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THERMAL NON-LINEAR EFFECTS ON UNDERWATER SOUND GENERATION BY LASER RADIATION

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In the present paper, the authors study the thermal non-linear effects on the excitation of sound generation which is applicable to the determination of the pressure in the direction of the laser beam and at an angle with the laser beam. An additional pressure term appears due to this non-linear effect. Considering the small amplitude waves, the impulse responses of the system corresponding to both the pressure terms are determined. The pressure corresponding to the main term is expressed as convolution between the impulse response and the time derivative of the laser intensity and that corresponding to the additional term is given as a convolution between the system's impulse response and double time derivative of the square of the energy released in time t. The effects of the thermal non-linearity on the sound pressure are studied, considering the different time profiles of the laser pulse. It is found that besides the thermal non-linearity, the pressure profile depends on laser parameters and the shape. It is found that the pressure profiles are drastically different for the small and long thermoacoustic arrays. The thermal non-linearity affects the pressure waveform and the effect is pronounced for a short source. The results obtained are compared with those of previous studies. A reasonable explanation of the experimental observations of anomalous behaviour of underwater sound generation is also provided, on the basis of thermal non-linearity.

1. INTRODUCTION

The excitation of underwater acoustic waves from an airborne laser source has been extensively studied theoretically as well as experimentally over two decades [1–3, and references therein]. The main advantage of this type of sound generation is that there is no need for having a real transducer in the water and the generated sound is usually very directive. By regulating the laser parameters it is possible to implement remote control on the frequency, directivity and intensity of sound waves in a liquid. The laser-induced sound waves in different media have a wide range of applications in many areas of science, namely optoacoustic concentrators [4], optoacoustic non-destructive testing [5], photoacoustic spectroscopy [6, 7], thermoacoustic medical diagnostics [7] and thin-film ultrasonic

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measurements [8]. It has also been suggested that the optoacoustic concentrator can be of great advantage to the destruction of internal formations of biological origin, e.g., kidney stones [4]. Laser-induced underwater sound waves have also been generated in sea water [9]. Some oceanographic experiments have also been made to monitor wind waves by optoacoustic sources [10]. Pierce and Hsieh [1] have proposed a sophisticated design of an optical system for laser-induced heating of the ocean surface, indicating the possibility of detecting thermoacoustic signals several kilometres away from the source. Experimental observations show that the peak amplitude of the thermoacoustic signal can be substantially increased by moving the laser beam at a velocity close to that of sound in water [12]. Berthelot and Sayal *et al*. have suggested that the narrow frequency band signals can be achieved with high repetition rate Gaussian [13] and half-sine [14] pulsed lasers by proper choice of laser parameters. Recently, Zavtrak [15] has suggested an acoustic laser with dispersed particles as an analog of a free-electron laser.

For moderate densities of released laser energy in the liquid, the thermal expansion mechanism is the main process to generate the sound in the medium. Within the framework of the linear model, very effective theories have been developed to explain the sound generation in different conditions. Attention has also been paid to the non-linear effects for the optical generation of acoustic waves in a liquid [16–18]. Vitshas *et al*. [16] studied the non-linear dependence of the amplitude of the acoustic signal on the energy density of the incident radiation and determined the energy threshold for the onset of evaporation and flashing. Bozhkov *et al*. [17] reported on the experimental observations of the non-linear evolution of large amplitude acoustic pulses in an aqueous solution of cupric chloride. They observed the formation of weak shock waves and non-linear temporal broadening of acoustic pulses. Davydov and Korchikov [18] investigated experimentally and theoretically the non-linear effects associated with waveform of the acoustic pulse generated in water with a suspension of solid material (carbon) during laser heating. It was found that the onset of a non-linear mechanism significantly affects the waveform of the acoustic pulse. Kolomenskii *et al*. [4] have theoretically and experimentally studied a non-linear optoacoustic concentrator, i.e., the production of intense acoustic pulses in a localized volume of a medium.

In the process of powerful laser beams impinging on water there is a considerable increase in the temperature of the water. As a result the thermodynamical parameters of the medium can no longer, in general, be considered as constant. Non-linear effects associated with the temperature dependence of thermodynamic coefficients become appreciable even for moderate released energy densities in the medium. A non-linear theory of the thermal mechanism of sound generation has been developed by Dunina *et al*. [19]. They took into account the non-linear effects associated with variation of thermal expansion coefficient during the absorption of a laser pulse. Using the farfield approximations they applied their theory only in the direction of laser beams.

In experiments the receiver is usually located in the direction making some angle with a laser beam incidence. In this paper, a theory of excitation of sound generation which includes the thermal non-linear effects is presented. The theory is applicable to determining the pressure in the direction of the laser beam and at an angle with the laser beam. In this investigation, the authors study the thermodynamic effect due to the variation of thermal expansion coefficient of water during the absorption of laser pulses for small amplitude waves and therefore neglect the non-linear hydrodynamic effects.

The plan of the paper is as follows: in section 2, the problem is formulated and the expression for pressure is obtained. The pressure expression contains an additional term due to the variation of thermal expansion coefficient. Following Berthelot and Busch-Vishniac [20] and considering the small amplitude acoustic waves, both the pressure terms are expressed in terms of impulse responses. In section 3, the expressions for impulse responses corresponding to both the pressure terms are derived. Section 4 is devoted to a discussion of the pressure wave forms for different time profiles of laser pulses. In section 5, the main conclusions are presented.

2. FORMULATION OF THE PROBLEM

The acoustic pressure pulses with amplitudes substantially smaller than ρc^2, where ρ is the equilibrium value of density, c is acoustic velocity, can be well described by linear acoustic theory, without considering the non-linear hydrodynamic effects. However, in some situations the non-linear thermodynamic effects may be significant even for small amplitude pressure pulses. The generation of small amplitude sound waves is considered by laser radiation normally incident on the free surface of water. Neglecting the non-linear hydrodynamic effects, the dynamics of water can be described by the continuity equation, equation of momentum and equation of state as follows:

$$\frac{\partial \rho'}{\partial t} + \rho(\nabla \cdot \mathbf{v}') = 0, \tag{1}$$

$$\rho \frac{\partial \mathbf{v}'}{\partial t} = -\nabla p, \tag{2}$$

$$p = c^2\rho' + \rho c^2 \beta(T)T', \tag{3}$$

where p is the change in pressure, $\mathbf{v}'$ and ρ' are changes in fluid velocity and density, β is the thermal expansion coefficient, the temperature $T = T_0 + T'$, where T_0 is the initial temperature and T' is its increment.

It is well known that the effect of temperature on density ρ and specific heat c_p of water is negligible in comparison with the thermal expansion coefficient. Therefore, ρ and c_p are treated as constant and β as a function of temperature. From equations (1)–(3), the following inhomogeneous wave equation is obtained to describe the generation of acoustic waves by laser radiation

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = -\rho \frac{\partial^2}{\partial t^2}(\beta T'). \tag{4}$$

The general solution of equation (4) can be given in the following form

$$p = \frac{\rho}{4\pi}\int_V \frac{1}{r}\frac{\partial^2}{\partial t^2}\beta T'(t')\,\mathrm{d}V, \tag{5}$$

where $t' = t - r/c$, r is the distance from the source to the observation point.

