be the acoustic velocity of the main mode (generated by the loudspeaker): that is, the pressure variation $p(\mathbf{r}, t)$ is replaced by its initial value at $t = 0$. At the resonant radian frequency $\omega_1 \cong \omega_1^0 - \gamma_1$, its real expression takes the form

$$p(\mathbf{r}, t) = A_1^c \cos(\omega_1 t)\Psi_1^c(r, \varphi, z), \tag{64a}$$

with

$$A_1^c = \rho_0 c^2 Q_0 \Psi_1^c(\mathbf{r}_s) \frac{\omega_1}{2\gamma_1 \omega_1^0} \cong \frac{\rho_0 c^2}{2\gamma_1} Q_0 \Psi_1^c(\mathbf{r}_s). \tag{64b}$$

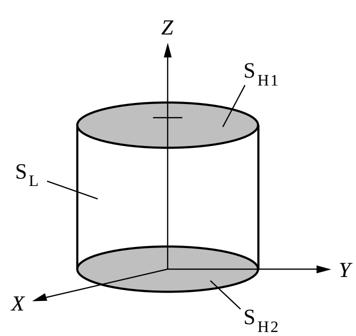

Figure 7. Walls S_{H1}, S_{H2}, and S_L of the cavity.

The real function for the laminar acoustic velocity can then be expressed as:

$$v_a(\mathbf{r}, t) = -\frac{1}{\rho_0}\frac{\sin(\omega_1 t)}{\omega_1}\frac{A_1^c}{\alpha_{0r}\sqrt{h\pi}}\left\{\begin{array}{l} \partial_r \mathrm{J}_1(k_{01} r)\cos\varphi \mathbf{e}_r \\ -\frac{1}{r}\mathrm{J}_1(k_{01} r)\sin\varphi \mathbf{e}_\varphi \\ \begin{cases} 0\mathbf{e}_z & \text{in the domain } D \\ (-v_h - v_v)\mathbf{e}_z & \text{over the surfaces } \mathrm{S}_{\mathrm{H1}} \text{ and } \mathrm{S}_{\mathrm{H2}} \end{cases} \end{array}\right\} \quad (65)$$

where $k_{01} = \gamma_{01}/R_0$ and S_{H1} and S_{H2} are the surfaces of the cavity located at $z = 0$ and $z = h$ (see Figure 7).

The thermal and vortical velocities are negligible in comparison with the laminar acoustic velocity in the whole volume under consideration except in the boundary layers near the walls. On these walls, the particle velocity vanishes: $\mathbf{v}_a + \mathbf{v}_h + \mathbf{v}_v = 0$. As the thermal and vortical velocities in the boundary layers represent a diffusing motion along the direction normal to the wall, these velocities have different expressions when they are expressed near the cylindrical lateral wall (S_L) or near the horizontal upper (S_{H1}) and lower (S_{H2}) surfaces of the cavity.

Therefore, the real expressions for the laminar thermal velocity are given near the wall S_{H1}, by

$$\mathbf{v}_h^{(\mathrm{H1})} = -\frac{l_h}{\rho_0 c}(\gamma - 1)\frac{A_1^c}{\alpha_{0r}\sqrt{h\pi}}\mathrm{e}^{\eta_h(z-h)} \times \left\{\begin{array}{l} \cos(\omega_1 t + \eta_h(z-h))\partial_r \mathrm{J}_1(k_{01} r)\cos\varphi \mathbf{e}_r \\ -\cos(\omega_1 t + \eta_h(z-h))\frac{1}{r}\mathrm{J}_1(k_{01} r)\sin\varphi \mathbf{e}_\varphi \\ +\eta_h[\cos(\omega_1 t + \eta_h(z-h)) - \sin(\omega_1 t + \eta_h(z-h))]\mathrm{J}_1(k_{01} r)\cos\varphi \mathbf{e}_z \end{array}\right\}, \quad (66)$$

