Journal of Sound and Vibration (1998) **212**(5), 855–874

MATHEMATICAL MODELS FOR THE APPARENT MASS OF THE SEATED HUMAN BODY EXPOSED TO VERTICAL VIBRATION

L. Wei and M. J. Griffin

Human Factors Research Unit, Institute of Sound and Vibration Research, University of Southampton, Southampton SO17 1BJ, England

(*Received* 10 *December* 1996, *and in final form* 19 *December* 1997)

Alternative mathematical models of the vertical apparent mass of the seated human body are developed. The optimum parameters of four models (two single-degree-of-freedom models and two two-degree-of-freedom models) are derived from the mean measured apparent masses of 60 subjects (24 men, 24 women, 12 children) previously reported. The best fits were obtained by fitting the phase data with single-degree-of-freedom and two-degree-of-freedom models having rigid support structures. For these two models, curve fitting was performed on each of the 60 subjects (so as to obtain optimum model parameters for each subject), for the averages of each of the three groups of subjects, and for the entire group of subjects. The values obtained are tabulated. Use of a two-degree-of-freedom model provided a better fit to the phase of the apparent mass at frequencies greater than about 8 Hz and an improved fit to the modulus of the apparent mass at frequencies around 5 Hz. It is concluded that the two-degree-of-freedom model provides an apparent mass similar to that of the human body, but this does not imply that the body moves in the same manner as the masses in this optimized two-degree-of-freedom model.

1. INTRODUCTION

The biodynamic responses of the human body influence the manner in which vibration causes discomfort and injury and interferes with activities. Purely numerical considerations of body dynamics have been reported, but the complexity of the human body dictates a vital role for experimentation in the development of understanding of human responses to vibration. There are currently insufficient data to derive a “complete” mathematical model of the movement of the body during exposure to vibration and also insufficient information to fully justify the form of complex models of body responses. The development of complex models of the responses of the body requires an understanding of the modes of oscillation of the body.

Biodynamic models may either seek to explain the form of body motion caused by vibration or seek to provide a simple mathematical summary of the effect of this response. For example, a model which explains the seat-to-head transmissibility of the human body will be exceedingly complex [1], but for some purposes a single-degree-of-freedom model may adequately summarize the transmissibility of a group of people [2]. The aim in this paper is to present a model for the driving point apparent mass of the seated human body without proposing the mechanisms and movements of the body responsible for this apparent mass.

Driving-point frequency response functions, such as mechanical impedance and apparent mass, have been determined at the seat–person interface for vertical whole-body

0022–460X/98/200855 + 20 $25.00/0/sv971473

vibration in various studies, but only a few investigations have resulted in a mathematical model or fully investigated the parameters of the model. The mechanical impedance of the human body could be represented by a discrete system of masses, springs and dampers (e.g., models proposed in references [3–6]) or a distributed parameter model (e.g., references [7, 8]). The number of degrees of freedom required in a model depends on the purpose of the model: a model explaining the motion of the human body will tend to be more complex than the simplest model giving an approximation to the driving point impedance. For example, the 15-degree-of-freedom model proposed by Nigam and Malik [9] and the finite element model derived by Kitazaki and Griffin [1] are overly complex for the predicton of the average point impedance of a person sitting in a single posture and exposed to a single type of motion. Unless the sophistication of complex models is used to predict variations in impedance (e.g., with variations in posture or vibration magnitude) or to predict the motions of other body parts, they appear to have no advantage over simple models. For a simple model of the driving point apparent mass, the motions of body parts which do not contribute to the driving point apparent mass over the frequency range of interest can be ignored. Further, it may also be possible to represent a complex motion by a simpler motion which gives a similar apparent mass. Unnecessarily complex models are unnecessarily difficult to calculate and tend to present unfounded speculation on how the body moves.

The main purpose of the present study was to obtain an improved model of the apparent mass of the seated human body for use in procedures for predicting seat transmissibility. In a previous experiment Fairley and Griffin [10] measured the apparent masses of 60 seated subjects and derived a single-degree-of-freedom model to fit the measured data. This model has been used successfully to predict seat transmissibility from measures of the dynamic stiffness and damping of seats [11]. However, seat transmissibilities obtained with human subjects often show evidence of a two-degree-of-freedom response in the human body. This study involved a re-analysis of the earlier data so as to obtain an improved fit to the measured apparent masses of subjects.

2. PREVIOUS EXPERIMENTAL RESULTS

The vertical (i.e., z-axis) whole-body driving-point apparent masses of 60 persons (12 children, 24 men, 24 women) were obtained with the subjects seated on a rigid force platform without a backrest [10]. Subjects were exposed to 1·0 ms^{-2} r.m.s. random vertical vibration over the range 0·25–20 Hz. The subjects sat in a normal upright posture with their feet supported on a footrest which vibrated in phase with the seat. The force platform incorporated quartz piezo-electric force transducers mounted at the corners of a rectangular welded steel frame (Kistler 9281B). The top plate of this platform, on which the subjects sat, was 0·02 m thick, 0·6 m wide and 0·4 m deep; it was 0·46 m above the footrest. The acceleration of the platform was measured on the top plate by using an accelerometer.

The apparent mass frequency response function was presented in preference to other force response relationships (such as mechanical impedance or dynamic stiffness) because at zero-frequency it indicates the static weight of a person on the seat. The apparent mass frequency response function is defined as:

$$\text{apparent mass }(\omega \mathrm{i}) = F(\omega \mathrm{i})/\ddot{x}(\omega \mathrm{i}), \qquad (1)$$

where $F(\omega \mathrm{i})$ is the force and $\ddot{x}(\omega \mathrm{i})$ is the acceleration measured in the force platform supporting the subject.