Using Taylor's expansion, the thermal expansion coefficient $\beta(T)$ is expanded as

$$\beta(T) = \beta_0 + \beta_1 T', \tag{6}$$

where

$$\beta_0 = \beta(T_0) \quad \text{and} \quad \beta_1 = (\mathrm{d}\beta/\mathrm{d}T)_{T=T_0}.$$

In the above equation the higher order terms are neglected in comparison with first order. If the characteristic energy release time is sufficiently small in comparison with the characteristic time of longitudinal heat diffusion, thermal conduction can be neglected and it can be considered the temperature increment T' is given by the expression

$$T' = q_1(t)/\rho c_p, \tag{7}$$

where $q_1(t)$ is the energy density released at time t. For a uniform distribution of light intensity in the cross-section of the laser beam, $q_1(t)$ is given by

$$q_1(t) = \frac{A\alpha}{S} E(t)\, e^{-\alpha z}, \tag{8}$$

where α is the absorption coefficient of water, S is the transverse cross-sectional area of the laser beam, $E(t)$ is the energy released in the water up to time t, and A is the transmission coefficient of light across the interface.

Using expressions (6–8) in equation (5), one obtains

$$p = p_1 + p_2, \tag{9}$$

where

$$p_1 = \frac{A\beta_0\alpha}{4\pi c_p} \int_V \frac{e^{-\alpha z}}{r} I_t(t')\, dS\, dz, \tag{10}$$

$$p_2 = \frac{A^2\beta_1\alpha^2}{2\pi\rho c_p^2} \int_V \frac{e^{-2\alpha z}}{r} [I^2(t') + E_S(t')I_t(t')]\, dS\, dz, \tag{11}$$

where $E_S(t) = E(t)/S$ is the surface energy density and $I(t)$ is the intensity of the laser beam at time t.

In the present investigation, only small amplitude waves are considered, i.e., $p_1, p_2 \ll \rho c^2$. The pressure term p_1 corresponds to the linear theory and the pressure term p_2 corresponds to non-linear effects associated with the variation of the thermal expansion coefficient during absorption of the laser light. The first term is proportional to the time derivative of the laser intensity and the second one is proportional to $[I^2(t) + E_S(t)I_t(t)]$ or to the double time derivative of the square of the energy released in time t.

The impulse responses $h_1(t)$ and $h_2(t)$ of the thermoacoustic system corresponding to pressure terms p_1 and p_2 can be defined in the following manner:

$$p_1(t) = h_1(t) * I_t(t), \tag{12}$$

$$p_2(t) = h_2(t) * [I^2(t) + E_S(t)I_t(t)]. \tag{13}$$

Here, $h_1(t)$ and $h_2(t)$ contain all of the spatial characteristics of the system, and $I_t(t)$ and $[I^2(t) + E_S(t)I_t(t)]$ contain all of the temporal information of the source corresponding to p_1 and p_2. The asterisk (*) denotes convolution over time variable only. If the laser beam is collimated in x and y and tapered in z, the spatial dependence of the system can be decomposed into its vertical and horizontal components, provided that the receiver is situated far away from the source [20]. Let $h_a(t)$ be the horizontal impulse response and $h'_{L1}(t)$ and $h'_{L2}(t)$ be the vertical impulse responses corresponding to pressure terms p_1 and p_2. Then, $p_1(t)$ and $p_2(t)$ can be written as

$$p_1(t) = h'_{L1}(t) * h_a(t) * I_t(t), \tag{14}$$

$$p_2(t) = h'_{L2}(t) * h_a(t) * [I^2(t) + E_S(t)I_t(t)]. \tag{15}$$

In the present analysis, it is assumed that the laser beam diameter is sufficiently small and the finite beam width effects are considered to be negligible. Therefore, one can put $h_a(t) \to S\delta(t)$. In this situation, $p_1(t)$ and $p_2(t)$ can be put in the following form

$$p_1(t) = h_{L1}(t) * I_t(t), \tag{16}$$

$$p_2(t) = h_{L2}(t) * [I^2(t) + E_S(t)I_t(t)], \tag{17}$$

where

$$h_{L1}(t) = Sh'_{L1}(t) \quad \text{and} \quad h_{L2}(t) = Sh'_{L2}(t).$$

In the next section, the expressions for the impulse responses $h_{L1}(t)$ and $h_{L2}(t)$ are derived.

3. IMPULSE RESPONSES $h_{L1}(t)$ AND $h_{L2}(t)$

For a sufficiently small laser beam diameter, the pressures p_1 and p_2 in equations (10) and (11) are given by

$$p_1 = \frac{A\beta_0\alpha S}{4\pi c_p}\left[\int_0^\infty \frac{e^{-\alpha z}}{r} I_t(t - r/c)\,dz - \int_{-\infty}^0 \frac{e^{\alpha z}}{r'} I_t(t - r/c)\,dz\right], \tag{18}$$

$$p_2 = \frac{A^2\beta_1\alpha^2 S}{2\pi\rho c_p^2}\left[\int_0^\infty \frac{e^{-2\alpha z}}{r}[I^2(t - r/c) + E_S(t - r/c)I_t(t - r/c)]\,dz - \int_{-\infty}^0 \frac{e^{2\alpha z}}{r'}[I^2(t - r/c) + E_S(t - r/c)I_t(t - r/c)]\,dz\right]. \tag{19}$$

The second integrals in equations (18) and (19) are the contributions of the mirror images above the pressure release boundary, corresponding to linear and non-linear terms, respectively.

If x_R, y_R and z_R denote the co-ordinates of the receiver, then

$$r^2 = (x_R - x_c)^2 + (y_R - y_c)^2 + (z_R - z)^2, \tag{20}$$

$$r'^2 = (x_R - x_c)^2 + (y_R - y_c)^2 + (z_R + z)^2, \tag{21}$$

where x_c and y_c are the centre co-ordinates of the beam. From equations (10) and (11) the impulse responses, h_{L1} and h_{L2}, corresponding to linear and non-linear pressure terms are given by

$$h_{L1} = \frac{A\beta_0\alpha S}{4\pi c_p}\int_z \frac{e^{-\alpha z}}{r}\delta(t - r/c)\,dz\text{—mirror image}, \tag{22}$$

$$h_{L2} = \frac{A^2\beta_1\alpha^2 S}{2\pi\rho c_p^2}\int_z \frac{e^{-2\alpha z}}{r}\delta(t - r/c)\,dz\text{—mirror image}. \tag{23}$$

To obtain the impulse responses $h_{L1}(t)$ and $h_{L2}(t)$, a standard relation

$$\delta f(z) = \left|\frac{df(z)}{dz}\right|^{-1}\delta(z - z_0), \tag{24}$$

is used, where z_0 is chosen such that $f(z_0) = 0$.

The receiver is considered to be located at a point (r_0, θ_0), where r_0 is the distance from the point where the laser beam impinges on the water surface (say O) to the receiver, and θ_0 is the angle between the beam direction and the line joining the point of receiver and O. Using the relations (20), (21) and (24), in equations (22) and (23), the impulse responses $h_{L1}(t)$ and $h_{L2}(t)$ are obtained in the simplified form as follows:

$$h_{L1}(t) = \frac{A\beta_0\alpha Sc}{4\pi c_p}\left\{\int_0^{r_0\cos\theta\beta_0} \frac{\mathrm{e}^{-\alpha z}}{|z - r_0\cos\theta_0|}\,\delta(z - (r_0\cos\theta_0 - \sqrt{(ct)^2 - r_0^2\sin^2\theta_0}))\,\mathrm{d}z\right.$$

$$+ \int_{r_0\cos\theta_0}^{2r_0\cos\theta_0} \frac{\mathrm{e}^{-\alpha z}}{|z - r_0\cos\theta_0|}\,\delta(z - (r_0\cos\theta_0 + \sqrt{(ct)^2 - r_0^2\sin^2\theta_0}))\,\mathrm{d}z$$

$$+ \int_{2r_0\cos\theta_0}^{\infty} \frac{\mathrm{e}^{-\alpha z}}{|z - r_0\cos\theta_0|}\,\delta(z - (r_0\cos\theta_0 + \sqrt{(ct)^2 - r_0^2\sin^2\theta_0}))\,\mathrm{d}z$$