and near the wall S_L by

$$\mathbf{v}_h^{(\mathrm{L})} = \frac{l_h}{\rho_0 c}(\gamma - 1)\frac{A_1^c \mathrm{J}_1(\gamma_{01})}{\alpha_{0r}\sqrt{h\pi}}\mathrm{e}^{\eta_h(r-R_0)} \times \left\{\begin{array}{l} \eta_h[\cos(\omega_1 t + \eta_h(r-R_0)) - \sin(\omega_1 t + \eta_h(r-R_0))]\cos\varphi \mathbf{e}_r \\ -\cos(\omega_1 t + \eta_h(r-R_0))\frac{1}{r}\sin\varphi \mathbf{e}_\varphi \\ +0\mathbf{e}_z \end{array}\right\} \quad (67)$$

with $\eta_h = \sqrt{\omega_1/2cl_h}$.

The real expressions for the vortical velocity are given near the wall S_{H1} by

$$\mathbf{v}_v^{(\mathrm{H1})} = \frac{1}{\rho_0} \frac{A_1^c}{\alpha_{0r}\sqrt{h\pi}} \mathrm{e}^{\eta_v (z-h)}$$

$$\times \left\{ \begin{array}{l} \partial_r \mathrm{J}_1(k_{01} r) \cos\varphi \left[\frac{1}{\omega_1} \sin(\omega_1 t + \eta_v (z-h)) + \frac{l_h}{c}(\gamma-1)\cos(\omega_1 t + \eta_v (z-h)) \right] \mathbf{e}_r \\ -\frac{1}{r} \mathrm{J}_1(k_{01} r) \sin\varphi \left[\frac{1}{\omega_1} \sin(\omega_1 t + \eta_v (z-h)) \right. \\ \qquad \left. + \frac{l_h}{c}(\gamma-1)\cos(\omega_1 t + \eta_v (z-h)) \right] \mathbf{e}_\varphi \\ + \frac{k_{01}^2}{2\eta_v} \mathrm{J}_1(k_{01} r) \cos\varphi \left[\left(\frac{1}{\omega_1} + \frac{l_h}{c}(\gamma-1) \right) \sin(\omega_1 t + \eta_v (z-h)) \right. \\ \qquad \left. + \left(\frac{-1}{\omega_1} + \frac{l_h}{c}(\gamma-1) \right) \cos(\omega_1 t + \eta_v (z-h)) \right] \mathbf{e}_z \end{array} \right\}, \tag{68}$$

and near the wall S_L by

$$\mathbf{v}_v^{(\mathrm{L})} = \frac{1}{\rho_0} \frac{A_1^c}{\alpha_{0r}\sqrt{h\pi}} \mathrm{J}_1(\gamma_{01}) \mathrm{e}^{\eta_v (r-R_0)}$$

$$\times \left\{ \begin{array}{l} \cos\varphi \frac{1}{2\eta_v r^2} \left[\left[\frac{1}{\omega_1} + \frac{l_h}{c}(\gamma-1) \right] \sin(\omega_1 t + \eta_v (r-R_0)) \right. \\ \quad \left. + \left[\frac{-1}{\omega_1} + \frac{l_h}{c}(\gamma-1) \right] \cos(\omega_1 t + \eta_v (r-R_0)) \right] \mathbf{e}_r \\ -\frac{1}{r} \sin\varphi \left[\frac{1}{\omega_1} \sin(\omega_1 t + \eta_v (r-R_0)) + \frac{l_h}{c}(\gamma-1)\cos(\omega_1 t + \eta_v (r-R_0)) \right] \mathbf{e}_\varphi \\ \quad + 0\mathbf{e}_z \end{array} \right\}, \tag{69}$$

with $\eta_v = \sqrt{\omega_1 / 2cl'_v}$.

Note that the three components of each velocity have been calculated, although some of them are *a priori* negligible in comparison with others.

