Journal of Sound and Vibration (1998) **212**(2), 357–364

ACTIVE CONTROL OF SOURCE SOUND POWER RADIATION IN UNIFORM FLOW

P. Joseph, C. L. Morfey and P. A. Nelson

Institute of Sound and Vibration Research, The University, Southampton SO17 1BJ, England

(*Received* 18 *August* 1995, *and in final form* 3 *December* 1997)

Many suggested applications of active noise control involve the presence of a background air flow such as in ventilation ducts, exhaust stacks and the radiation from turbofan engines. This paper is a theoretical study aimed at assessing the effect of a uniform flow on the ability of simple sources actively to control free field sound power radiation. In particular, the minimum sound power radiated by two point volume velocity sources situated in an unbounded uniform fluid which moves with a uniform steady velocity is determined. Also the maximum sound power that can be absorbed by a point volume velocity source from an incident plane wave is determined.

1. INTRODUCTION

Many applications for which active control has been investigated have as an additional complication the presence of a background flow. Such applications include, for example, the suppression of noise transmitted along ventilation ducts and exhaust stacks, or the reduction in level of fan tones radiated from aircraft turbofan engines. These examples encompass a wide range of flow speeds, from Mach numbers of up to 0·1 in the case of ventilation ducts, to about 0·5 in the intake of a turbofan engine on an aeroplane on approach. This paper is concerned with the influence of a steady, uniform, subsonic flow on the ability of idealized elementary volume velocity sources to control the sound power radiated by other, similar sources in a free field. Two model problems investigated previously for a medium at rest [1, 2] are analyzed, but with the important addition of a mean flow.

2. ACOUSTIC ENERGY IN UNIFORM FLOW

The definition of acoustic energy in a fluid medium in motion is far less straightforward than for a medium at rest; the fundamental difficulties involved are made clear by Morfey [3]. The difference in the exact expressions for the acoustic energy density $E(\mathbf{r}, t)$ and acoustic intensity $\mathbf{I}(\mathbf{r}, t)$ in a moving fluid, and the corresponding quantities for the mean background flow (i.e., without sound) involve second order terms that, in general, will not be products of two first order terms that can be deduced from the linearized equations of fluid mechanics. Any other definition of sound energy in fluid flow must be arbitrary and will not be a true measure of mean energy flux carried by the acoustic disturbance. Nevertheless, a definition of acoustic energy in flow which possesses the continuity property $\nabla \cdot \mathbf{I} = 0$ under steady state conditions in a source free region is extremely useful.

0022–460X/98/170357 + 08 $25.00/0/sv971461

The following discussion of acoustic energy in the presence of flow appears to have gained widest acceptance since it satisfies a conservation law under a wide range of conditions. However, the analysis presented here will be confined to ideal fluid flows, whereas in the more general treatment by Morfey [3] account is taken of dissipative processes, viscous forces and entropy production. An addition made here to Morfey's analysis is the inclusion of a volume velocity source term which is of particular interest in what follows.

The starting point of the analysis is an energy balance equation for isentropic and irrotational flows. This is obtained by combining the linearized equations of mass and momentum conservation to give [4]

$$\partial E/\partial t + \nabla \cdot \mathbf{I} \approx \rho_0\,(H'q' + \mathbf{m}' \cdot \mathbf{f}'), \tag{1}$$

where ρ_0, $H'(\mathbf{r}, t)$ and $\mathbf{m}'(\mathbf{r}, t)$ are the ambient mass density, fluctuating total enthalpy and mass flux respectively and $q'(\mathbf{r}, t)$ and $\mathbf{f}'(\mathbf{r}, t)$ are the source terms of fluctuating volume velocity and body force distribution per unit volume. The generalized acoustic energy density and intensity can be expressed in terms of the fluctuating pressure p', ambient sound speed c_0, particle velocity $\mathbf{u}'$ and mean flow velocity $\mathbf{V}$ in the form [3]

$$E = \frac{p'^2}{2\rho_0\,c_0^2} + \left(\frac{\rho_0\,\mathbf{u}'^2}{2} + \frac{p'\mathbf{u}' \cdot \mathbf{V}}{c_0^2}\right), \qquad \mathbf{I} = H'\mathbf{m}' = \left(\frac{p'}{\rho_0} + \mathbf{u}' \cdot \mathbf{V}\right)\left(\rho_0\,\mathbf{u}' + \frac{p'\mathbf{V}}{c_0^2}\right). \tag{2, 3}$$