$$\left.- \int_{-\infty}^{0} \frac{\mathrm{e}^{\alpha z}}{|z - r_0\cos\theta_0|}\,\delta(z - (r_0\cos\theta_0 - \sqrt{(ct)^2 - r_0^2\sin^2\theta_0}))\,\mathrm{d}z\right\}, \tag{25}$$

$$h_{L2}(t) = \frac{A^2\beta_1\alpha^2 Sc}{2\pi\rho c_p^2}\left\{\int_0^{r_0\cos\theta_0} \frac{\mathrm{e}^{-2\alpha z}}{|z - r_0\cos\theta_0|}\,\delta(z - (r_0\cos\theta_0 - \sqrt{(ct)^2 - r_0^2\sin^2\theta_0}))\,\mathrm{d}z\right.$$

$$+ \int_{r_0\cos\theta_0}^{2r_0\cos\theta_0} \frac{\mathrm{e}^{-2\alpha z}}{|z - r_0\cos\theta_0|}\,\delta(z - (r_0\cos\theta_0 + \sqrt{(ct)^2 - r_0^2\sin^2\theta_0}))\,\mathrm{d}z$$

$$+ \int_{2r_0\cos\theta_0}^{\infty} \frac{\mathrm{e}^{-2\alpha z}}{|z - r_0\cos\theta_0|}\,\delta(z - (r_0\cos\theta_0 + \sqrt{(ct)^2 - r_0^2\sin^2\theta_0}))\,\mathrm{d}z$$

$$\left.- \int_{-\infty}^{0} \frac{\mathrm{e}^{2\alpha z}}{|z - r_0\cos\theta_0|}\,\delta(z - (r_0\cos\theta_0 - \sqrt{(ct)^2 - r_0^2\sin^2\theta_0}))\,\mathrm{d}z\right\}. \tag{26}$$

The four integrals in equations (25) and (26) represent impulse responses from the four source regions. The array response can also be represented by regions in time instead of space. The impulse responses are divided into four time regions:

(1) $t \leqslant r_0\sin\theta_0/c$.

In this time region, the impulse responses reduce to

$$h_{L1}(t) = 0 \quad \text{and} \quad h_{L2}(t) = 0, \tag{27}$$

which state that acoustic emission from the array has to travel a distance $r_0\sin\theta_0$ before reaching the observation point. Therefore, the impulse responses corresponding to p_1 and p_2 are zero for time $t \leqslant r_0\sin\theta_0/c$.

(2) $r_0\sin\theta_0/c < t < r_0/c$.

During the time interval $r_0\sin\theta_0/c < t < r_0/c$, the acoustic response corresponds to the disturbances coming from regions 0 to $r_0\cos\theta_0$ and $r_0\cos\theta_0$ to $2r_0\cos\theta_0$ of the thermoacoustic source, therefore the impulse response h_{L1} corresponding to p_1 is given by the first two integrals in equation (25).

$$h_{L1}(t) = \frac{2K\,\mathrm{e}^{-\alpha r_0 \cos\theta_0}}{\sqrt{(ct)^2 - r_0^2 \sin^2\theta_0}} \cosh\left[\alpha\sqrt{(ct)^2 - r_0^2\sin^2\theta_0}\right], \tag{28}$$

where

$$K = A\beta_0 \alpha Sc/4\pi c_p.$$

Similarly, the impulse response h_{L2} corresponding to p_2 is given by the first two integrals in equation (26) as

$$h_{L2}(t) = \frac{2K_1\,\mathrm{e}^{-2\alpha r_0 \cos\theta_0}}{\sqrt{(ct)^2 - r_0^2 \sin^2\theta_0}} \cosh\left[2\alpha\sqrt{(ct)^2 - r_0^2\sin^2\theta_0}\right], \tag{29}$$

where

$$K_1 = A^2\beta_1\alpha^2 Sc/2\pi\rho c_p^2.$$

(3) $t = r_0/c$.

The pressure release boundary condition implies that the acoustic contribution from the point at $z = 0$ is zero, and therefore the acoustic response at time $t = r_0/c$ is given by the wavelet coming from the point $z = 2r_0\cos\theta_0$. Therefore, the impulse response h_{L1} corresponding to p_1 is given by the second integral in equation (25) as

$$h_{L1}(t) = \frac{K\,\mathrm{e}^{-2\alpha r_0\cos\theta_0}}{r_0\cos\theta_0}. \tag{30}$$

Similarly,

$$h_{L2}(t) = \frac{K_1\,\mathrm{e}^{-4\alpha r_0\cos\theta_0}}{r_0\cos\theta_0}. \tag{31}$$

(4) $t > r_0/c$.

For time $t > r_0/c$, the impulse response h_{L1} corresponding to p_1 is given by the last two integrals in equation (25).

$$h_{L1}(t) = -\frac{2K\,\mathrm{e}^{-\alpha\sqrt{(ct)^2 - r_0^2\sin^2\theta_0}}}{\sqrt{(ct)^2 - r_0^2\sin^2\theta}} \sinh(\alpha r_0\cos\theta_0). \tag{32}$$

Similarly,

$$h_{L2}(t) = -\frac{2K_1\,\mathrm{e}^{-2\alpha\sqrt{(ct)^2 - r_0^2\sin^2\theta_0}}}{\sqrt{(ct)^2 - r_0^2\sin^2\theta_0}} \sinh(2\alpha r_0\cos\theta_0). \tag{33}$$

Therefore, the impulse responses can be written as follows:

$$h_{L1}(t) = \begin{cases} 0 & \text{for } t \leqslant t_1, \\ 2K\,\mathrm{e}^{-\Gamma}\mu^{-1}\cosh(\alpha\mu) & \text{for } t_1 < t < t_0, \\ K\,\mathrm{e}^{-2\Gamma}(r_0\cos\theta_0)^{-1} & \text{for } t = t_0, \\ -2K\,\mathrm{e}^{-\alpha\mu}\mu^{-1}\sinh(\Gamma) & \text{for } t > t_0, \end{cases} \tag{34}$$

$$h_{L2}(t) = \begin{cases} 0 & \text{for } t \leqslant t_1, \\ 2K_1\, e^{-2\Gamma}\mu^{-1} \cosh(2\alpha\mu) & \text{for } t_1 < t < t_0, \\ K_1\, e^{-4\Gamma}(r_0 \cos\theta_0)^{-1} & \text{for } t = t_0, \\ -2K_1\, e^{-2\alpha\mu}\mu^{-1} \sinh(2\Gamma) & \text{for } t > t_0, \end{cases} \tag{35}$$

where $t_0 = r_0/c,\ t_1 = r_0 \sin\theta_0/c,\ \Gamma = \alpha r_0 \cos\theta_0,\ \mu = c\sqrt{t^2 - t_1^2}$,

$$K = \frac{A\beta_0 \alpha c S}{4\pi c_p} \quad \text{and} \quad K_1 = \frac{A^2 \beta_1 \alpha^2 c S}{2\pi\rho c_p^2}.$$

The effect of thermal non-linearity can be characterized by the quantity

$$N \approx \frac{A\beta_1 E_S \alpha}{\beta_0 c_p \rho}, \tag{36}$$

which describes the ratio between the second and first terms in equation (9), using equations (16), (17), (34) and (35). Therefore, the effect of thermal non-linearity can be neglected only if $N \ll 1$.

4. PRESSURE WAVEFORMS FOR DIFFERENT TIME PROFILES OF LASER PULSE

In this section, the effects of thermal non-linearity on the pressure waveforms for different time profiles of laser are discussed. The three types of time profiles of laser pulse are considered.