4.3.3. *The contribution of the Coriolis acceleration*

The coupling factor $Co_1^s(t)$ (equation (58c)), which accounts for the effects of the Coriolis acceleration, can be separated into three integrals which correspond to the three "components" of the particle velocity: namely,

$$Co_1^s(t) = Co_{1a}^s(t) + Co_{1h}^s(t) + Co_{1v}^s(t), \tag{70a}$$

with

$$Co_{1a}^{s}(t) = 2\rho_0 \iiint_V \operatorname{div}(\mathbf{\Omega} \times \mathbf{v}_a)\Psi_1^s(\mathbf{r})\,\mathrm{d}\mathbf{r}, \tag{70b}$$

$$Co_{1h}^{s}(t) = 2\rho_0 \iiint_V \operatorname{div}(\mathbf{\Omega} \times \mathbf{v}_h)\Psi_1^s(\mathbf{r})\,\mathrm{d}\mathbf{r}, \tag{70c}$$

$$Co_{1v}^{s}(t) = 2\rho_0 \iiint_V \operatorname{div}(\mathbf{\Omega} \times \mathbf{v}_v)\Psi_1^s(\mathbf{r})\,\mathrm{d}\mathbf{r}. \tag{70d}$$

Upon using the relationship

$$\operatorname{div}(\mathbf{\Omega} \times \mathbf{v})\Psi_1^s(\mathbf{r}) = \operatorname{div}[(\mathbf{\Omega} \times \mathbf{v})\Psi_1^s(\mathbf{r})] - (\mathbf{\Omega} \times \mathbf{v}) \cdot \mathbf{grad}\, \Psi_1^s(\mathbf{r}), \tag{71}$$

and Ostrogradsky's law, the integral $Co_1^s(t)$ takes the form

$$\iiint_D \operatorname{div}(\mathbf{\Omega} \times \mathbf{v})\Psi_1^s(\mathbf{r})\,\mathrm{d}\mathbf{r} = \iiint_D \operatorname{div}[(\mathbf{\Omega} \times \mathbf{v})\Psi_1^s(\mathbf{r})]\,\mathrm{d}\mathbf{r} - \iiint_D (\mathbf{\Omega} \times \mathbf{v}) \cdot \mathbf{grad}\, \Psi_1^s(\mathbf{r})\,\mathrm{d}\mathbf{r}$$

$$= \iint_{\partial D} (\mathbf{\Omega} \times \mathbf{v})\Psi_1^s(\mathbf{r})\,\mathrm{d}S - \iiint_D (\mathbf{\Omega} \times \mathbf{v}) \cdot \mathbf{grad}\, \Psi_1^s(\mathbf{r})\,\mathrm{d}\mathbf{r} \tag{72}$$

and as the particle velocity vanishes on the walls, it simplifies to

$$Co_1^s(t) = -2\rho_0 \iiint_V (\mathbf{\Omega}(\mathbf{r}, t) \times \mathbf{v}) \cdot \mathbf{grad}\, \Psi_1^s(\mathbf{r})\,\mathrm{d}\mathbf{r} = 2\rho_0 \iiint_V (\mathbf{grad}\, \Psi_1^s(\mathbf{r}) \times \mathbf{v}) \cdot \mathbf{\Omega}(\mathbf{r}, t)\,\mathrm{d}r. \tag{73}$$

The function $\Phi_{Co_1^s}(t)$ can then be calculated by replacing $Co_1^s(t)$ by this expression (73) in equation (59c).