Fairley and Griffin calculated a "normalized apparent mass" for each subject by dividing the apparent mass of each subject by the apparent mass at 0·5 Hz. The normalized apparent masses calculated from their 60 subjects are shown in Figure 1. The dynamic properties (i.e., mass, stiffness and damping) of a structure may be determined from suitable experimental frequency response data [12]. However, current experimental data are insufficient to define the relevant movements of the human body during vibration and, therefore, they are also insufficient to determine the relevant masses, stiffness and damping

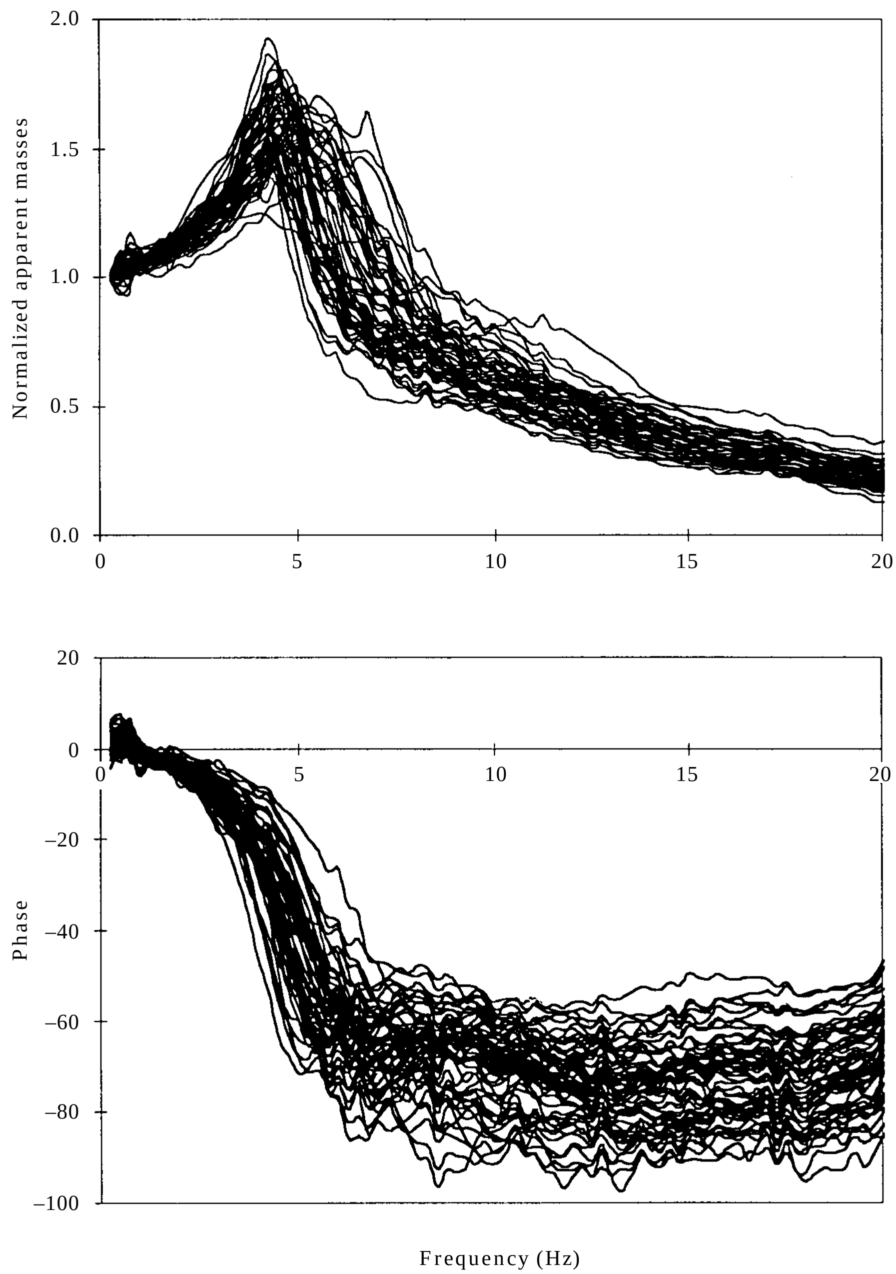

Figure 1. Normalized apparent masses of 60 seated subjects in the vertical axis (data from reference [10]).

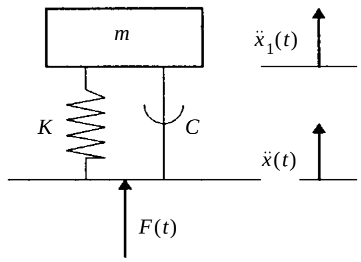

Figure 2. Single-degree-of-freedom model (model 1a).

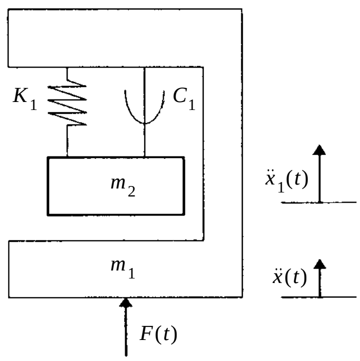

Figure 3. Single-degree-of-freedom model with rigid support (model 1b).

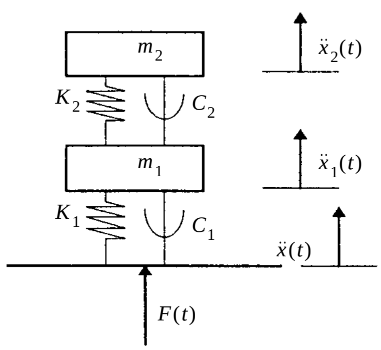

Figure 4. Two-degree-of-freedom model (model 2a).

of a structure that moves like the body during vibration. The experimental data suggest that just one or two degrees of freedom might accurately represent a subject's apparent mass over the 0–20 Hz frequency range; it is therefore reasonable to seek a model which does not move internally the same as the human body but has the same apparent mass. In this study, the simplest possible mathematical models having similar apparent masses to those of human subjects were sought. It appears from Figure 1 that some subjects have apparent masses showing one degree of freedom while others show two degrees of freedom. For the present study, all reasonable one- and two-degree-of-freedom systems were investigated as representations of subject apparent mass.

