Journal of Sound and Vibration (1998) **212**(4), 637–647

GEOMETRIC DIFFUSION OF VIBRATIONAL ENERGY AND COMPARISON WITH THE VIBRATIONAL CONDUCTIVITY APPROACH

A. LE BOT

Laboratoire de Tribologie et Dynamique des Systèmes LTDS-UMR 5513 *CNRS*, *École Centrale de Lyon*, 36 *Avenue Guy de Collongue*, *B.P.* 163, *F*-69131 *Ecully*, *Cedex*, *France*

(*Received* 19 *March* 1997, *and in final form* 1 *December* 1997)

The vibrational conductivity approach is sometimes used to evaluate the spatial partition of the energy density of dynamical structural/acoustic systems in the high frequency range. This is a significant improvement on the Statistical Energy Analysis which provides only a single energy value per sub-system. However, this model is based on the underlying assumption that the wave field is constructed as a superposition of plane waves. This hypothesis may fail for largely non-diffuse fields. This paper is devoted to the study of other types of waves. The fields are still described in terms of energy quantities which are solved by using a differential equation written along the "streamlines of energy". Results depend strongly on the geometry of these streamlines. Whenever this geometry is known, for instance for plane, cylindrical and spherical waves, the differential equation may be solved. The plane wave case is in good agreement with the vibrational conductivity approach, whereas a large class of other waves are generated by this equation. Some numerical simulations illustrate these facts.

1. INTRODUCTION

It is well-known that finite element methods or boundary element methods applied to the solution of classical dynamical equations of motion are limited in frequency owing to the increasing number of degrees of freedom. This is the reason why non-conventional models such as *Statistical Energy Analysis* (SEA) have been studied and are successful today. Among several improvements of this last method, the vibrational conductivity approach may be used to model the spatial partition of the energy density inside each sub-system [1]. The main advantage of the use of this diffusion equation compared to the solving of classical governing equations lies in its low numerical cost. Moreover, thanks to a heat conduction analogy [1], it becomes possible to re-employ thermal softwares to solve vibrational problems in high frequency range.

However, some recent investigations [2, 3] show that the asymptotic behaviour of the energy density predicted by using the thermal analogy is in contradiction with the asymptotic behaviour of the energy density deduced from the equations of motion. The diffusion equation is usually constructed on the plane wave assumption. Therefore it cannot be correctly applied to situations where other types of waves dominate: infinite systems and heavily damped systems for instance. Actually, the applicability of the vibrational conductivity approach to one-dimensional systems seems to be agreed by a large consensus of those interested in that domain. Direct proofs of the diffusion equation

0022–460X/98/190637 + 11 $25.00/0/sv971442

based on analytical developments are available [4]. But the generalization of the diffusion equation to two-dimensional systems leads to the limitation mentioned above.

In this paper, the diffusion of the energy in multi-dimensional systems is studied by examining the geometry of the lines of propagation of energy. These curves are characterized by a geometric factor which depends on the position. Moreover, for any systems which behave like waveguides for two travelling waves, a simplified energy equation is derived. Plane waves are embedded as a particular case but this equation also includes other types of waves, such as cylindrical and spherical waves.

2. A REVIEW OF THE DIFFUSION EQUATION

This section summarizes the main steps of the derivation of the diffusion equation of vibrational conductivity. The complete formulations may be found in different papers cited below.

Two continuous energy quantities are involved in contrast to the discrete variables appearing in SEA: the total energy density W, which is a scalar field, and the active energy flow $\mathbf{I}$, which is a vector field. These quantities, introduced in the framework of this model, may be related with classical energy quantities deduced from motion equations in various ways. The idea common to all these interpretations is that energies considered here are classical energies from which all details irrelevant in a high frequency view, are removed. Usually, local averages over time, space, frequency or ensemble are involved.

The first step in deriving the energy equation is the local energy balance for a non-loaded region,

$$\mathbf{div} \cdot \mathbf{I} + p_{diss} = 0, \tag{1}$$

where p_{diss} is the power density being dissipated. The damping model adopted here is the same as in SEA: power density being dissipated is proportional to the energy density. Hence

$$p_{diss} = \eta \omega W, \tag{2}$$

where η is the hysteretic damping loss factor and ω is the circular frequency. The validity of this relationship has been discussed in the literature about SEA.