4.3.4. *The contribution of the angular acceleration*

The factor $\Phi_{Aa_1^s}(t)$ (equation (59d)) is determined in the same manner as the Coriolis factor (equation (59c)) in section (4.3.3). The time derivative of the angular velocity $\mathbf{\Omega}(r, z, t)$ is expressed as

$$\partial_t\, \mathbf{\Omega}(r, z, t) = \mathbf{\Omega}_0 \left(1 + \frac{R_0}{r} \sum_{m=1,3\ldots} \sum_{n=1,2\ldots} \frac{8}{m\pi} \sin\left(\frac{m\pi}{h} z\right) \frac{\mathrm{J}_1(\lambda_n r/R_0)}{\lambda_n \mathrm{J}_0(\lambda_n)} \mathrm{e}^{-k_{mn}^2 \nu t}\right)\delta(t)$$

$$- U(t)\mathbf{\Omega}_0 \frac{R_0}{r} \sum_{m=1,3\ldots} \sum_{n=1,2\ldots} k_{mn}^2 \nu \frac{8}{m\pi} \sin\left(\frac{m\pi}{h} z\right) \frac{\mathrm{J}_1(\lambda_n r/R_0)}{\lambda_n \mathrm{J}_0(\lambda_n)} \mathrm{e}^{-k_{mn}^2 \nu t}, \tag{74}$$

where $\delta(t)$ is the Dirac distribution. Then, the integral $Aa_1^s(t)$ (equation (58d)) can be written as

$$Aa_1^s(t) = +\rho_0 \iiint_V \partial_t\, \mathbf{\Omega} \cdot (\mathbf{grad}\ \Psi_1^s(\mathbf{r}_0) \times \partial_t^{-1}\, \mathbf{v})\, \mathrm{d}\mathbf{r}_0, \tag{75}$$

and the function $\Phi_{Aa_1^s}(t)$ is calculated by introducing expression (75) in equation (59d). Note that the contribution of the first term of the left side of equation (74) is null because the particle displacement $\partial_t^{-1}\,\mathbf{v}$ vanishes at time $t = 0$.

4.3.5. *The contribution of the centripetal acceleration*

The centripetal acceleration has no acoustic influence, because the intergral $Ce_1^s(t)$ and then the function $\Phi_{ce_1^s}(t)$ are equal to zero. This can be demonstrated in the following manner.

The integral $Ce_1^s(t)$ (58e) can be expressed in the same way as both integrals (58c) and (58d):

$$Ce_1^s(t) = -\rho_0 \iiint_V \mathbf{\Omega} \times (\mathbf{\Omega} \times \partial_t^{-1}\, \mathbf{v}) \cdot \mathbf{grad}\ \Psi_1^s(\mathbf{r}_0)\, \mathrm{d}\mathbf{r}_0. \tag{76}$$

The relationship $\mathbf{\Omega} \times (\mathbf{\Omega} \times \partial_t^{-1}\, \mathbf{v}) = -\Omega^2(\partial_t^{-1}\, v_r\, \mathbf{e}_r + \partial_t^{-1}\, v_\varphi\, \mathbf{e}_\varphi)$ permits one to write the integrand as

$$\mathbf{\Omega} \times (\mathbf{\Omega} \times \partial_t^{-1}\, \mathbf{v}) \cdot \mathbf{grad}\ \Psi_1^s(\mathbf{r}_0) = -\Omega^2\left(\partial_t^{-1}\, v_r\, \partial_r\, \Psi_1^s(\mathbf{r}) + \partial_t^{-1}\, v_\varphi\, \frac{1}{r}\, \partial_\varphi\, \Psi_1^s(\mathbf{r})\right). \tag{77}$$

The expressions for the velocities (equations (65)–(69)), and that for the eigenfunction $\Psi_1^s(\mathbf{r})$ (equation (54)), show that this last equation (77) includes the scalar product of orthogonal eigenfunctions (with $\cos\varphi$ and $\sin\varphi$). Hence, the coefficient $Ce_1^s(t)$ and the function $\Phi_{Ce_1^s}(t)$ vanish.

The analytical expressions for the functions $\Phi_{Co_1^s}(t)$ and $\Phi_{Aa_1^s}(t)$ have been determined after some lengthy calculations. Nevertheless, some spatial integrals involving Bessel functions had to be calculated numerically as no analytical solutions were found for them.