3. DERIVATION OF MATHEMATICAL IMPEDANCE MODELS

3.1. SINGLE-DEGREE-OF-FREEDOM MODELS

A simple linear single-degree-of-freedom model (model 1a) is shown in Figure 2. The mass, m, represents the weight of the person which is supported by tissues represented by the spring, K, and damping, C.

The equations of motion of this model are

$$m\ddot{x}_1 = F(t), \qquad m\ddot{x}_1 + C(\dot{x}_1 - \dot{x}) + K(x_1 - x) = 0. \tag{2, 3}$$

The only force which can be transmitted to the model is $F(t)$. Invoking the Laplace transform, one obtains for the steady state case:

$$F(s) = ms^2 x_1(s). \tag{4}$$

The acceleration and the velocity of the model, when transformed, will be

$$\ddot{x}(s) = s^2 x(s), \qquad \dot{x}(s) = sx(s). \tag{5}$$

In order to arrive at a term that corresponds to the apparent mass one seeks to solve for $x_1(s)$ in terms of $x(s)$ by Newton's second law of motion:

$$m\ddot{x}_1 = K(x - x_1) + C(\dot{x} + \dot{x}_1). \tag{6}$$

Upon taking the Laplace transforms and substituting ωi for s, the model in the frequency domain becomes

$$x_1(\omega \mathrm{i}) = \frac{K + C\omega \mathrm{i}}{K - m\omega^2 + C\omega \mathrm{i}}\, x(\omega \mathrm{i}). \tag{7}$$

Substituting for $x_1(\omega \mathrm{i})$ above gives

$$F(\omega \mathrm{i}) = \left(\frac{m(K + C\omega \mathrm{i})}{K - m\omega^2 + C\omega \mathrm{i}}\right) \ddot{x}(\omega \mathrm{i}). \tag{8}$$

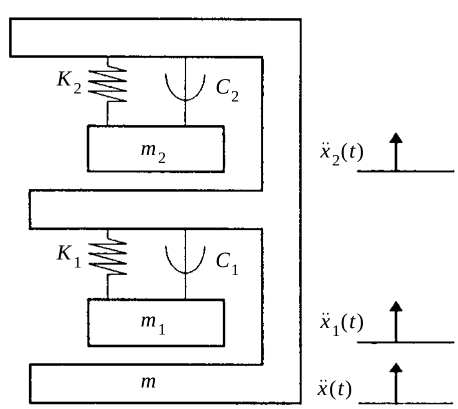

Figure 5. Two-degree-of-freedom model with rigid support (model 2b).

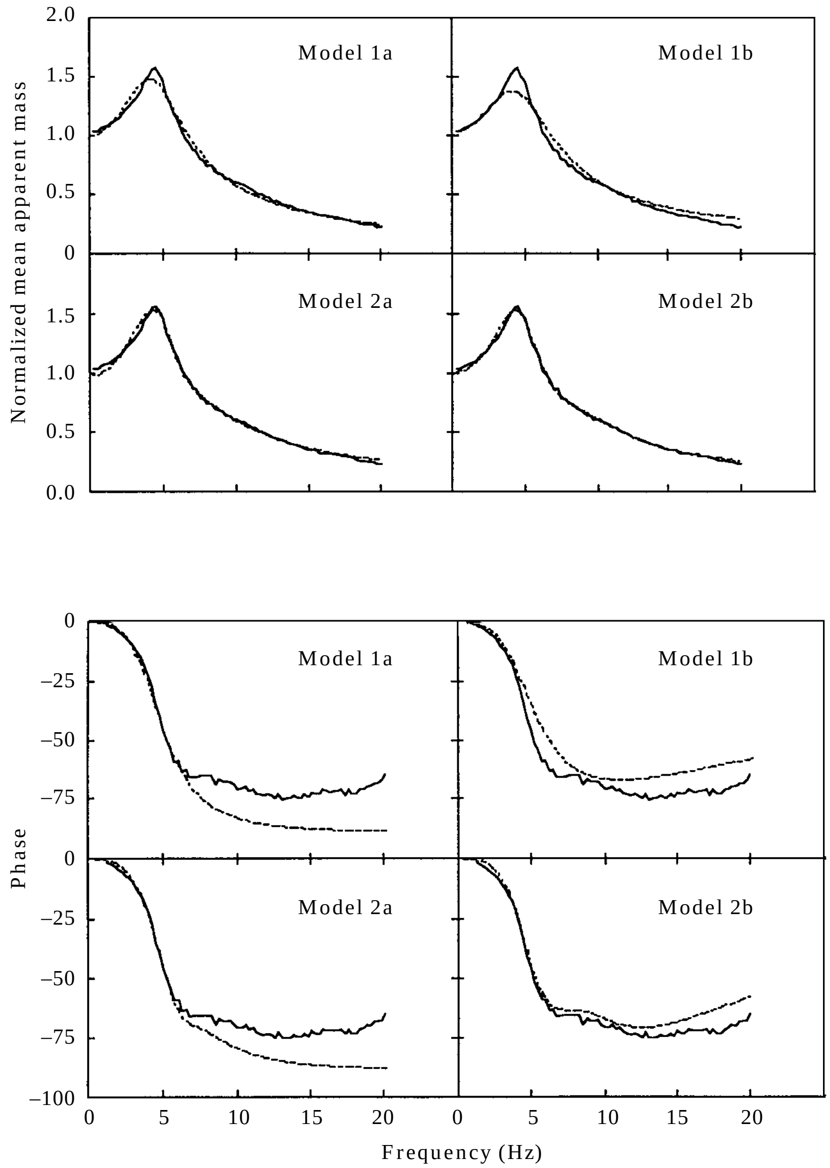

Figure 6. Mean modulus and phase of the normalized apparent masses of 60 subjects compared with optimized responses of models 1a, 1b, 2a and 2b fitted by minimizing the error in the modulus (—, experimental data: --------, fitted curves).