Finally, a local relationship expresses the energy flow in terms of energy density,

$$\mathbf{I} = (-c_g^2/\eta\omega)\,\mathbf{grad}\ W, \tag{3}$$

where c_g is the group velocity of the wave at hand. This relationship has been widely applied for one-dimensional cases [1, 5]. In reference [4], explicit calculations based on analytical solutions of governing equations for rods and beams, clearly establish the latter expression. The generalization to two-dimensional structures has been implemented in reference [1]. In references [6, 7], this relationship is demonstrated for plane waves and in references [6–8] several proofs have been proposed for wave fields built as a superposition of plane waves. An analogy with Fourier's thermal law is often stated: the energy propagates from high levels to low levels.

Substituting equations (2, 3) into the energy balance (1) yields

$$(-c_g^2/\eta\omega)\Delta W + \eta\omega W = 0. \tag{4}$$

Equation (4) is analogous to the steady state heat conduction equation with a convective term. The solutions of this equation have a slow space evolution in opposition with the energy densities predicted from the classical governing equations. The diffusion equation (4) predicts only the macroscopic evolution of the energy density without any detail on

the smallest disturbances which are not useful for medium and high frequency models. This is the sense of the local space averages introduced in reference [4]. Alternatively, the solution of this diffusion equation may be viewed as the frequency average of the energy density deduced from classical governing equations. As a compensation for this loss of information, a significant gain is obtained in the required computation time which allows the solving of vibrational problems up to high frequencies.

3. GEOMETRIC ANALYSIS OF STREAMLINES

In this section, a particular generalized co-ordinate system is attached to any problem. On the one hand, the diffusion equation will be rewritten in this system and, on the other hand an original equation will be derived in this system.

Consider the vector field **I**. It is a continuous function except at singularities originating from driving forces for instance. Where the field is continuous, it defines a family of lines of flow which are lines at every point tangent to the vector at that point. These curves are the lines of propagation of the energy.

In all examples which are of concern later on, the differential equations for these streamlines can be integrated. The algebraic equations obtained involve one or two parameters depending on the dimension. Furthermore, by integrating the differential equations of the family of surfaces perpendicular to the lines of flow, an additional parameter is obtained which matches with the arc length s measured along the lines. The set of these parameters can be viewed as a curvilinear co-ordinate system. The streamlines are then a co-ordinate line. More details about this procedure can be found in reference [10].

Let **t** denote the unit vector tangent to the streamlines. Obviously, at any point, **I** and **t** are collinear vectors. Then, $\mathbf{I} = I\mathbf{t}$ where I is the magnitude of **I**. In terms of the curvilinear co-ordinates, the divergence of a vector field $\mathbf{T} = T\mathbf{t}$ which is at every point collinear with the vector **t**, is

$$\mathbf{div} \cdot \mathbf{T} = \partial T/\partial s + \Gamma T, \tag{5}$$

where Γ is a geometric factor which depends only on the local geometry of the streamlines.

In order to obtain a geometric interpretation of this factor Γ, one can apply equation (5) to the vector field **t** itself. Then,

$$\mathbf{div} \cdot \mathbf{t} = \Gamma. \tag{6}$$

Now, referring to the divergence theorem, one finds

$$\int_V \mathbf{div} \cdot \mathbf{t} \, \mathrm{d}V = \oint_S \mathbf{t} \cdot \mathbf{n} \, \mathrm{d}S, \tag{7}$$

where V is a volume enclosed by the surface S and **n** is the outward unit vector normal to S. Then, for an infinitesimal volume V, $\mathbf{div} \cdot \mathbf{t}$ is found to represent the flux or net outflow per unit volume of the vector **t** from the surface S:

$$\mathbf{div} \cdot \mathbf{t} = \frac{1}{V} \oint_S \mathbf{t} \cdot \mathbf{n} \, \mathrm{d}S. \tag{8}$$

Finally, choose V as a beam of streamlines limited by two sections S and S' as shown in Figure 1. As the vectors **t** and **n** are perpendicular along the streamlines, the scalar product $\mathbf{t} \cdot \mathbf{n}$ is non-zero only on the two sections. Since the volume is $V = S\Delta s$, one has

$$\Gamma = \frac{1}{S}\frac{\Delta S}{\Delta s}, \tag{9}$$

where $\Delta S = S' - S$. Then the factor Γ can be interpreted as the rate of relative increase of the cross-section of a beam of streamlines. This clarifies the dependence on the local geometry of such streamlines.