The approach due to Levine [5] is now followed for computing the sound power output for the point monopole source $q'(\mathbf{r}, t) = Q'(t)\delta(\mathbf{r} - \mathbf{r}_s)$ and point dipole source $\mathbf{f}'(\mathbf{r}, t) = \mathbf{F}'(t)\delta(\mathbf{r} - \mathbf{r}_s)$, where $Q'(t)$ and $\mathbf{F}'(t)$ are their respective source strengths per unit volume and δ is the Dirac delta function. For irrotational, isentropic flows, equation (1) is a statement of energy conservation. This equation can be integrated over a fixed control volume V surrounding the sources to give the following equation which specifies an exact balance between the rate of change of acoustic energy within V, plus the outflow of acoustic energy from the bounding surface S and the net supply of acoustic energy by the sources:

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_V E\,\mathrm{d}V = \int_V \rho_0\,(H'q' + \mathbf{m}' \cdot \mathbf{f}')\,\mathrm{d}V - \int_S \mathbf{I} \cdot \hat{\mathbf{n}}\,\mathrm{d}S. \tag{4}$$

The surface integral follows from Gauss' divergence theorem applied to $\mathbf{I}$, where S is the surface bounding V and $\hat{\mathbf{n}}$ is an outward-pointing unit normal vector. For statistically time stationary signals, the total time averaged energy in V remains constant so that the total time averaged radiated sound power may be obtained by integrating the normal component of acoustic intensity over S, which yields

$$\bar{W} = \int_S \bar{\mathbf{I}} \cdot \hat{\mathbf{n}}\,\mathrm{d}S = \rho_0\,\overline{(H'(\mathbf{r}_s)Q' + \mathbf{m}'(\mathbf{r}_s) \cdot \mathbf{F}')}, \tag{5}$$

where the overbar denotes time averaging. The sifting property of the Dirac delta function has been used to reduce the integral of the source distribution over the volume to a straightforward evaluation of the acoustic pressure and particle velocity at the source point $\mathbf{r}_s$. Equation (5) may be expressed in terms of the acoustic pressure and particle velocity [4] to become

$$\bar{W} = \overline{(p'(\mathbf{r}_s) + \rho_0\,\mathbf{V} \cdot \mathbf{u}'(\mathbf{r}_s))Q'} + \overline{(\mathbf{u}'(\mathbf{r}_s) + p'(\mathbf{r}_s)\mathbf{V}/\rho_0\,c_0^2) \cdot \mathbf{F}'}. \tag{6}$$

This expression involves only quantities at the source location which constitute the source loading and consequently avoids the potentially cumbersome integration of far field acoustic intensity suggested by equation (4).

In an irrotational flow, $\nabla \times \mathbf{u}' = 0$, and the acoustic particle velocity $\mathbf{u}'$ and acoustic pressure p' for an inviscid fluid can be deduced from the velocity potential by using

$$\mathbf{u}' = \nabla\phi', \qquad p' = -\rho_0\,\mathrm{D}\phi'/\mathrm{D}t = -\rho_0\,(\partial\phi'/\partial t + \mathbf{V}\cdot\nabla\phi'). \tag{7, 8}$$