The term in parentheses being the ratio of $F(\omega i)$ to $\ddot{x}(\omega i)$, is called the apparent mass:

$$m_a(\omega i) = \frac{F(\omega i)}{\ddot{x}(\omega i)} = \frac{mK + imC\omega}{K - m\omega^2 + iC\omega}, \qquad |m_a| = \sqrt{\frac{(mK)^2 + (mC\omega)^2}{(K - m\omega^2)^2 + (C\omega)^2}}, \quad (9, 10)$$

$$\theta = a\tan(mC\omega/mK) - a\tan\{C\omega/(K - m\omega^2)\}. \quad (11)$$

Here m_a is the apparent mass and θ is the phase angle between force and acceleration.

It is difficult to make a real model like that shown in Figure 2 as there is no support for the mass other than the spring and damper and therefore no constraint to prevent rotational modes of vibration. An alternative single-degree-of-freedom model (model 1b) is shown in Figure 3. In this model, the mass of the person is divided into two parts: a support structure, m_1, and a sprung mass, m_2. If a dummy were manufactured according

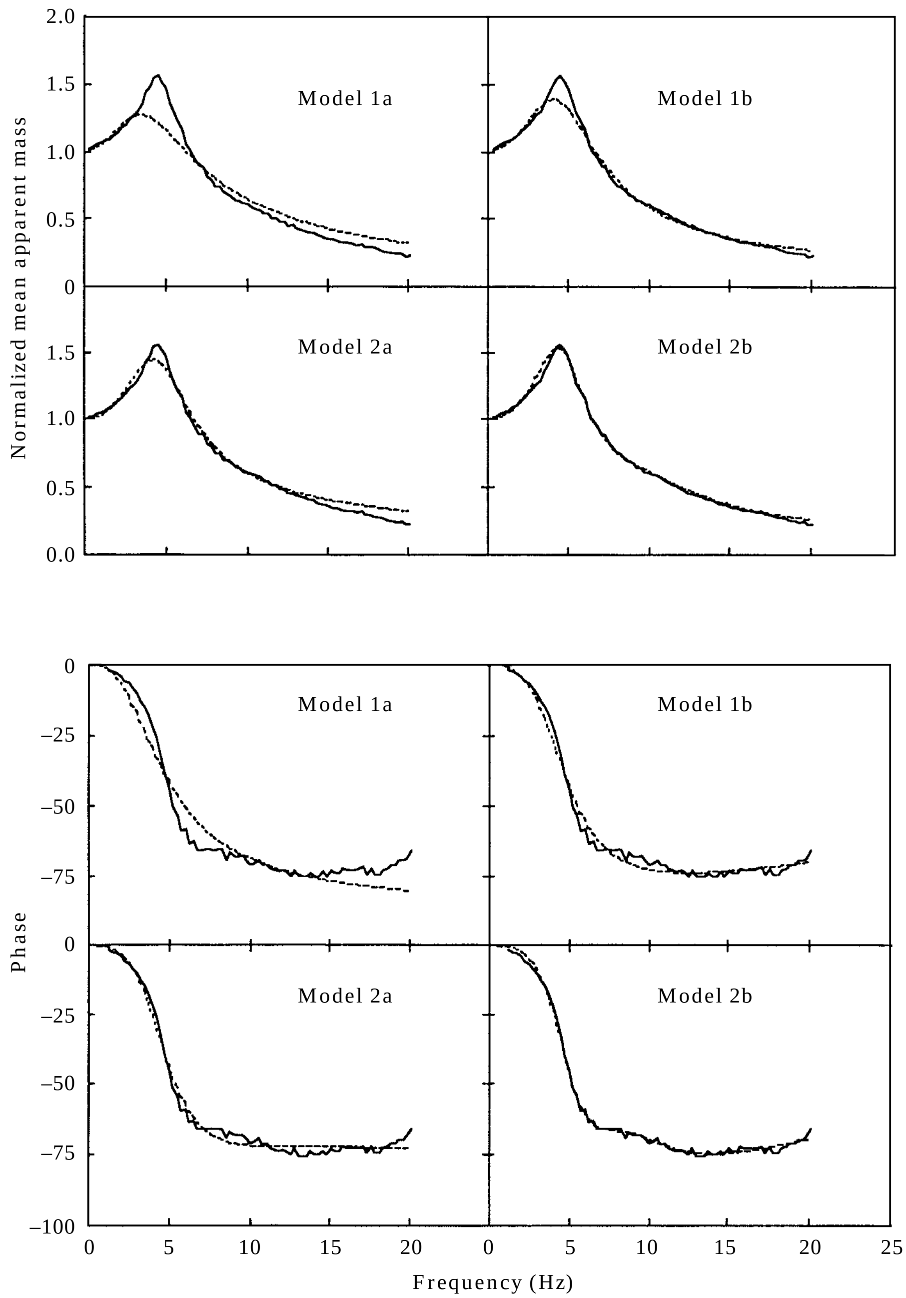

Figure 7. Mean modulus and phase of the normalized apparent masses of 60 subjects compared with optimized responses of models 1a, 1b, 2a and 2b fitted by minimizing the error in the phase (—, experimental data; --------, fitted curves).