Later on, interest will be in the study of the propagation of the energy along a particular streamline. So, this streamline has an arc length s and a geometric factor $\Gamma(s)$ which depends only on the position s. Moreover, ordinary derivatives which respect to s are used in place of partial derivatives. Now, one can re-write equations (3, 4) along this line of propagation of energy. Using the well-known expressions of the gradient and the Laplacian in generalized co-ordinate system [10] and remembering that the only non-vanishing component of the intensity vector is the first one, one obtains, first,

$$I(s) = \frac{-c_g^2}{\eta\omega}\,\mathrm{d}W/\mathrm{d}s \tag{10}$$

for the expression of the energy flow in terms of energy density, and second,

$$\frac{\mathrm{d}^2W}{\mathrm{d}s^2} + \Gamma(s)\frac{\mathrm{d}W}{\mathrm{d}s} - \left(\frac{\eta\omega}{c_g}\right)^2 W(s) = 0 \tag{11}$$

for the differential equation on the energy density for a non-loaded region.

4. TRAVELLING WAVES ALONG STREAMLINES

In this section, an alternative point of view is examined, leading to an energy equation different from equation (11).

Now, attention is focused on special cases where the field can be considered as a superposition of just two travelling waves: a s-positive travelling wave noted with a superscript $+$ and an s-negative one noted with a superscript $-$. This restriction includes all systems which behave as wave guides but also certain systems with a particular symmetry. The partial energy densities associated separately with these travelling waves are denoted by W^+ and W^- and the partial energy flows are denoted by I^+ and I^-.

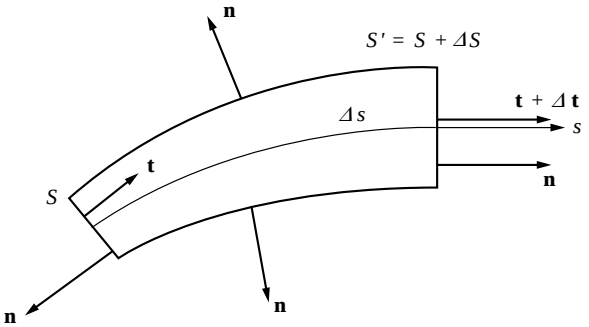

Figure 1. Geometric interpretation of the factor Γ.

As a travelling wave is a particular solution of the equation of motion, the power balance (1) may still be applied to one travelling wave:

$$\mathrm{d}I^{\pm}/\mathrm{d}s + \Gamma(s)I^{\pm}(s) + p^{\pm}_{diss} = 0 \tag{12}$$

for a non-loaded region. As one is concerned with travelling waves, simple relationships exist between energy flows and energy densities:

$$I^{\pm}(s) = \pm c_g\, W^{\pm}(s). \tag{13}$$

The minus sign before the group velocity on the right side stems from the direction of propagation. The power densities being dissipated are modelled as in SEA, so that

$$p^{\pm}_{diss} = \eta\omega W^{\pm}. \tag{14}$$

Now, substituting equations (14, 13) into the power balances (12) yields

$$I^{\pm}(s) = \frac{-c_g^2}{\eta\omega}\left[\frac{\mathrm{d}W^{\pm}}{\mathrm{d}s} + \Gamma(s)W^{\pm}(s)\right]. \tag{15}$$

When both waves travel simultaneously along the streamline, the complete energy density W is equal to the sum of the partial energy densities W^+, W^- of each separate wave plus an additional interference term. For a high frequency model, this interference term can be removed. This is a usual approximation in the high frequency literature especially in acoustics with ray methods. This approximation, for instance, may be justified by considering that the energy quantities are averaged over a small space domain [4]. An alternative view is to consider ensemble averages on different parameters as explained in reference [11]. It turns out that whatever interpretation is retained, it leads to simple relationships between the total energy quantities and partial ones:

$$W(s) = W^+(s) + W^-(s), \qquad I(s) = I^+(s) + I^-(s). \tag{16}$$

Then, a linear superposition principle for energy quantities is valid.