Substituting equations (7) and (8) into equation (6) yields the sound power output due to the point monopole and dipole sources as

$$\bar{W} = -\rho_0\,\overline{\frac{\partial\phi'(\mathbf{r}_s)}{\partial t}\,\tilde{Q}'} + \overline{\left(\nabla\phi'(\mathbf{r}_s) - \left(\frac{\partial\phi'(\mathbf{r}_s)}{\partial t} + \mathbf{V}\cdot\nabla\phi'(\mathbf{r}_s)\right)\frac{\mathbf{V}}{c_0^2}\right)\cdot\mathbf{F}'}\,. \tag{9}$$

The first term on the right side of equation (9) for the contribution to the total sound power by the monopole source has previously been derived by Levine [5] by following a different, but essentially equivalent, procedure. This first term has no explicit dependence on the flow velocity $\mathbf{V}$. Velocity potential is therefore a more natural quantity for describing energy produced by volume velocity sources, unlike the second term for the dipole source sound power output which is now of greater algebraic complexity than the earlier expression of equation (6).

3. FREE FIELD MONOPOLE SOURCE RADIATION IN UNIFORM FLOW

The unsteady velocity potential $\phi'(\mathbf{r}, t)$ due to a point monopole velocity source, situated at $\mathbf{r}_s$, with source strength density Q', satisfies the inhomogeneous convected wave equation

$$\nabla^2\phi' - \frac{1}{c_0^2}\left(\frac{\partial}{\partial t} + V_x\frac{\partial}{\partial x}\right)^2\phi' = -Q'(t)\delta(\mathbf{r} - \mathbf{r}_s). \tag{10}$$

The free field solution $\phi'(\mathbf{r}, \omega) = \tilde{\phi}(\mathbf{r})\,\mathrm{e}^{\mathrm{j}\omega t}$ for harmonically time varying sources $Q'(t) = \tilde{Q}\,\mathrm{e}^{\mathrm{j}\omega t}$, is given by [5]

$$\tilde{\phi}(\mathbf{r}) = -\tilde{Q}\,\frac{\mathrm{e}^{-\mathrm{j}k(-M_x x + \sqrt{x^2 + (1 - M_x^2)y^2})/(1 - M_x^2)}}{4\pi\sqrt{x^2 + (1 - M_x^2)y^2}}, \tag{11}$$

where (x, y) denotes the co-ordinates of the point defined by the vector $(\mathbf{r} - \mathbf{r}_s)$, $\tilde{Q}$ is the complex monopole source strength and $M_x = V_x/c_0$. Equation (11) can be re-written more compactly in terms of a new set of co-ordinates [6] defined by $\bar{x} = x/\beta^2$, $\bar{y} = y/\beta$ and $\bar{R} = (\bar{x}^2 + \bar{y}^2)^{1/2}$, to give

$$\tilde{\phi}(\mathbf{r}) = -\tilde{Q}\,\mathrm{e}^{\mathrm{j}M_x k\bar{x} - \mathrm{j}k\bar{R}}/4\pi\beta^2\bar{R}, \tag{12}$$

where β is the Lorentz factor $\beta = \sqrt{1 - M_x^2}$. The advantage of using the "similarity variables" $(\bar{x}, \bar{y}, \bar{R})$ is that many formulas involving mean flow have the same functional form as for those with no mean flow, except that there is always an additional factor $\mathrm{e}^{\mathrm{j}kM_x\bar{x}}$.

4. MINIMUM SOUND POWER OUTPUT OF TWO POINT SOURCES IN UNIFORM FLOW

In Figure 1 are depicted two point volume velocity sources separated by a distance d with their axis at an angle θ to the x direction. The sources radiate into an unbounded uniform fluid which moves with a uniform steady velocity $\mathbf{V} = (V_x, 0, 0)$. At a single frequency ω, the first term on the right side of equation (9) for monopole source sound power output averaged over a period becomes $\bar{W} = \frac{1}{2}\,\omega\rho_0\,\mathrm{Im}\,\{\tilde{\phi}^*(\mathbf{r}_s)\tilde{Q}\}$. The total sound power radiated by the source pair may be deduced from the velocity potential at the respective source locations in order to give

$$\bar{W} = (\omega\rho_0\,/2)\,\mathrm{Im}\,\{(\tilde{\phi}_p^*\,(\mathbf{r}_p) + \tilde{\phi}_s^*\,(\mathbf{r}_p))\tilde{Q}_p + (\tilde{\phi}_p^*\,(\mathbf{r}_s) + \tilde{\phi}_s^*\,(\mathbf{r}_s))\tilde{Q}_s\}. \tag{13}$$