TABLE 1

Single-degree-of-freedom model 1b fit to the experiment curves

Subject	Sex	Age	K_1 (N/m)	C_1 (Ns/m)	M_1 (kg)	M_2 (kg)	Total mass (kg)
1	M	26	34142	1187	1·3	44·6	45·9
2	M	16	41151	1122	8·8	35·6	44·4
3	M	39	71772	1845	21·3	86·7	108·0
4	M	38	62976	1631	4·3	52·8	57·1
5	M	34	34653	1312	5·0	47·8	52·8
6	M	33	29409	675	12·9	31·0	43·9
7	M	29	54623	1658	11·7	60·5	72·2
8	M	25	35756	1009	13·0	39·1	52·1
9	M	45	36286	898	15·3	32·7	48·0
10	M	51	66748	1705	0·1	65·8	65·9
11	M	16	38962	985	5·9	35·6	41·5
12	M	27	34822	954	17·2	39·0	56·2
13	M	56	70926	1447	6·8	59·0	65·8
14	M	17	54085	1475	1·4	59·4	60·8
15	M	69	71813	1173	20·9	34·7	55·6
16	M	27	46384	1797	2·1	51·3	53·4
17	M	39	66593	1377	4·7	51·5	56·2
18	M	39	66803	1833	3·4	80·4	83·8
19	M	50	42940	1286	12·2	46·7	58·9
20	M	45	77829	2345	2·1	76·2	78·3
21	M	17	48025	1165	13·6	46·5	60·1
22	M	23	42443	1083	3·1	43·9	47·0
23	M	23	52609	1204	17·3	40·7	58·0
24	M	17	63948	1636	0·9	43·3	44·2
Mean of 24 men			51987	1366	8·6	50·2	58·8
25	F	24	26951	957	4·8	38·9	43·7
26	F	56	48045	1217	11·7	48·1	59·8
27	F	22	58890	1486	0·5	45·3	45·8
28	F	45	40143	1565	3·1	48·6	51·7
29	F	55	58186	1277	1·5	39·7	41·2
30	F	52	37755	1792	0·4	52·8	53·2
31	F	25	36342	1170	12·1	39·9	52·0
32	F	23	38886	1925	1·1	52·2	53·3
33	F	40	32252	621	18·2	33·5	51·7
34	F	23	32174	935	13·8	36·5	50·3
35	F	17	45515	1403	2·2	50·7	52·9
36	F	35	38227	1178	15·4	41·2	56·6
37	F	25	43578	976	16·7	34·5	51·2
38	F	39	35351	1076	8·2	39·5	47·7
39	F	21	46037	1577	2·6	53·2	55·8
40	F	38	39493	823	11·9	28·8	40·7
41	F	24	50712	1172	9·6	38·4	48·0
42	F	31	30671	880	17·4	47·1	64·5
43	F	59	52524	1319	1·6	58·7	60·3
44	F	21	52151	1003	12·1	31·8	43·9
45	F	41	35154	826	14·9	37·4	52·3
46	F	38	38850	1435	0·1	50·2	50·3
47	F	22	39338	1909	0·8	42·9	43·7
48	F	31	42586	994	18·8	40·8	59·6
Mean of 24 women			41659	1230	8·3	42·9	51·3
49	F	10	34387	421	12·5	19·4	31·9
50	F	11	32487	762	7·0	26·4	33·4

Continued

TABLE 1 (*continued*)

Subject	Sex	Age	K_1 (N/m)	C_1 (Ns/m)	M_1 (kg)	M_2 (kg)	Total mass (kg)
51	F	7	24887	511	2·1	21·3	23·4
52	F	9	37977	923	2·2	31·1	33·3
53	F	11	36203	992	3·8	30·6	34·4
54	F	14	28960	734	5·5	24·7	30·2
55	M	11	35428	753	3·1	28·2	31·3
56	M	13	47668	1564	0·3	50·9	51·2
57	M	12	25937	820	11·1	34·9	46·0
58	M	13	31973	607	14·2	30·9	45·1
59	M	8	33395	718	3·5	27·5	31·0
60	M	13	31032	1387	0·3	41·4	41·7
Mean of 12 children			33361	849	5·5	30·6	36·1
Mean of 60 subjects			44130	1485	7·8	43·4	51·2
Fit mean of 60 subjects			44115	1522	4·1	46·7	50·8

to this model, a constraint mechanism would be required to ensure that the sprung mass m_2 moved only in the vertical direction.

The response of this system is given by

$$F(t) = m_1\ddot{x} + m_2\ddot{x}_1, \qquad m_2\ddot{x}_1 = K_1(x - x_1) + C_1(\dot{x} - \dot{x}_1). \tag{12, 13}$$

Based on the same analysis as above, the apparent mass is

$$m_a(\omega \mathrm{i}) = \left[m_1 + m_2\left(\frac{K_1 + C_1\omega \mathrm{i}}{K_1 - m_2\omega^2 + C_1\omega \mathrm{i}}\right)\right], \qquad |m_a| = \sqrt{\frac{D1^2 + E1^2}{A1^2 + B1^2}}, \tag{14, 15}$$

$$\theta = a\tan(E1/D1) - a\tan(B1/A1). \tag{16}$$

Here

$$A1 = K_1 - m_2\omega^2, \qquad B1 = C_1\omega,$$

$$D1 = ((m_1 + m_2)K_1 - m_1 m_2\omega^2), \qquad E1 = (m_1 + m_2)\omega C_1.$$

3.2. TWO-DEGREE-OF-FREEDOM MODELS

Measurements of the mechanical impedance of the human body usually show evidence of a two-degree-of-freedom response (see, e.g., Figure 1 of reference [13]). For this reason, a two-degree-of-freedom system is also developed here.

Figure 4 shows a serial two-degree-of-freedom discrete model (model 2a). The motion equations of the system are

$$m_2\ddot{x}_2 + K_2(x_2 - x_1) + C_2(\dot{x}_2 - \dot{x}_1) = 0, \tag{17}$$

$$m_1\ddot{x}_1 + K_1(x_1 - x) + C_1(\dot{x}_1 - \dot{x}) + K_2(x_1 - x_2) + C_2(\dot{x}_1 - \dot{x}_2) = 0, \tag{18}$$

$$F(t) = K_1(x - x_1) + C_1(\dot{x} - \dot{x}_1) = m_1\ddot{x}_1 + m_2\ddot{x}_2. \tag{19}$$