By adding equations (15), a relationship between I and W is obtained,

$$I(s) = \frac{-c_g^2}{\eta\omega}\left[\frac{\mathrm{d}W}{\mathrm{d}s} + \Gamma(s)W(s)\right], \tag{17}$$

which is quite different from equation (10). Finally, by introducing this relationship into the energy balance, one has

$$\frac{\mathrm{d}^2W}{\mathrm{d}s^2} + 2\Gamma(s)\frac{\mathrm{d}W}{\mathrm{d}s} + \left[\frac{\mathrm{d}\Gamma}{\mathrm{d}s} + \Gamma^2(s) - \left(\frac{\eta\omega}{c_g}\right)^2\right]W(s) = 0. \tag{18}$$

At this stage, it can be noticed that equations (17, 18) strongly depend on the geometric factor Γ which depends upon the geometry of the streamlines. So equations (17, 18) cannot be solved without the knowledge of this. These equations require one to know *a priori* the geometry of the streamlines. In other words, equations (17, 18) contain information about the magnitude of energy density and energy flow but not about the direction of the latter. However, in certain cases of simple geometry, this factor is known. Then calculations are possible. This situation is similar to Bernoulli's equation in fluid mechanics. Bernoulli's equation describes the energy balance along a streamline. Each time that such a streamline is known (pipes, emptying of tanks and so on), Bernoulli's equation provides a solution of the problem. However with very rare exceptions, irrotational motion for instance, Bernoulli's equation cannot be generalized over the whole domain.

5. PARTICULAR GEOMETRIES

In this section, solutions of equations (17, 18) are sought for some particular geometries and compared with the solutions of equations (10, 11).

5.1. PLANE WAVES

One-dimensional systems behave as waveguides. The energy propagates along a beam of parallel curves. So, the geometric factor Γ vanishes and the energy equations (10, 11) become:

$$I(s) = \frac{-c_g^2}{\eta\omega}\frac{\mathrm{d}W}{\mathrm{d}s}, \qquad \frac{\mathrm{d}^2W}{\mathrm{d}s^2} - \left(\frac{\eta\omega}{c_g}\right)^2 W(s) = 0. \tag{19, 20}$$

In the same way, the energy equations (17, 18) lead to exactly the same reduced form.

The general solution of this set of equations is

$$W(s) = A^+ \,\mathrm{e}^{-(\eta\omega/c_g)s} + A^- \,\mathrm{e}^{(\eta\omega/c_g)s} \tag{21}$$

for the energy density and

$$I(s) = c_g\,[A^+ \,\mathrm{e}^{-(\eta\omega/c_g)s} - A^- \,\mathrm{e}^{(\eta\omega/c_g)s}] \tag{22}$$

for the energy flow. A^+ and A^- are two arbitrary constants which have to be determined by using appropriate boundary conditions usually expressed on I [4].

5.2. CYLINDRICAL WAVES

Consider a two-dimensional axisymmetric system. The factor Γ is equal to $1/s$ and the energy equations (10, 11) become

$$I(s) = \frac{-c_g^2}{\eta\omega}\frac{\mathrm{d}W}{\mathrm{d}s}, \qquad \frac{\mathrm{d}^2W}{\mathrm{d}s^2} + \frac{1}{s}\frac{\mathrm{d}W}{\mathrm{d}s} - \left(\frac{\eta\omega}{c_g}\right)^2 W(s) = 0. \tag{23, 24}$$

The general solution of this set of equations is

$$W(s) = A^+ K_0\left(\frac{\eta\omega}{c_g}s\right) + A^- I_0\left(\frac{\eta\omega}{c_g}s\right) \tag{25}$$

for the energy density and

$$I(s) = c_g\left[A^+ K_1\left(\frac{\eta\omega}{c_g}s\right) - A^- I_1\left(\frac{\eta\omega}{c_g}s\right)\right] \tag{26}$$

for the energy flow. In these relationships K_i and I_i denote respectively the modified Bessel functions of first and second kind of order i.