In this expression, $\tilde{Q}_p$ and $\tilde{Q}_s$ denote the respective complex primary and secondary source strengths and, for example, $\tilde{\phi}_s\,(\mathbf{r}_p)$ refers to the velocity potential at the primary source location due to the secondary source. Using equation (12) to evaluate the velocity potential terms appearing in equation (13) leads to the following expression for the total sound power:

$$\bar{W} = \tfrac{1}{2}\,Z_M\,[|\tilde{Q}_p\,|^2 + |\tilde{Q}_s\,|^2 + 2\,\mathrm{Re}\,\{\tilde{Q}_s^*\,\tilde{Q}_p\,\mathrm{e}^{-\mathrm{j}kM_x\,\bar{d}_x}\}(\sin k\bar{d})/k\bar{d}], \tag{14}$$

where $\bar{d} = (\bar{d}_x^2 + \bar{d}_y^2)^{1/2}$, $\bar{d}_x = d\cos\theta/\beta^2$ and $\bar{d}_y = d\sin\theta/\beta$. Re-writing the primary and secondary complex source strengths in magnitude and phase form, $\tilde{Q}_p = |\tilde{Q}_p\,|\,\mathrm{e}^{\mathrm{j}\sigma_p}$, $\tilde{Q}_s = |\tilde{Q}_s\,|\,\mathrm{e}^{\mathrm{j}\sigma_s}$ and completing the square gives

$$\bar{W} = \tfrac{1}{2}\,Z_M\,[|\tilde{Q}_p\,|^2 + (|\tilde{Q}_s\,| + |\tilde{Q}_p\,|\cos(\sigma_p - \sigma_s - kM_x\,\bar{d}_x)\,(\sin k\bar{d})/k\bar{d})^2$$
$$- |\tilde{Q}_p\,|^2\cos^2(\sigma_p - \sigma_s - kM_x\,\bar{d}_x)\,((\sin k\bar{d})/k\bar{d})^2]. \tag{15}$$

The magnitude of the secondary source strength $|\tilde{Q}_s\,|$ that removes the second term in equation (15) is $|\tilde{Q}_s\,| = -|\tilde{Q}_p\,|\cos(\sigma_p - \sigma_s - kM_x\,\bar{d}_x)\,(\sin k\bar{d})/k\bar{d}$, which leaves

$$\bar{W} = \bar{W}_p\,(1 - \cos^2(\delta - kM_x\,\bar{d}_x)\,(\sin^2 k\bar{d})/(k\bar{d})^2), \tag{16}$$

where $\delta = \sigma_p - \sigma_s$ is the difference in phase between the two sources. The term $\bar{W}_p$ is the sound power output due to the primary source in the absence of the secondary source and is given by

$$\bar{W}_p = \tfrac{1}{2}\,Z_M\,|\tilde{Q}|^2, \qquad \text{where } Z_M = Z_0\,\beta^{-2}, \quad Z_0 = \omega^2\rho_0\,/4\pi c_0, \tag{17}$$

in which Z_0 is the radiation resistance of a point monopole source radiating into a free field at rest [5]. The presence of a uniform flow therefore increases $\bar{W}_p$ (through convective amplification) by the factor β^{-2}, compared with the power produced in a medium at rest [7]. The phase relationship of the secondary source to the primary source that minimizes the total radiated sound power is readily identified from equation (16) as $\sigma_s = \sigma_p + kM_x\,\bar{d}_x$. The optimal secondary source strength $\tilde{Q}_{so}$, and the minimum sound power output $\bar{W}_o$, are therefore given by

$$\tilde{Q}_{so} = -\tilde{Q}_p\,\mathrm{e}^{-\mathrm{j}M_x\,\bar{d}_x}(\sin k\bar{d})/k\bar{d}, \qquad \bar{W}_o = \bar{W}_p\,(1 - (\sin^2 k\bar{d})/(k\bar{d})^2). \tag{18, 19}$$