The apparent mass is

$$m_a(\omega \mathrm{i}) = \frac{DD + EEi}{AA + BBi}, \qquad |m_a| = \sqrt{\frac{DD^2 + EE^2}{AA^2 + BB^2}}, \tag{20, 21}$$

$$\theta = a\tan(EE/DD) - a\tan(BB/AA). \tag{22}$$

TABLE 2

Two-degree-of-freedom model 2b fit to the experiment curves

Subject	Sex	Age	K_1 (N/m)	C_1 (Ns/m)	K_2 (N/m)	C_2 (Ns/m)	M (kg)	M_1 (kg)	M_2 (kg)	Total mass (kg)
1	M	26	19480	379	33076	462	6·5	26·9	14·4	47·8
2	M	16	24769	495	32408	725	6·5	23·3	14·3	44·1
3	M	39	45996	654	74312	1181	15·2	56·3	30·7	102·2
4	M	38	46964	832	42583	433	8·4	38·2	9·4	56·0
5	M	34	28370	773	24781	277	8·2	37·3	6·6	52·1
6	M	33	27466	475	32673	262	6·9	29·1	6·4	42·5
7	M	29	43452	950	42592	521	10·6	48·4	11·5	70·5
8	M	25	32558	702	43856	370	4·7	36·7	8·4	49·8
9	M	45	24075	937	19473	341	10·6	18·4	18·5	47·5
10	M	51	62452	1539	8579	32	5·4	61·0	2·2	68·6
11	M	16	33378	617	26446	179	5·5	30·2	5·1	40·8
12	M	27	27158	444	61480	789	7·3	30·8	15·2	53·2
13	M	56	58555	866	39418	428	8·6	48·1	9·6	66·4
14	M	17	33199	584	38179	589	9·7	36·1	16·1	61·9
15	M	69	69489	1106	10696	1944	1·5	37·3	15·5	54·3
16	M	27	23999	450	57886	925	7·2	27·6	19·5	54·3
17	M	39	51950	774	41713	391	6·0	40·6	10·4	57·0
18	M	39	46763	723	66688	789	10·2	50·0	20·6	80·8
19	M	50	42940	1286	23580	62	12·2	46·7	4·4e-8	58·9
20	M	45	38524	694	63705	1419	10·1	37·3	30·1	77·5
21	M	17	39619	654	56222	639	5·5	39	12·7	57·2
22	M	23	33234	489	48621	465	4·7	33·8	10·8	49·3
23	M	23	44330	713	76006	949	3·4	36·9	15·9	56·1
24	M	17	38983	522	63842	463	8·3	26·8	11·9	47·0
Mean 24 men			39071	736	42867	609	7·6	37·4	13·2	58·2
25	F	24	14024	212	32783	671	6·2	19·9	16·6	42·7
26	F	56	58657	1392	21911	236	9·4	30·7	23·1	63·3
27	F	22	57632	801	23411	269	9·3	19·4	18·6	47·4
28	F	45	28143	780	32115	455	6·9	34·6	10·1	51·6
29	F	55	32519	479	38142	348	8·8	23·0	10·4	42·2
30	F	52	29715	762	50028	466	5·5	36·1	10·8	52·5
31	F	25	35293	981	27790	127	7·0	39·1	3·9	50·0
32	F	23	24838	592	54586	625	7·8	30·6	14·3	52·7
33	F	40	27850	427	45774	706	8·6	29·6	10·7	48·9
34	F	23	29510	1201	9738	137	7·2	30·0	11·7	48·8
35	F	17	23768	325	62185	910	9·3	28·2	21·6	59·1
36	F	35	35475	779	52313	402	7·2	37·8	9·0	54·0
37	F	25	42718	791	44800	273	8·4	35·0	6·1	49·5
38	F	39	30298	635	34964	258	5·1	33·7	7·2	45·9
39	F	21	53928	1276	12718	156	5·6	33·7	16·7	56·0
40	F	38	34767	604	32934	357	7·0	26·2	5·7	38·9
41	F	24	50012	1402	9332	76	5·4	33·7	7·5	46·6
42	F	31	27309	618	38489	526	10·0	42·0	10·2	62·2
43	F	59	36931	578	37228	560	9·1	39·2	14·1	62·5
44	F	21	51014	930	40675	494	4·3	33·0	5·4	42·7
45	F	41	23601	350	35771	736	9·6	25·6	14·3	49·5
46	F	38	33723	886	27262	204	5·3	39·7	5·6	50·6
47	F	22	29733	1291	20719	134	4·6	34·8	4·6	44·0
48	F	31	36403	1720	18585	224	6·6	32·4	18·5	57·6
Mean 24 women			35328	825	33511	340	7·3	32·0	11·5	50·8
49	F	10	37407	537	37561	199	4·1	23·1	4·1	31·3
50	F	11	31410	617	14398	55	5·0	25·6	1·9	32·5

Continued

TABLE 2 (*continued*)

Subject	Sex	Age	K_1 (N/m)	C_1 (Ns/m)	K_2 (N/m)	C_2 (Ns/m)	M (kg)	M_1 (kg)	M_2 (kg)	Total mass (kg)
51	F	7	24672	500	261	8.8	0·9	21·2	1·1	23·2
52	F	9	32118	814	87658	120	2·1	30·8	0·5	33·4
53	F	11	24752	1002	8911	43	2·4	29·8	3·7	35·9
54	F	14	26590	568	19975	159	3·5	22·9	3·2	29·6
55	M	11	35428	754	74657	1271	3·1	28·2	1·1e-8	31·3
56	M	13	36735	744	45494	355	7·3	35·0	9·5	51·8
57	M	12	25934	820	14442	455	4·1	34·9	7·0	46·0
58	M	13	29360	446	45128	653	4·3	29·5	9·9	43·7
59	M	8	33395	718	95584	9·6	3·6	27·5	1·1e-7	31·1
60	M	13	23199	682	25296	168	6·9	28·8	5·8	41·6
Mean 12 children			30083	683	39114	291	3·9	28·1	3·9	36·0
Mean 60 subjects			35776	761	38374	458	6·7	33·4	10·7	50·8
Fit mean of 60 subjects			35007	815	33254	484	5·6	36·2	8·9	50·7