In contrast, the reduced forms of equations (17, 18) are

$$I(s) = \frac{-c_g^2}{\eta\omega}\left[\frac{\mathrm{d}W}{\mathrm{d}s} + \frac{1}{s}W\right], \qquad \frac{\mathrm{d}^2W}{\mathrm{d}s^2} + \frac{2}{s}\frac{\mathrm{d}W}{\mathrm{d}s} - \left(\frac{\eta\omega}{c_g}\right)^2 W(s) = 0. \tag{27, 28}$$

Equation (28) is different from equation (24), with the factor $2/s$ instead of $1/s$. This difference stems from the relationship (27) which clearly shows that the energy flow is not

proportional to the gradient of energy density. Thus, the analogy with Fourier's law, established for plane waves, is no longer valid.

The general solution of this set of equations is

$$W(s) = A^+ \frac{e^{-(\eta\omega/c_g)s}}{s} + A^- \frac{e^{(\eta\omega/c_g)s}}{s} \tag{29}$$

for the energy density and

$$I(s) = c_g \left[A^+ \frac{e^{-(\eta\omega/c_g)s}}{s} + -A^- \frac{e^{(\eta\omega/c_g)s}}{s} \right] \tag{30}$$

for the energy flow.

One can now compare some asymptotic developments of these solutions. The general solution of the governing equation for an infinite membrane is $H_0^{(2)}(ks)$ where k is the wavenumber and $H_0^{(2)}$ the Hankel function of the second kind and order zero. As the kinetic energy density is proportional to the square modulus of the displacement, an asymptotic development of the Hankel function for large arguments leads to an energy density $W(s) \propto e^{-\eta k_0 s}/s$ where k_0 is the undamped wavenumber. Moreover, for a membrane the group velocity is $c_g = \omega/k_0$, and then $W(s) \propto e^{-\eta\omega s/c_g}/s$. This result is in agreement with the first term of equation (29). The decrease is like $1/s$. A similar calculation for infinite plates without evanescent waves should give the same result. On the other hand, a far field development of the first term of solution (25) corresponding to an outgoing wave, is $W(s) \propto e^{-\eta\omega s/c_g}/\sqrt{s}$. The decrease is like $1/\sqrt{s}$. This disagreement indicates that the diffusion equation does not correctly predict the direct field.

5.3. SPHERICAL WAVES

The argument developed in the previous sub-section can be applied to the case of spherical waves. The factor Γ is equal to $1/s^2$ and equations (10, 11) take the particular forms

$$I(s) = \frac{-c_g^2}{\eta\omega} \frac{dW}{ds}, \qquad \frac{d^2W}{ds^2} + \frac{2}{s}\frac{dW}{ds} - \left(\frac{\eta\omega}{c_g}\right)^2 W(s) = 0. \tag{31, 32}$$

The general solution of this set of equations is

$$W(s) = A^+ \frac{e^{-(\eta\omega/c_g)s}}{s} + A^- \frac{e^{(\eta\omega/c_g)s}}{s} \tag{33}$$

for the energy density and

$$I(s) = c_g \left[A^+ \frac{e^{-(\eta\omega/c_g)s}}{s}\left(1 + \frac{c_g}{\eta\omega s}\right) + -A^- \frac{e^{(\eta\omega/c_g)s}}{s}\left(1 - \frac{c_g}{\eta\omega s}\right) \right] \tag{34}$$

for the energy flow.

The reduced forms of equations (17, 18) are

$$I(s) = \frac{-c_g^2}{\eta\omega}\left[\frac{dW}{ds} + \frac{2}{s}W\right], \qquad \frac{d^2W}{ds^2} + \frac{4}{s}\frac{dW}{ds} + \left[\frac{2}{s^2} - \left(\frac{\eta\omega}{c_g}\right)^2\right] W(s) = 0. \tag{35, 36}$$

Once again, the two latter equations are different from the two former. In particular, as already remarked for cylindrical waves, the energy flow is not proportional to the gradient of the energy density.