4.1. CASE I: PARABOLIC LASER PULSE

First, a parabolic time profile laser pulse is considered:

$$I(t) = \frac{6E_0 t}{S\tau_p^2}\left(1 - \frac{t}{\tau_p}\right), \tag{37}$$

where E_0 and τ_p are, respectively, the laser pulse total energy and pulse width. The coefficient of the profile is so chosen that at time $t = \tau_p$, the energy $E(t) = E_0$, the total energy of the laser pulse. The pressure waveforms are obtained by using equations (16), (17), (34) and (35) in equation (9). Corresponding to linear theory, only the term p_1 is considered.

In Figures 1 and 2, the pressure waveforms with time are plotted for $T_0 = 10$ and $5°$C, respectively. It is assumed that the observation point is fixed at a position ($r_0 = 2$ m, $\theta_0 = 30°$). For the numerical analysis, the following values are used: the laser pulse width $\tau_p = 0{\cdot}08$ ms, laser pulse radius $a = 0{\cdot}5$ mm, laser energy density = 2 MJ/m^2, and the value of $\alpha = 17$ Np/m, which corresponds to the effective length, $L = \alpha^{-1}$, of the source 5·88 cm. Two positive peaks are obtained in the pressure waveforms. The dashed curves corresponding to linear theory can be understood easily by impulse response of the system and the time profile of the laser. For the parabolic laser pulse the derivative of the intensity I_t decreases linearly from a maximum positive value to a minimum negative value, becoming zero at $t = \tau_p/2$, which is greater than $(t_0 - t_1)$, the time duration of the positive impulse response. The impulse response is zero for time $t \leqslant t_1$, hence, zero pressure is obtained for time $t \leqslant t_1$. During $t_1 < t < t_0$, positive impulse response convolve with positive I_t and positive pressure is obtained. For $t > t_0$, impulse response is negative and its magnitude decreases with time. Now, looking at impulse response and I_t, it is very easy to see that for time greater than t_0, first negative pressure is obtained (the dip at

$t \approx t_0 + \tau_p/2$) and after a certain time positive pressure is again established. The second positive peak is obtained at $t \approx t_0 + \tau_p$. For time above this, only the negative impulse (which is decreasing in magnitude with time) convolve with I_t. The continuous curves in Figures 1 and 2 correspond to the present theory. It can be seen that for a chosen length of the thermoacoustic source and energy density of the laser, the effect of thermal non-linearity is significant at temperature $T_0 = 10°\text{C}$ (Figure 1), and is pronounced at temperature $T_0 = 5°\text{C}$ (Figure 2).

Different types of pressure waveform can be obtained by changing the laser pulse width or by changing the effective length of the thermoacoustic source. In Figures 3 and 4, the pressure waveforms are shown for very small and very large thermoacoustic sources, respectively. From the dashed curve of Figure 3, it can be seen that the pressure waveform of the small laser source corresponding to the linear theory is proportional to the second time derivative of the laser pulse intensity, which is consistent with previous studies [1]. It can be explained by the impulse response approach: the impulse response of a very small thermoacoustic source behaves as the time derivative of a delta function centred at $t = t_0$. Now it is clear that the pressure is proportional to

$$I_t(t) * \delta_t(t - t_0) = I_{tt}(t - t_0). \tag{38}$$

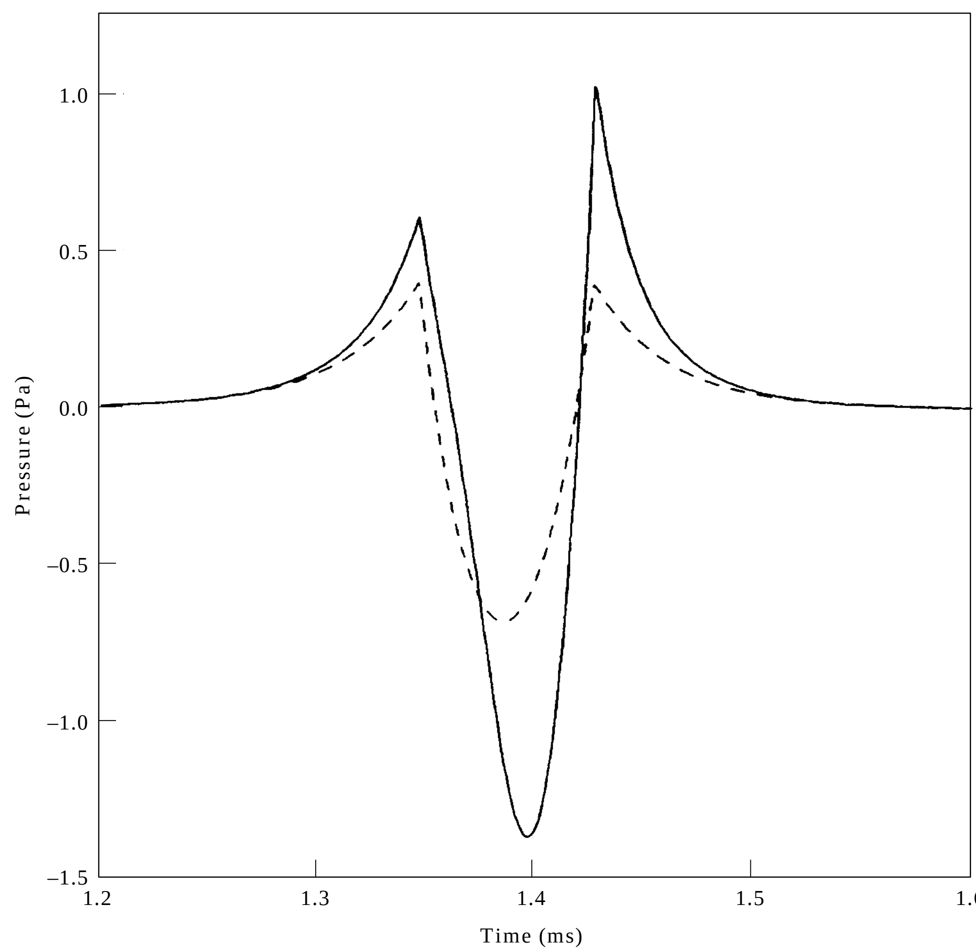

Figure 1. Pressure waveforms for parabolic laser pulse equation (37), for $r_0 = 2$ m, $\theta_0 = 30°$, laser beam radius $a = 0{\cdot}5$ mm, $\alpha = 17$ Np/m, $\tau_p = 0{\cdot}08$ ms and $T_0 = 10°\text{C}$ for: linear model (-----); present theory (——).