Here

$$AA = m_1 m_2 \omega^4 - (m_1 K_2 + m_2 K_1 + m_2 K_2 + C_1 C_2)\omega^2 + K_1 K_2,$$
$$BB = (C_1 K_2 + C_2 K_1)\omega - (m_1 C_2 + m_2 C_1 + m_2 C_2)\omega^3,$$
$$DD = (m_1 + m_2)K_1 K_2 - (m_1 C_1 C_2 + m_2 C_1 C_2 + m_1 m_2 K_1)\omega^2,$$
$$EE = (m_1 + m_2)(C_1 K_2 + C_2 K_1)\omega - m_1 m_2 C_1 \omega^3.$$

A two-degree-of-freedom system having a support structure is shown in Figure 5 (i.e., model 2b). It has two mass–spring systems, m_1 and m_2, supported on the support mass, m. It is tempting to assume that the mass m_2 consists of the masses of the head and the upper torso while the mass m_1 represents the main part of the body and the mass m comprises the skeleton [4]. However, the models derived in this paper are not intended to represent the locations or mechanisms of body movement: the models show "equivalent mechanical systems" only in that they have a mechanical impedance similar to that of the human body.

The equations for vertical axial motion of the model in Figure 5 are

$$F(t) = m\ddot{x} + m_1\ddot{x}_1 + m_2\ddot{x}_2, \tag{23}$$

$$m_1\ddot{x}_1 = K_1(x - x_1) + C_1(\dot{x} - \dot{x}_1), \qquad m_2\ddot{x}_2 = K_2(x - x_2) + C_2(\dot{x} - \dot{x}_2). \quad (24,\ 25)$$

The solution of there equations has the form

$$m_a(\omega \mathrm{i}) = \frac{D + E + (F + G)\mathrm{i}}{A + B_\mathrm{i}}, \qquad |m_a| = \sqrt{\frac{(D + E)^2 + (F + G)^2}{A^2 + B_2}}, \quad (26,\ 27)$$

$$\theta = a\tan\{(F + G)/(D + E)\} - a\tan(B/A). \tag{28}$$

Here

$$A = K_1 K_2 - \omega^2(K_1 m_2 + K_2 m_1) + m_1 m_2 \omega^4 - C_1 C_2 \omega^2,$$
$$B = (K_1 C_2 + K_2 C_1)\omega - (m_1 C_2 + m_2 C_1)\omega^3,$$
$$D = (m + m_1 + m_2)K_1 K_2 - (m m_2 K_1 + m m_1 K_2 + m_1 m_2 K_1 + m_1 m_2 K_2)\omega^2,$$
$$E = m m_1 m_2 \omega^4 - (m C_1 C_2 + m_1 C_1 C_2 + m_2 C_1 C_2)\omega^2,$$
$$F = (m + m_1 + m_2)(K_1 C_2 + K_2 C_1)\omega,$$
$$G = -(m m_1 C_2 + m m_2 C_1 + m_1 m_2 C_2 + m_1 m_2 C_1)\omega^3.$$

4. FITTING THE MATHEMATICAL MODELS TO THE EXPERIMENTAL DATA

4.1. FITTING TO THE MEAN RESPONSES

The optimum forms of the models shown in Figures 2–5 were determined by curve fitting to the experimental data obtained by Fairley and Griffin [10]. The least square error