The general solution of equations (35, 36) is

$$W(s) = A^+ \frac{e^{-(\eta\omega/c_g)s}}{s^2} + A^- \frac{e^{(\eta\omega/c_g)s}}{s^2}, \qquad I(s) = c_g \left[A^+ \frac{e^{-(\eta\omega/c_g)s}}{s^2} + -A^- \frac{e^{(\eta\omega/c_g)s}}{s^2} \right]. \qquad (37, 38)$$

The decrease is like $1/s$ according to equation (33) and $1/s^2$ according to equation (37). But it is well-known that the acoustical energy decreases like $1/s^2$ in unbounded space.

6. NUMERICAL SIMULATIONS

The first simulation concerns a circular membrane with radius s_{max}. Three calculations have been carried out. The first one is a classic calculation. The governing equation is solved and then a linear combination $aH_0^{(1)}(ks) + bH_0^{(2)}(ks)$ is obtained for the transverse displacement. The transverse displacement is assumed to be known at s_{min} and to be zero at s_{max}. The constants a and b are then determined. Energy density and energy flow are deduced from the transverse displacement. Note that the hysteretic damping η is introduced in the expression of the tension of the membrane, which becomes a complex number. Secondly, the solutions (25, 26) of the diffusion equation are involved. Boundary conditions are the following: the energy flow vanishes at s_{max} and is assumed to be known at s_{min}. Obviously, the numerical value of the energy flow at s_{min} is estimated from the classical simulation. Finally, the third calculation is carried out with solutions (29, 30) in a similar manner as the previous one. These calculations have been applied to an axisymmetric membrane with group velocity $c_g = 340$ m/s, frequency $f = 1000$ Hz, damping loss factor $\eta = 0{\cdot}05$, $s_{min} = 0{\cdot}25$ wavelength and radius $s_{max} = 5$ wavelengths. Results are shown in Figure 2. The energy density predicted by the diffusion equation (4) is underestimated near the excitation point and overestimated in the far field. The decrease of this solution is clearly too weak. This shortcoming, emphasized in this numerical simulation of circular systems, is however observed for some square systems [3]. In opposition, the energy density predicted by the energy equation (18) is a smooth estimation

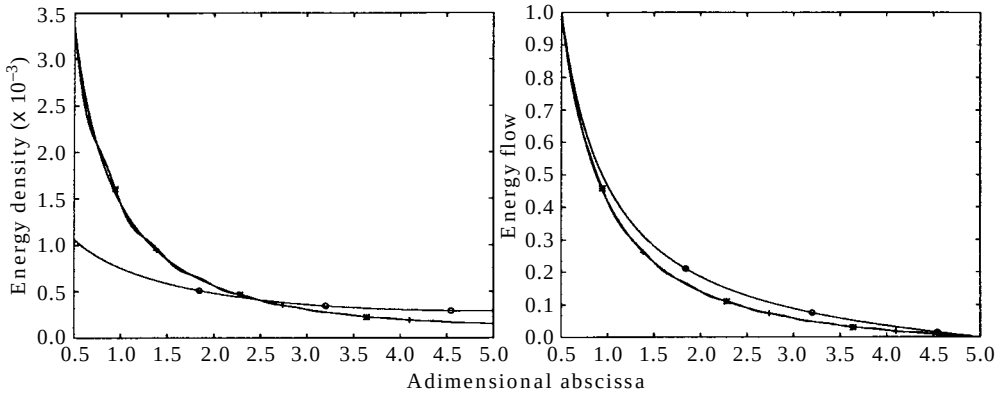

Figure 2. Comparison of energy densities and energy flows evaluated with three different methods for an axisymmetric membrane. *, Equation of motion; ○, diffusion equation (4); +, energy equation (18).