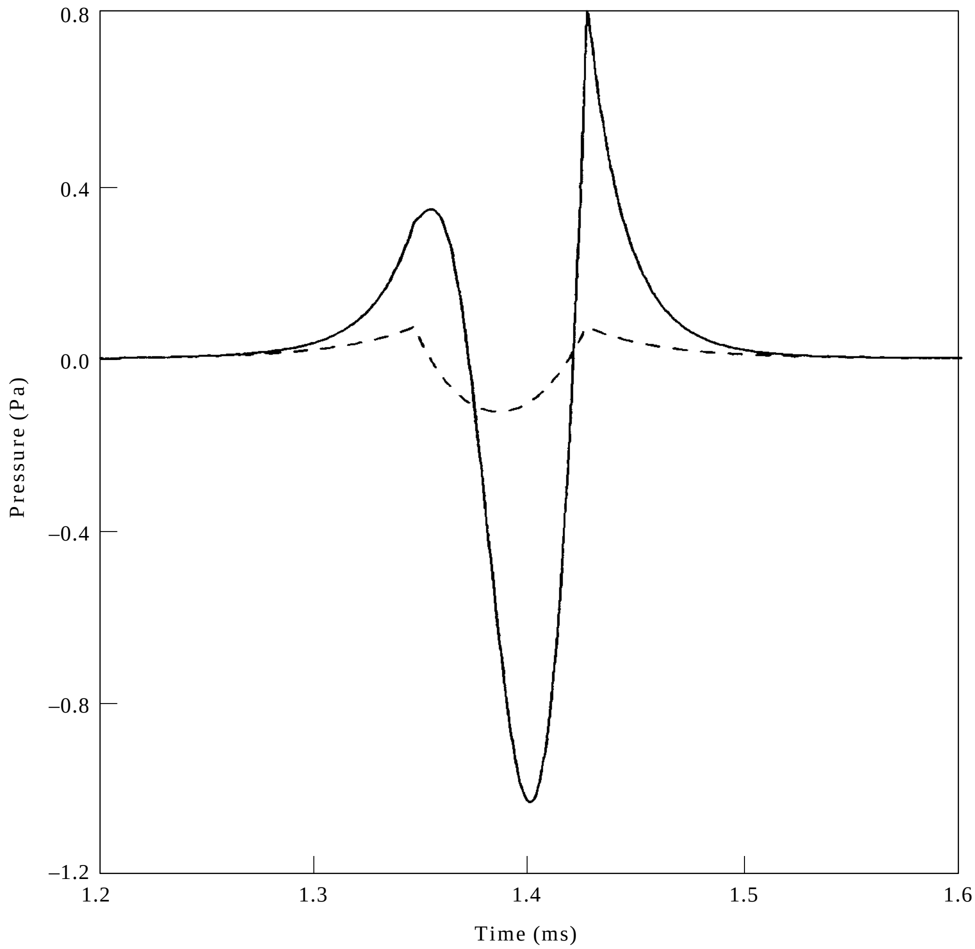

Figure 2. Pressure waveforms for parabolic laser pulse equation (37), for $r_0 = 2$ m, $\theta_0 = 30°$, laser beam radius $a = 0{\cdot}5$ mm, $\alpha = 17$ Np/m, $\tau_p = 0{\cdot}08$ ms and $T_0 = 5°$C for: linear model (-----); present theory (——).

Comparing Figures 1 and 3, it can be observed that the pressure signal generated by the short source is received later in comparison to the longer source, even though the receiver is situated at the same point for both cases. The physical reason is that for a very long source, i.e., for a small α, the first signal travels the shortest distance, $r_0 \sin \theta_0$, to reach the receiver. However, for a small effective length of the source, i.e., for a large α, due to the smallness of array, the first wavelet has to travel a great distance, and hence it reaches the receiver at a later time.

Figure 3, shows that for a large α, i.e., for a very small length source, for a chosen energy density of the laser pulse the thermal non-linear effects are very significant even at temperature $T_0 = 20°$C. The physical reason for this is that for a very large α of the medium, the laser source heats a small length column and hence a small volume of the medium. Therefore, the temperature variation during the absorption of the laser source is large which increases the thermal non-linear effect.

From the dashed curve of Figure 4, it can be observed that the pressure waveform of the long thermoacoustic source is proportional to the inverted laser pulse intensity, which is consistent with previous studies [1]. It can be seen that the thermal nonlinear effects are not very significant at high temperatures. This is because of the small variation of the temperature of the medium due to the long length column of the medium. However, the thermal non-linear effects cannot be neglected near 4°C even for a long length source. In that situation, the variation of the thermal expansion coefficient with temperature cannot be neglected in comparison to the thermal expansion coefficient of water at an initial temperature.

From equation (36), it can be seen that the effect of thermal non-linearity is significant for large energy density of the laser beam and also for large β_1/β_0. For water, the ratio β_1/β_0 becomes very large for $T_0 \approx 4$°C. Therefore, for water the effect of thermal non-linearity is strongest for a temperature close to 4°C. The effect of thermal non-linearity is increased for strong absorption of laser radiation in liquids, e.g., containing suitable dyes or for CO_2 laser in comparison to Nd-YAG laser.

For water, the thermal expansion coefficient changes sign at about 4°C. Above this temperature water expands on heating and below it contracts. However, the experimental

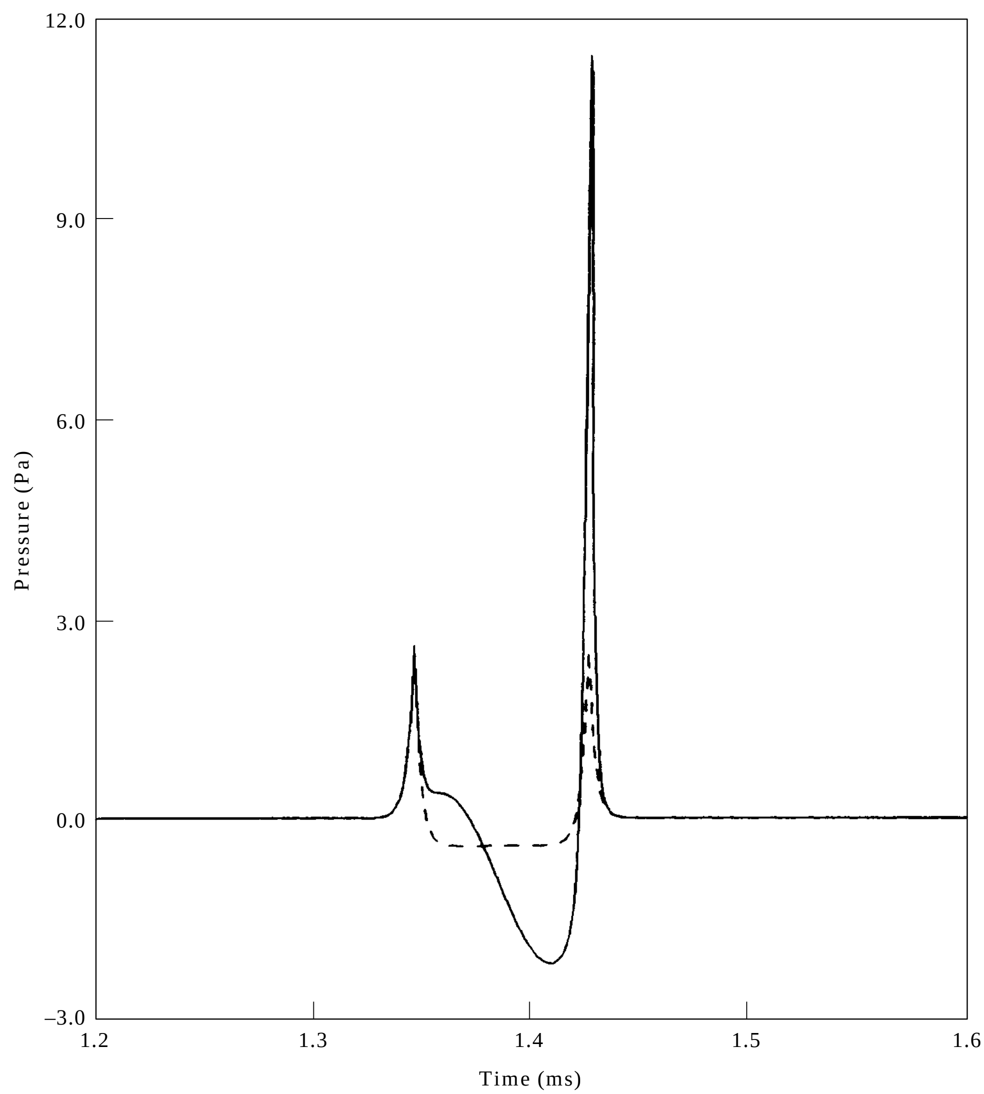

Figure 3. Pressure waveforms for parabolic laser pulse equation (37), for $r_0 = 2$ m, $\theta_0 = 30°$, laser beam radius $a = 0{\cdot}5$ mm, $\alpha = 200$ Np/m, $\tau_p = 0{\cdot}08$ ms and $T_0 = 20$°C for: linear model (-----); present theory (——).