5. RESULTS AND CONCLUSION

A discussion of the transient behaviour of the cylindrical acoustic gyro in detail is beyond the scope of this paper. Thus the aim has been only to address new requirements that have to be taken into account in the design of acoustic gyros and that arose during the analysis presented in this paper. So far there is no "model" available to describe the response of the acoustic gyro for strong variations of the rotation rates. The design requirements, for the optimum response of the gyro, had concerned only the steady response and was mainly governed by the quality factor of the cavity which depends on the acoustic modes generated, the dimensions of the cavity, the ambient temperature, the static pressure, the specific heat ratio, the Prandtl number, and so on [1].

Now, one can identify more design requirements for the "optimum" response of the gyro. An example of the contribution of the Coriolis effect to the transient response of the gyro is given in Figure 8, showing the sensitivity increasing regularly. As expected, the asymptotic value of this sensitivity obtained for the transient response is exactly equal to the one obtained from the method used to calculate the steady state behaviour, which was itself in very good agreement with experimental results [2]. This transient response is clearly governed by the velocity distribution (with circular streamlines), generated by the rotation

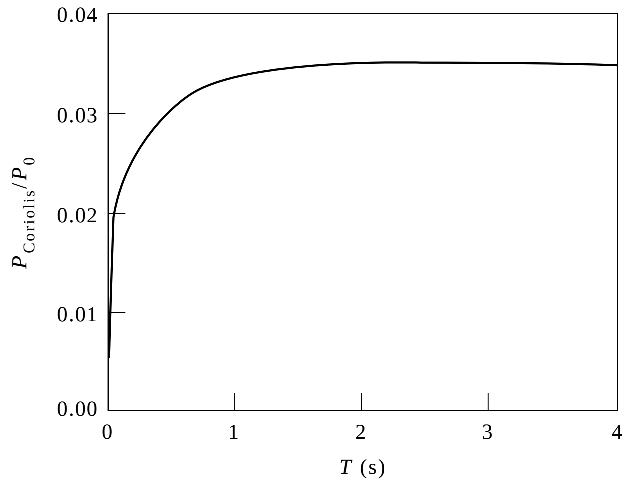

Figure 8. Root mean square of the amplitude (linear scale) of the acoustic pressure due to the Coriolis mode coupling, normalized to the acoustic pressure generated by the loudspeaker (sensitivity), as a function of time. Cavity filled with SF6: $h = 2 \times 10^{-3}$ m; $R_0 = 7 \times 10^{-3}$ m.

of the rigid boundaries, which reaches steady motion after roughly one second for a cavity 2 mm high.

The contribution of the term which involves the time rate of change of the angular velocity ($\partial\Omega/\partial t$), proportional to the acoustic particle displacement, reaches its maximum value at the beginning of the transient and vanishes as the unsteady rotational velocity gradient vanishes. The acoustic effect of the third inertial term linked to the centripetal acceleration is negligible as long as the rotation rates remain low enough so that the inertial acoustic modes coupling remains linear.

The parameters which govern the transient responses mentioned above depend strongly on both the dimensions of the cavity and the shear viscosity coefficient of the fluid (among others). These parameters permit the modification of the shape of the transient response, in such a way (for example) that the stabilization time can be reduced.

ACKNOWLEDGMENTS

This work was supported by the Direction des Recherches et Etudes Techniques (D.R.E.T., Délégation Générale pour l'Armement); the authors are indebted to E. Pleska for encouragement. They wish also to express gratitude to B. Chaumet, SEXTANT-Avionique company, for very helpful discussions.