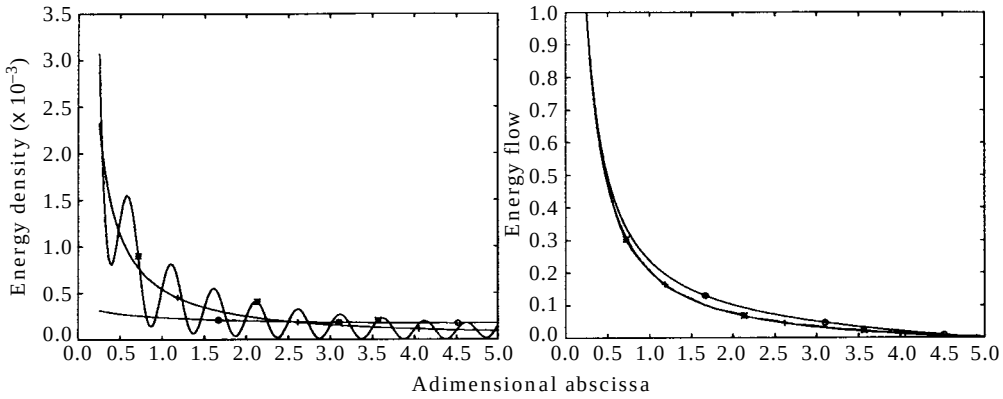

Figure 3. Comparison of energy densities and energy flows evaluated with three different methods for an axisymmetric plate. *, Equation of motion; ○, diffusion equation (4); +, energy equation (18).

of the classic response. This result well agrees with the averaging procedure over a wavelength introduced by Wohlever and Bernhard [4].

The second simulation is very similar to the first. The system studied is a circular plate excited at its centre and clamped at its edge. The transverse displacement is now a linear combination of four functions $aH_0^{(1)}(ks) + bH_0^{(2)}(ks) + cI_o(ks) + dK_0(ks)$. As the plate is clamped at s_{max}, the displacement and the slope (first derivative of displacement) are set to zero. Furthermore, the displacement is assumed to be known at s_{min} and the bending moment to vanish modelling a plate with a hole. The four constants a, b, c and d are determined and the energy quantities are computed. The other calculations are performed in a similar way as for membrane case. The results are shown in Figure 3 for the following values: group velocity $c_g = 680$ m/s (two times phase velocity), frequency $f = 1000$ Hz, damping loss factor $\eta = 0{\cdot}05$, Poisson's ratio $\nu = 0{\cdot}3$, $s_{min} = 0{\cdot}25$ wavelength and radius $s_{max} = 5$ wavelengths. The conclusions are the same as in the membrane case. However, the oscillation magnitudes of the energy density due to interference are larger than in the membrane case.

The third numerical simulation concerns an acoustical enclosure. The acoustic potential with a spherical symmetry is $a\,e^{-iks}/s + b\,e^{iks}/s$. The constants are that the pressure is known at s_{min} and that the normal velocity vanishes at s_{max}, which is the boundary condition

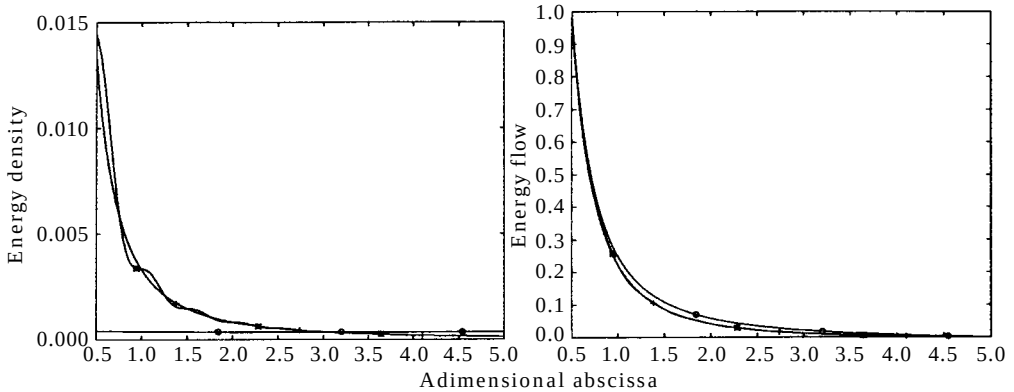

Figure 4. Comparison of energy densities and energy flows evaluated with three different methods for a spherical acoustical enclosure. *, Equation of motion; ○, diffusion equation (4); +, energy equation (18).