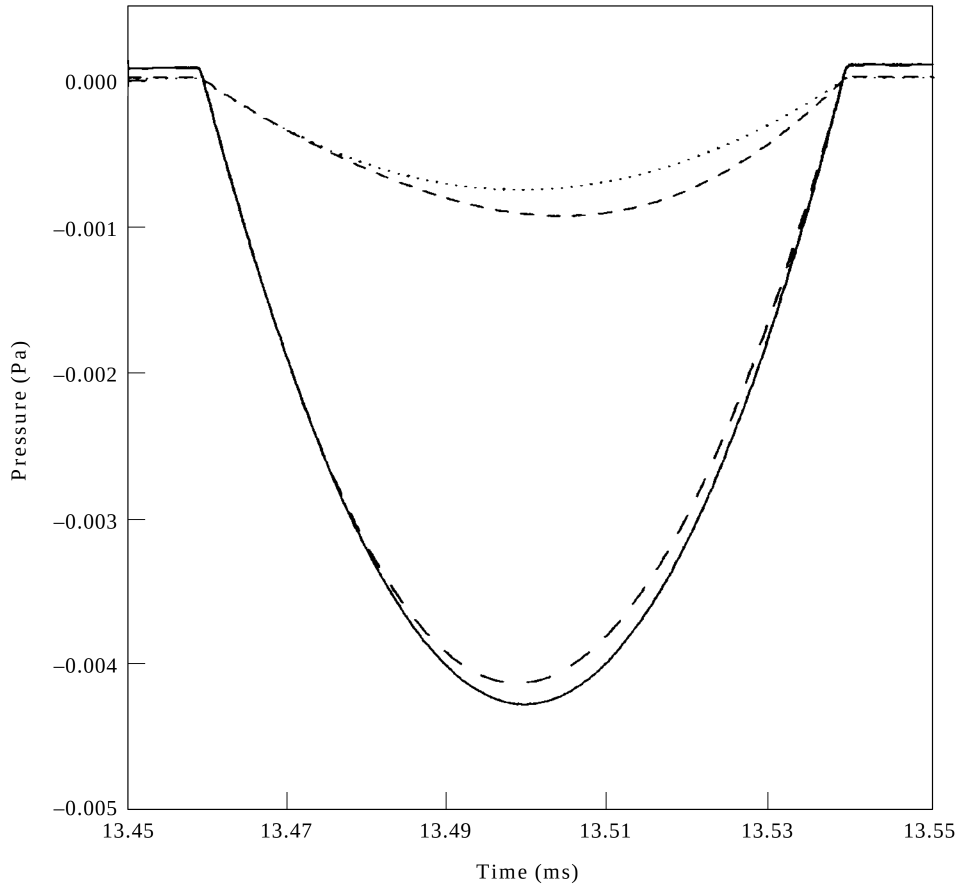

Figure 4. Pressure waveforms for parabolic laser pulse equation (37), for $r_0 = 20$ m, $\theta_0 = 30°$, laser beam radius $a = 0{\cdot}5$ mm, $\alpha = 0{\cdot}5$ Np/m, $\tau_p = 0{\cdot}08$ ms: $T_0 = 10°$C corresponding to linear model (——) and according to present theory (——); $T_0 = 5°$C corresponding to linear model (· · · · · · ·) and according to present theory (-----).

observations show that the laser water interaction exhibits the so-called anomalous behaviour [21]: the sound signal falls and vanishes and changes sign quite below 4°C, at about 2°C. This anomalous behaviour of laser-induced sound generation near 4°C can be explained on the basis of thermal non-linearity. At $T_0 \approx 4°$C, β_0 becomes zero and the sound pressure corresponding to linear theory vanishes. However, considerable pressure is still obtained because of thermal non-linearity. Below this temperature, the non-linear parameter N (given by equation (36)) becomes negative. The pressure corresponding to the linear term has an opposite sign to that corresponding to the non-linear term. The total pressure should vanish and change sign at a critical temperature T_c when $N \approx -1$, i.e.,

$$\frac{A\beta_1 E_S \alpha}{\beta_0 c_p \rho} \approx -1. \tag{39}$$

For example, for Nd-YAG laser with energy density 0·5 MJ/m^2, the sound pressure would change sign at about 2°C. It should be noted that the critical temperature T_c may fall below this value, for example for large values of E_S and α.

4.2. CASE II: MODULATED LASER PULSE

In this subsection, the results of the pressure waveforms are presented for a modulated laser pulse given by

$$I(t) = \frac{25E_0}{2\tau_p^2 S}\, t\, e^{-5t/\tau_p}(1 - \cos(\omega t)), \tag{40}$$

where ω is the angular modulation frequency.

For the numerical analysis, laser energy density is chosen to be 2 MJ/m^2. Other parameters are as shown in Figures 5 and 6. For a chosen length of the source $L = 5{\cdot}88$ cm, the pressure waveform appears as shown in Figure 5. It can be seen that the pressure waveform has a periodic nature due to the modulation of the intensity of the laser pulse and a roughly exponential decay in the amplitude corresponding to the exponential decay in the laser intensity is obtained. By comparing Figures 5 and 6, the amplitude of the pressure waveform can be seen to increase due to the thermal non-linearity.

Berthelot and Busch-Vishniac [12, 20] have found that for a stationary laser source, their theory, which does not consider thermal non-linear effects, underestimates the observed pressure. In reference [20], for a stationary laser source, the experimentally measured

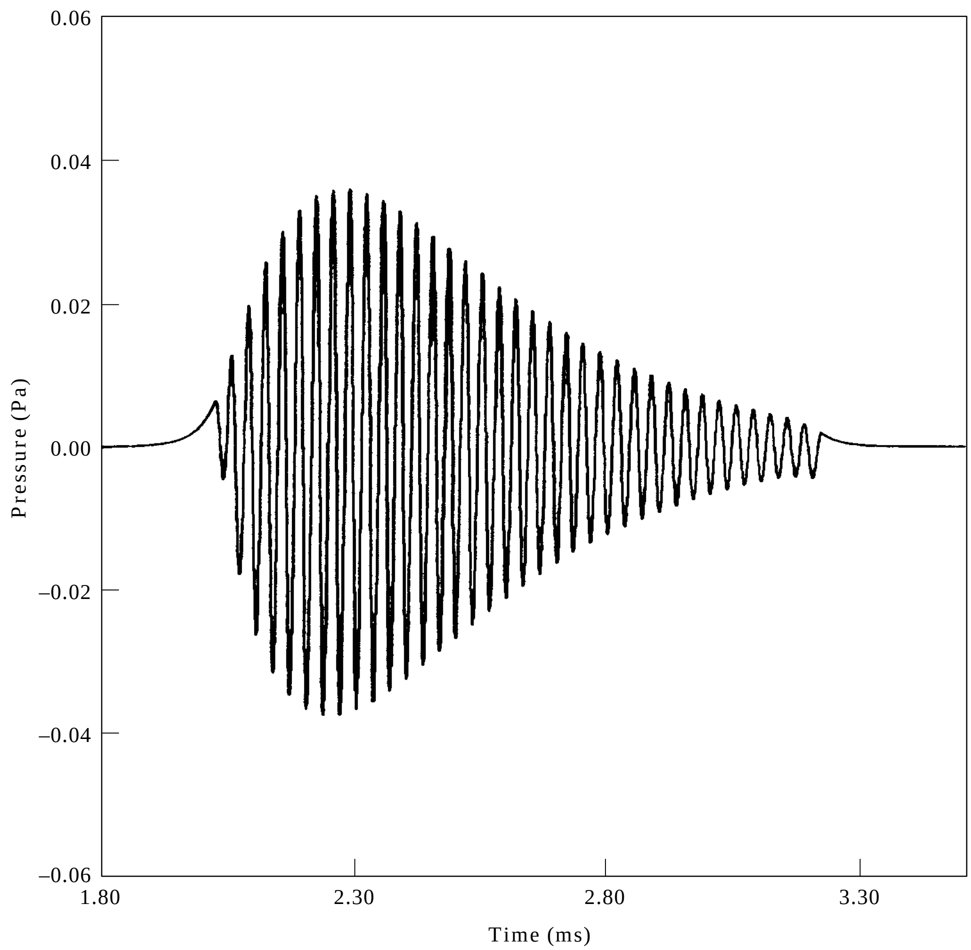

Figure 5. Pressure waveform according to linear model for modulated laser pulse equation (40), for $r_0 = 3$ m, $\theta_0 = 25°$, laser beam radius $a = 0{\cdot}5$ mm, $\alpha = 17$ Np/m, $\tau_p = 1{\cdot}2$ ms, frequency $f = 30$ kHz and $T_0 = 10°$C.