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APPENDIX: LIST OF SYMBOLS

A_1^c	amplitude of the initial acoustic field generated by the loudspeaker
c	adiabatic speed of the sound
C_p	heat coefficient at constant pressure per unit of mass of the fluid
$\mathrm{d}_t = \mathrm{d}/\mathrm{d}t$	material derivative
$\partial_t = \partial/\partial t$	time partial derivative
∂_n	spatial derivative normal to the walls
$\mathrm{d}_t^{(i)}$	material derivative relative to the inertial frame
$\mathrm{d}_t^{(m)} = \mathrm{d}_t$	material derivative relative to the moving frame
D	integration domain, the bulk of the cavity
∂D	surface containing the domain D (the walls of the cavity)
$\mathbf{e}_r, \mathbf{e}_\theta, \mathbf{e}_z$	unit vectors of the system of cylindrical co-ordinates
$G(\mathbf{r}, t; \mathbf{r}_0, t_0)$	Green function in the time domain
h	height of the cavity
i	$i = \sqrt{-1}$
J_0	Bessel function of the first kind and order zero
J_1	Bessel function of the first kind and first order
$k = \omega/c$	wavenumber associated to the adiabatic motion
$k_\perp$	component normal to the wall of the wavenumber $\mathbf{k}$
k_{01}	$k_{01} = \gamma_{01}/R_0$
l'_v	$l'_v = \mu/\rho c$, characteristic length linked to viscosity
l_h	$l_h = \lambda_h/\rho c C_p$, characteristic length linked to heat conduction
n_r, n_φ, n_z	quantum numbers associated to the modes of acoustic pressure inside the cavity
$\mathbf{OM} = \mathbf{r}$	mean position vector of a fluid element
$\mathbf{OP}$	instantaneous position vector of a fluid element including the acoustic motion
$p(\mathbf{r}, t)$	pressure variation
P	pressure associated to the circular fluid motion
P_T	whole pressure associated to the fluid motion
q_s	rate of volume creation of fluid per unit volume of the loudspeaker
Q_0	amplitude of q_δ
r	radial co-ordinate of the point M inside the cavity
$\mathbf{r}$	position vector of the point M inside the cavity
R_0	radius of the cylindrical cavity
S_T	entropy
$\mathrm{S}_{\mathrm{H1}}, \mathrm{S}_{\mathrm{H2}}$	upper and lower "horizontal" walls of the cavity
S_L	lateral cylindrical wall of the cavity
T	temperature
$U(t)$	Heaviside step function
$\mathbf{v}$	particle velocity $\mathbf{v} = \mathrm{d}_t\,\mathbf{MP} = \mathbf{v}_a + \mathbf{v}_h + \mathbf{v}_v$
$\mathbf{v}_a$	laminar acoustic velocity
$\mathbf{v}_h$	laminar entropic velocity
$\mathbf{v}_v$	vortical velocity
$\mathbf{V}_T$	whole particle velocity
$\mathbf{V}$	velocity of the fluid (without acoustic movement)
β	boundary specific admittance
$\gamma = C_p/C_v$	specific heat ratio
$\boldsymbol{\gamma}_a$	factor of angular acceleration
$\boldsymbol{\gamma}_{co}$	Coriolis acceleration
$\boldsymbol{\gamma}_{ce}$	centripetal acceleration
η	coefficient of bulk viscosity of the fluid
η_h^{-1}	thickness of the thermal boundary layer

η_v^{-1}	thickness of the viscous boundary layer
λ	coefficient of thermal conductivity of the fluid
μ	coefficient of shear viscosity of the fluid
$v = \mu/\rho_0$	coefficient of cinematic viscosity of the fluid
$\boldsymbol{\xi} = \mathbf{MP}$	particle displacement
ρ	density variation
ρ_T	whole density of the fluid
ρ_0	mean density of the fluid
χ_T	isothermal compressibility
$\omega = kc$	radian frequency
$\boldsymbol{\Omega}_0$	angular velocity of the walls of the cavity
$\boldsymbol{\Omega}(r, z, t)$	angular velocity of the fluid inside the cavity.