for a perfect reflection. Then, energy density and energy flow are deduced. The following numerical values are used: group velocity $c_g = 340$ m/s, frequency $f = 1000$ Hz, damping loss factor $\eta = 0{\cdot}008$, $s_{\min} = 0{\cdot}5$ wavelength and radius $s_{\max} = 5$ wavelengths. Figure 4 highlights that the shortcoming of the diffusion equation observed in two-dimensional systems is more important for three-dimensional acoustical enclosures.

7. CONCLUSION

In this study, a system of energy equations has been proposed to model the spread of energy throughout multi-dimensional systems in the high frequency range. The derivation of these equations matches the one proposed by Nefske and Sung [1] for the special one-dimensional case. But significant differences appear for other dimensions.

The analysis of the spread of the energy rests on the geometry of the lines of propagation of energy. A geometric factor summarizes the local geometry of these curves. The energy equations strongly depend on this geometric factor. Obviously, solving these equations is possible only if this geometric factor is known. In fact, these equations are able to predict the magnitude of the energy but not its direction of propagation. This is an important limitation. But, there exist at least some cases of symmetry for which the geometry of the streamlines are known. The plane wave solution matches with the one deduced from the vibrational conductivity approach. Moreover, cylindrical and spherical waves can be dealt with. This is an improvement on the diffusion equation. It should be remarked that plane waves satisfy the analogy with Fourier's thermal conduction law but not other types of waves.

In conclusion, the application of the vibrational conductivity approach to multi-dimensional systems may encounter some difficulties due to the underlying plane wave representation. For some particular geometries, other kinds of wave may dominate. It is then important to account correctly for the geometric factor. This is the purpose of the energy equation which has been proposed here.

ACKNOWLEDGMENT

The author gratefully acknowledges Mr E. Luzzato (EDF-DER) for his scientific collaboration, his advice and encouragement.

REFERENCES

1. D. J. Nefske and S. H. Sung 1987 *NCA Publication* **3**. Power flow finite element analysis of dynamic systems: basic theory and application to beams.
2. R. S. Langley 1991 *Journal of Sound and Vibration* **150**, 47–65. Analysis of beam and plate vibrations by using the wave equation.
3. R. S. Langley 1995 *Journal of Sound and Vibration* **182**, 637–657. On the vibrational conductivity approach to high frequency dynamics for two-dimensional structural components.
4. J. C. Wohlever and R. J. Bernhard 1992 *Journal of Sound and Vibration* **153**, 1–19. Mechanical energy flow models of rods and beams.
5. S. A. Rybak, V. D. Belov and B. D. Tartakovskii 1977 *Soviet Physics Acoustics* **23**, 115–119. Propagation of vibrational energy in absorbing structures.
6. O. M. Bouthier and R. J. Bernhard 1995 *Journal of Sound and Vibration* **182**, 129–147. Simple models of energy flow in vibrating membranes.

7. O. M. Bouthier and R. J. Bernhard 1995 *Journal of Sound and Vibration* **182**, 149–164. Simple models of energy flow in vibrating plates.
8. M. Djimadoum and J. L. Guyader 1995 *Proceedings of Inter-Noise* '95, *Newport Beach*, *CA*. Possibilities to generalize the heat transfer approach to vibration of plates problems.
9. M. N. Ichchou and L. Jezequel 1996 *Journal of Sound and Vibration* **195**, 679–685. Comments on simple models of the energy flow in vibrating membranes and on simple models of the energetics of transversely vibrating plates.
10. P. M. Morse and H. Feshbach 1953 *Methods of Theoretical Physics*. New York: McGraw-Hill.
11. R. S. Langley 1994 *Journal of Sound and Vibration* **178**, 483–500. Spatially averaged frequency response envelopes for one- and two-dimensional structural components.