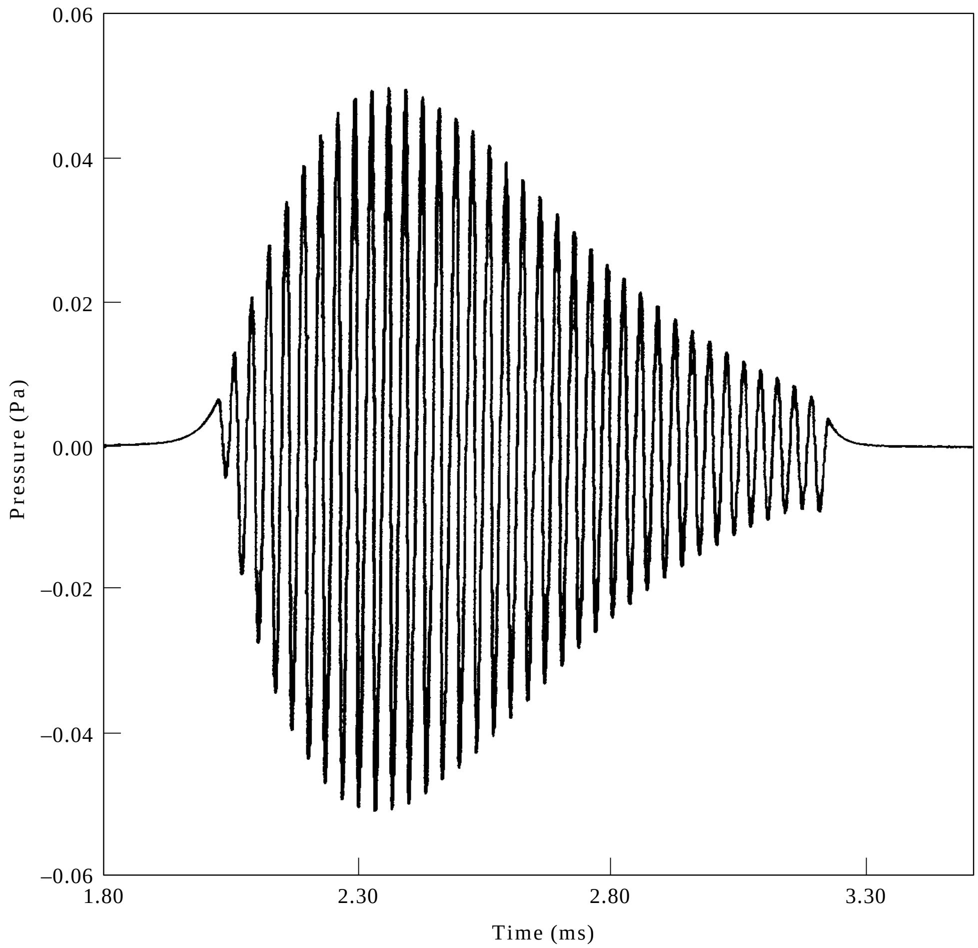

Figure 6. Pressure waveform according to the present theory for modulated laser pulse equation (40), for $r_0 = 3$ m, $\theta_0 = 25°$, laser beam radius $a = 0{\cdot}5$ mm, $\alpha = 17$ Np/m, $\tau_p = 1{\cdot}2$ ms, frequency $f = 30$ kHz and $T_0 = 10°$C.

acoustic level was 116·7 dB re: 1 μPa, whereas the theory predicted peak level of 109·4 dB re: 1 μPa. In reference [12], the predicted peak level was 123·1 dB re: 1 μPa, the average difference between experimental levels and theoretical levels was about 4·6 dB, and in general the theory of Berthelot and Busch-Vishniac underestimates the acoustic level obtained experimentally. According to the present theory, the above discrepancies can be substantially attributed to thermal non-linearity. However, the results concerned could not be compared quantitatively due to non-availability of temperature data in references [12, 20].

It may be noted that the effect of thermal non-linearity on the pressure waveform in the case of a moving laser source is insignificant. Because of the motion, the laser cannot deliver sufficient energy to a specific portion of the medium so that its temperature could be increased considerably.

The thermal non-linearity has been found to also produce the second harmonics of the modulation frequency. Near 4°C, the second harmonics are very significant. The physical reason for the production of the second harmonics is the weak non-linear self-interaction of the laser pulse fundamental harmonics.

Figure 7 shows that the pressure waveform of the long thermoacoustic array for a modulated laser source is proportional to an inverted laser pulse which is consistent with previous studies [1, 20] and the discussion of section 4.1. It is also seen in the same figure that the effect of thermal non-linearity becomes considerable in the neighbourhood of 4°C.

4.3. CASE III: GAUSSIAN LASER PULSE

In this subsection, the pressure waveform for a Gaussian laser pulse is discussed; this is given by

$$I(t) = \sqrt{\frac{5}{\pi}} \frac{E_0}{S\tau_p} \mathrm{e}^{-5(t-\tau_p)^2/\tau_p^2} \qquad 0 \leqslant t \leqslant 2\tau_p. \tag{41}$$

Figure 8 shows the pressure waveforms for a very short laser source; the length L of the array has been chosen to be 2 cm. The dashed curve is drawn neglecting the thermal non-linear effects while the continuous curve includes these effects. The pressure corresponding to the linear theory (dashed curve) is proportional to the double time derivative of the laser intensity, which is consistent with previous studies [1]. There are two