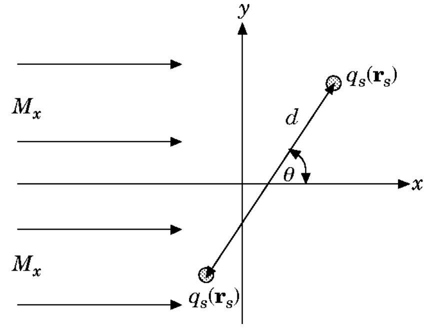

Figure 1. Two free field point monopole sources in a uniform flow.

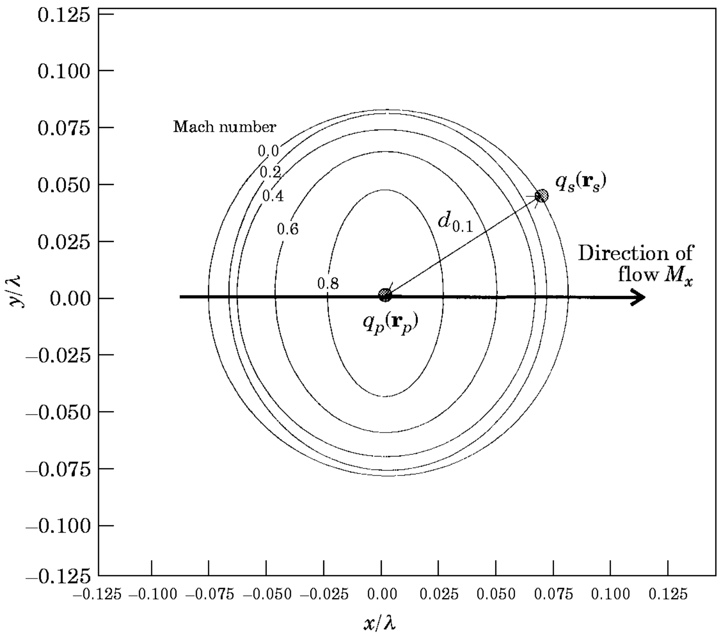

Figure 2. Contours of secondary source position relative to the primary source in a uniform flow for which total sound power reduction is 10 dB at Mach numbers 0·0, 0·2, 0·4, 0·6 and 0·8.

Equation (19) for the minimum sound power output in uniform flow in the co-ordinate system ($\bar{x}$, $\bar{y}$, $\bar{R}$) has exactly the same functional form as the zero flow result [1], with $\bar{d}$ replacing d, the physical distance between the two sources. This observation is surprising in view of the way the flow velocity enters the definition of acoustic intensity of equation (3). The definition of sound power in flow adopted in the paper and the subsequent co-ordinate transformations defined above generalizes the earlier result derived by Nelson *et al*. [1] to include the influence of a mean flow. Likewise, this approach suggests a way of including the effects of a mean flow on the minimum sound power emitted from more complicated source geometries [1].

The transformations $\bar{d}$, $\bar{d}_x$ and $\bar{d}_y$ defined earlier may be combined to give $\bar{d}/d = \beta^{-2}\sqrt{\cos^2\theta + \beta^2 \sin^2\theta}$. This expression has been used together with the result that $k\bar{d}_{0\cdot 1} \approx 0{\cdot}56$ in equation (19) gives $\bar{W}_0/\bar{W}_p = 0{\cdot}1$ to compute contours of secondary source position relative to that of the primary source at which the total sound power output is reduced by 10 dB. The results are presented in Figure 2 for Mach numbers in the range between 0 and 0·8. The units of distance are normalized to the zero flow acoustic wavelength $\lambda = c_0/f$. It is shown in Figure 2 that fluid flow past the source pair is consistently detrimental to the effectiveness of active sound power minimization. Increasing the flow speed reduces the range over which the point secondary source can reduce the radiation resistance of the neighbouring primary source and hence reduce its sound power output. The sources are required to be closer together than when in a medium at rest to achieve the same sound power reduction in the flow. The greatest sound power reductions are obtained when the axis of the source pair is perpendicular to the flow direction, and least reductions obtained when the axes are aligned with the flow direction. Note also the symmetry of the contours, suggesting that equal sound power reductions are obtained from the secondary source located at the same distance upstream, and downstream, of the primary source.

The source separation distance $d = d_\Delta$ at which the sound power output is reduced by a factor of Δ may be estimated from the small argument approximation $1 - (\sin^2 x)/x^2 = x^2/3 + O(x^4)$ applied to equation (19). The result is

$$\frac{d_\Delta}{\lambda} = \frac{\sqrt{3\Delta}}{2\pi}\frac{\beta^2}{\sqrt{M_x^2\cos^2\theta + \beta^2}} + O(\Delta^{3/2}). \tag{20}$$

This equation can be compared with the general polar equation (r, θ) of an ellipse with semi-major axis a (in the direction of $\theta = \pi/2$), semi-minor axis b and eccentricity ε $r = ab/\sqrt{a^2\varepsilon^2\cos^2\theta + b^2}$. Secondary source position contours of sound power reductions by a factor of Δ are therefore approximately elliptical with dimensions

$$a = \sqrt{3\Delta}\beta/k, \qquad b = \sqrt{3\Delta}\beta^2/k, \qquad \text{and} \qquad \varepsilon = M_x. \tag{21–23}$$

Volume velocity sources aligned in the direction of the flow are therefore required to be closer together by a factor of $\sqrt{(1 - M_x^2)}$ in order to achieve the same maximum sound power reduction produced by the source pair when their axis is perpendicular to the direction of flow, and by a factor of $(1 - M_x^2)$ when the sources are aligned with the flow.

5. ACTIVE ABSORPTION OF SOUND POWER FROM A FREE FIELD PLANE WAVE IN UNIFORM FLOW

In this section a brief examination is presented of a problem closely related to the case investigated in section 3, but with the field due to the point primary source replaced by a plane progressive wave propagating at an arbitrary angle to the direction of flow (see Figure 3). The secondary source can now no longer act at the primary source to reduce its radiation efficiency, which is likely to be the case for a large complex radiator. The secondary source is therefore restricted to extracting sound power from the plane wave with the purpose of reducing its pressure amplitude in the far field. The aim in this section is to quantify the influence of a uniform flow on the efficiency of a simple point volume velocity source in absorbing acoustic power from the plane wave. The identical problem for a medium at rest is presented by Nelson *et al.* [1].

A progressive plane wave with velocity potential amplitude $\tilde{\phi}_{po}$ propagating at an angle θ to the x-axis is of the form

$$\tilde{\phi}_p = \tilde{\phi}_{po}\, e^{jM_x k\bar{x} - jk\sqrt{\bar{x}^2\cos^2\theta + \bar{y}^2\cos^2\theta}}. \tag{24}$$

The total sound power radiated by the secondary source $\bar{W}_s$ is the sum of the contributions due to secondary source radiating in isolation, plus the change to the source loading from the incoming plane wave

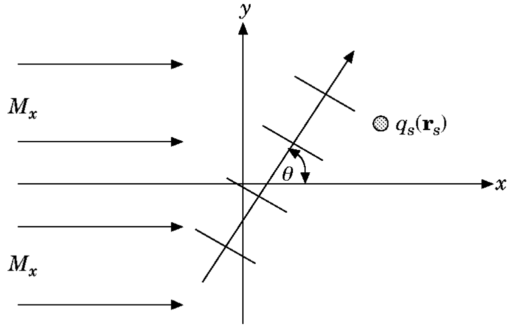

Figure 3. Free field point monopole source in a plane wave field with a uniform flow.