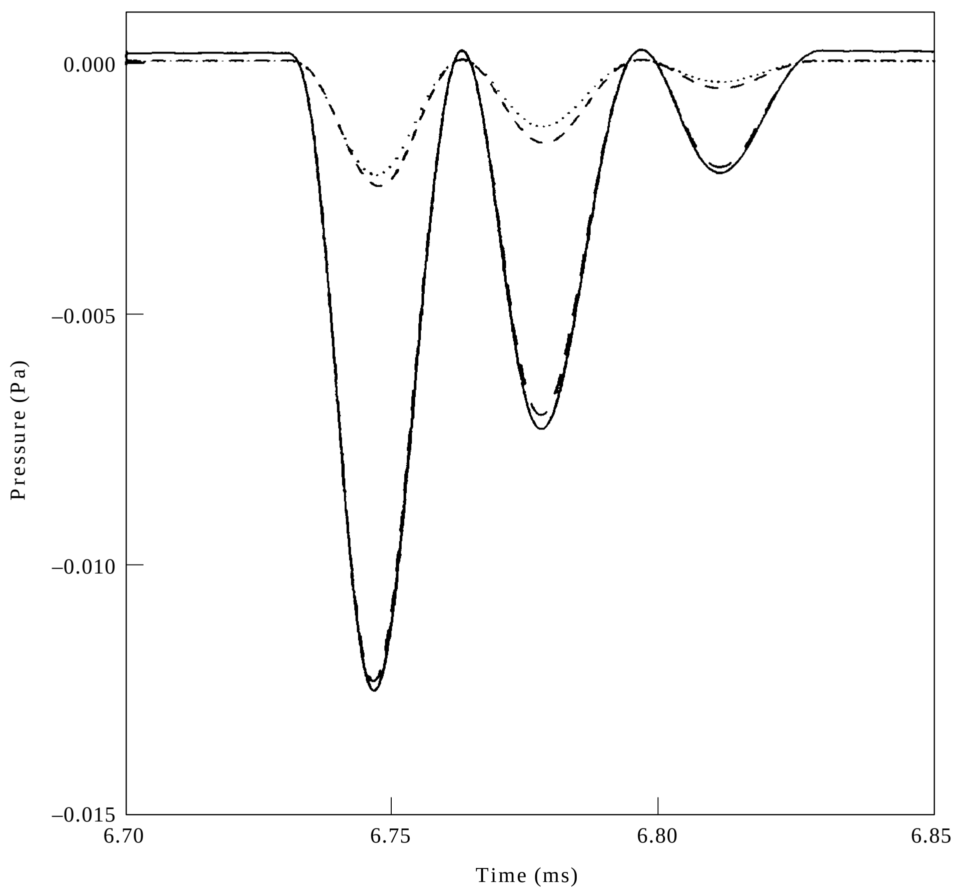

Figure 7. Pressure waveforms for modulated laser pulse equation (40), for $r_0 = 10$ m, $\theta_0 = 25°$, laser beam radius $a = 0{\cdot}5$ mm, $\alpha = 0{\cdot}8$ Np/m, $\tau_p = 0{\cdot}1$ ms, frequency $f = 30$ kHz; $T_0 = 10$°C corresponding to linear model (– – – –) and according to present theory (——); $T_0 = 5$°C corresponding to linear model (· · · · · · ·) and according to present theory (-----).

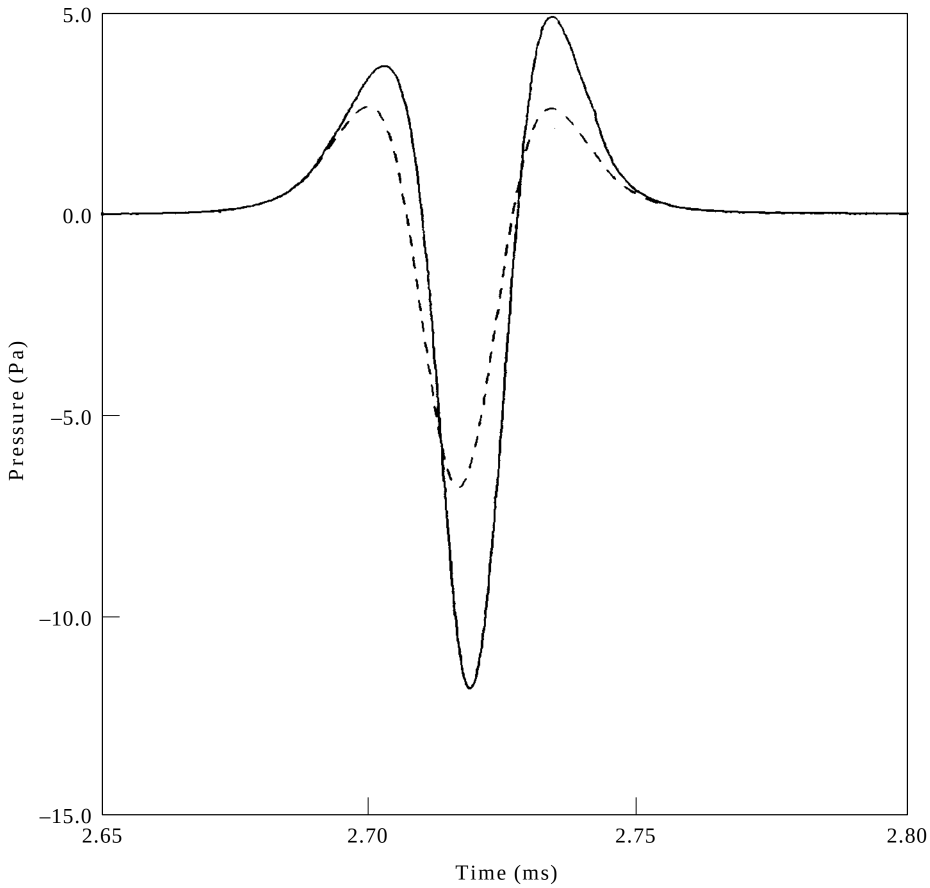

Figure 8. Pressure waveforms for Gaussian laser pulse equation (41), for $r_0 = 4$ m, $\theta_0 = 60°$, laser beam radius $a = 0{\cdot}5$ mm, $\alpha = 50$ Np/m, laser energy density = 2 MJ/m^2, laser time width $2\tau_p = 0{\cdot}05$ ms and $T_0 = 20°$C for: linear model (-----); present theory (——).

positive symmetric peaks of the pressure and a negative pressure dip at $t = t_0 + \tau_p$. The curve is symmetric about the $t = t_0 + \tau_p$ axis, which can also be explained by equations (38) and (41). Comparing both the curves in Figure 8, it is clear that for the chosen energy density of the laser beam, the thermal non-linear effects significantly modify the pressure pulse form even at temperature $T_0 = 20°$C. The negative pressure dip appears a little later. The pressure waveform becomes asymmetric, the second positive pressure peak becomes larger than the first positive pressure peak. The physical reason for this asymmetry is given as follows: as time increases, the laser source delivers more energy to the medium; consequently the temperature variation will be larger and thereby the thermal non-linear effects will be stronger. For a long thermoacoustic source excited by a Gaussian laser pulse, similar results are found as discussed earlier for parabolic and modulated laser pulses.

In the present study, the finite beam width effects have been neglected, which is fully justified for a sufficiently small laser beam diameter as considered in the above numerical analyses. For the finite sized laser beam one should also consider the effect of horizontal impulse response that includes diffraction effects [20]. In the present investigation, the small amplitude waves have been studied and the non-linear hydrodynamic effects neglected. For the large amplitude waves one should consider the exact fluid equations instead of equations (1)–(3). For large amplitude sound waves, the non-linear hydrodynamic effects may give rise to coherent non-linear structures like shock waves, solitons and vortices, which may be of great use in communication. Using the asymptotic method, the problem of small but finite amplitude non-linear sound waves will be addressed in the future.

5. CONCLUSIONS

In the present investigation, considering the thermal non-linear effects, the excitation of underwater sound generation has been studied. The theory is applicable to determining the pressure in the direction making an angle with a laser beam as well as in the laser beam direction, while Dunina *et al*. [19] have applied their theory only in the laser beam direction. Some interesting results for long and short laser pulses have been discussed and a comparison made with previous studies.

The effect of thermal non-linearity depends mainly on the temperature of the medium, surface energy density of the laser beam and length of thermoacoustic source. For water, the thermal non-linearity plays a significant role near a temperature of 4°C.

Considering the different time profiles of the laser pulse, it has been shown that the pressure waveforms are significantly different for long and small arrays. It is found that the thermal non-linearity affects the pressure waveform and the effect is pronounced for a short source.

For the modulated laser pulse, it is found that the thermal non-linearity also produces second harmonics, which is quite significant near 4°C.

A reasonable explanation has been provided for the experimental observation of the anomalous behaviour of laser water interaction around 4°C, on the basis of thermal non-linearity.

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