$$\bar{W}_s = \tfrac{1}{2} Z_M |\tilde{Q}_s|^2 + (\omega\rho_0/2)\,\mathrm{Im}\{\tilde{\phi}_p^*(\mathbf{r}_s)\tilde{Q}_s\}. \quad (25)$$

The minimum secondary source power output can be readily deduced by following a procedure identical to that outlined in section 4 to solve for the optimal source strength $\tilde{Q}_{so}$ and the maximum sound power of absorption $\bar{W}_{sa}$. This gives

$$\tilde{Q}_{so} = \pi c_0 \beta^2 \tilde{\phi}_p(\mathbf{r}_s)/2\omega, \qquad \bar{W}_{sa} = -\tfrac{1}{2}\pi\rho_0 c_0 \beta^2 |\tilde{\phi}_{po}(\mathbf{r}_s)|^2. \quad (26, 27)$$

The velocity potential amplitude $\tilde{\phi}_{po}$ is *independent* of the flow speed as shown by the form of the inhomogeneous wave equation specified by equation (10). The presence of a uniform flow therefore reduces the sound power that the point source can absorb by the factor $\beta^2 = (1 - M_x^2)$: the same factor which amplifies its sound power radiation. Thus, sound power radiation and sound power absorption are essentially reciprocal processes. Active sound power absorption may not, therefore, be an efficient noise control strategy in very high speed flows. The effect of flow on the maximally absorbing source can be expressed as

$$\bar{W}_{sa}(M_x) = \bar{W}_{sa}(0)(1 - M_x^2). \quad (28)$$

The optimal sound power absorbed by a monopole source is *independent* of the angle between the direction of the flow field and the direction of acoustic propagation in the radiation field. This is because velocity potential amplitude, which constitutes the source loading for sound power radiation in uniform flow according to equation (24), does not depend on flow speed.

6. CONCLUSIONS

The presence of a uniform flow has been shown to have a consistently adverse effect on the ability of simple volume velocity sources to control free field sound power radiation. The influence of convection on the ability of volume velocity sources to control the radiation from other volume velocity sources, and also to absorb the sound power from an incident plane progressive wave, has been shown to vary as $O(M^2)$. Only applications for which the flow speed is very high are therefore significantly affected by the presence of a uniform flow, such as, for example, in the case of the turbofan engine.

ACKNOWLEDGMENTS

The sponsorship of this work by DRA Pyestock is gratefully acknowledged. The authors would like to thank Dr John Chapman for suggesting the use of the "similarity transformations" used in this paper. In particular, the authors are indebted to an anonymous reviewer of an earlier draft of this paper for his invaluable comments.

REFERENCES

1. P. A. NELSON, A. R. D. CURTIS, S. J. ELLIOTT and A. J. BULLMORE 1987 *Journal of Sound and Vibration* **105**, 173–178. The minimum sound power output of free field point sources and the active control of sound.
2. P. A. NELSON, A. R. D. CURTIS and S. J. ELLIOTT 1986 *Proceedings of InterNoise* 1986, *Cambridge*, *Massachusetts*, 601–606. On the active absorption of sound.
3. C. L. MORFEY 1971 *Journal of Sound and Vibration* **14**(2), 159–170. Acoustic energy in non-uniform flows.
4. P. JOSEPH, C. L. MORFEY and P. A. NELSON 1996 *Proceedings of the* 2nd *AIAA/CEAS Aeroacoustics Conference*. Active control of source sound power output in uniform flow.

5. H. LEVINE 1980 *Journal of the Acoustical Society of America* **67**, 1935–1946. Output of acoustical sources.
6. C. J. CHAPMAN 1994 *Internal report*, *DRA*, *Farnborough*, *England*. Doppler factors and similarity rules for the general solution of the convected wave equation.
7. C. L. MORFEY and H. K. TANNA 1971 *Journal of Sound and Vibration* **15**, 325–351. Sound radiation from a point force in circular motion.