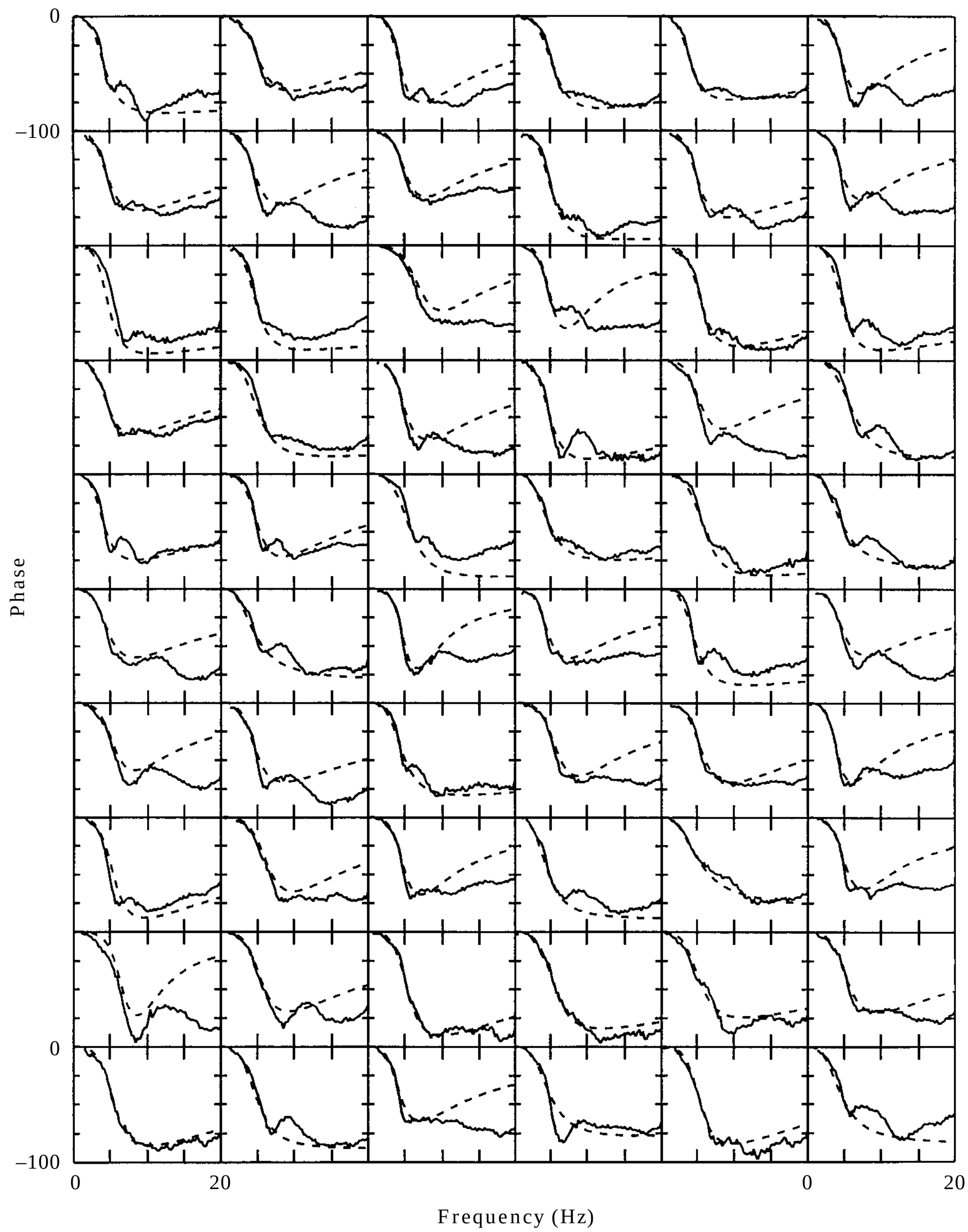

Figure 8.(a). *Caption on opposite page*.

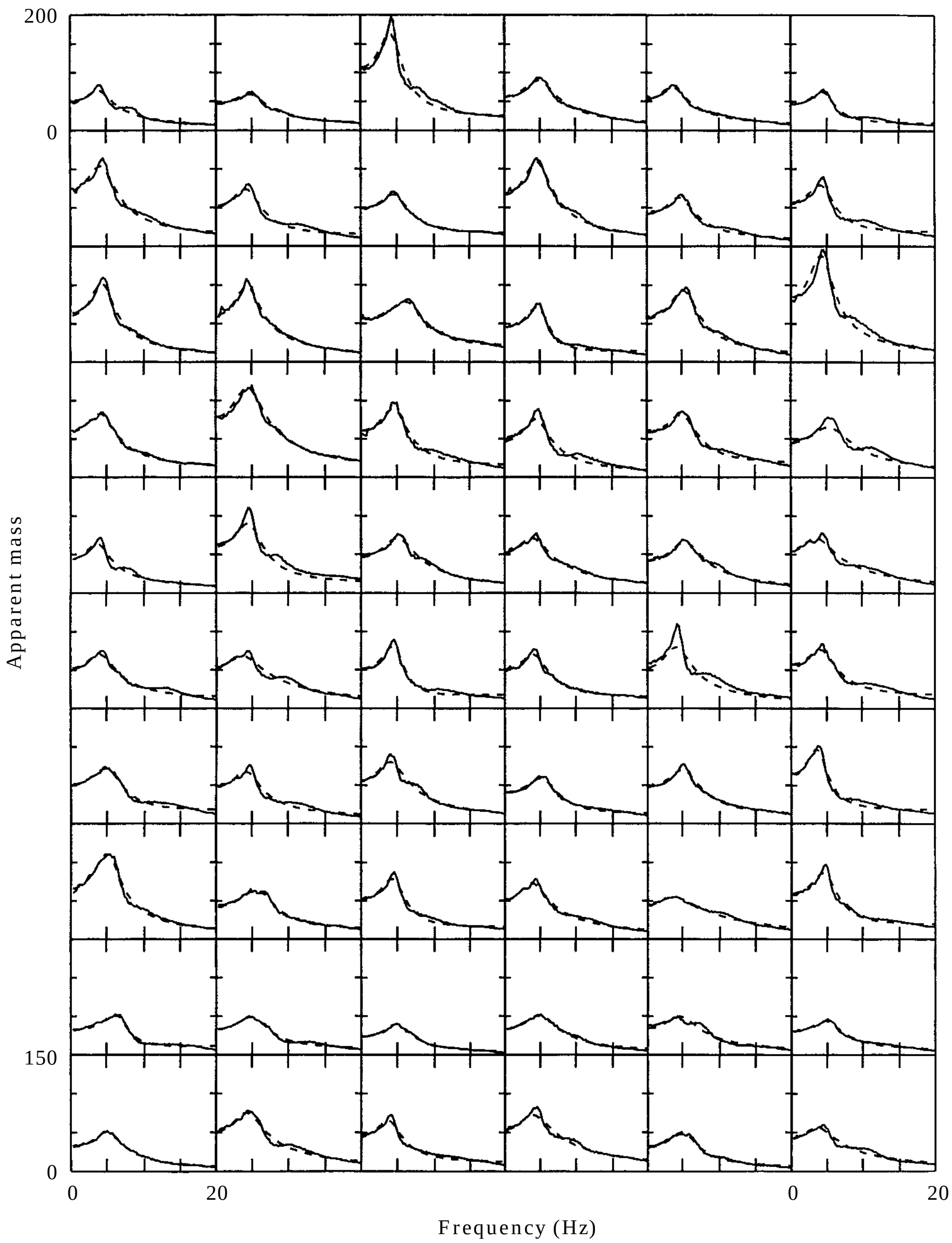

Figure 8(b)

Figure 8. Comparison of measured modulus and phase of apparent mass compared with values fitted fitted using model 1b (——, experimental curves; --------, fitted curves).

method and an optimization algorithm were utilized [14]. The parameters in the equations of each model were refined to minimize the function

$$\text{error} = \frac{1}{N}\sum_{i=1}^{N}(m_{af}(i) - m_a(i))^2, \tag{29}$$

where $m_{af}(i)$ is the modulus of the apparent mass from the curve fit at the ith frequency point and $m_a(i)$ is the modulus of the apparent mass from the measured data. With values of the parameters chosen at random used for starting values, the parameters were varied systematically by using an optimization algorithm [14]. The curves corresponding to the modulus and phase of the normalized apparent mass of each of the models are compared with the corresponding mean of the measured normalized apparent masses of the 60 subjects in Figure 6.

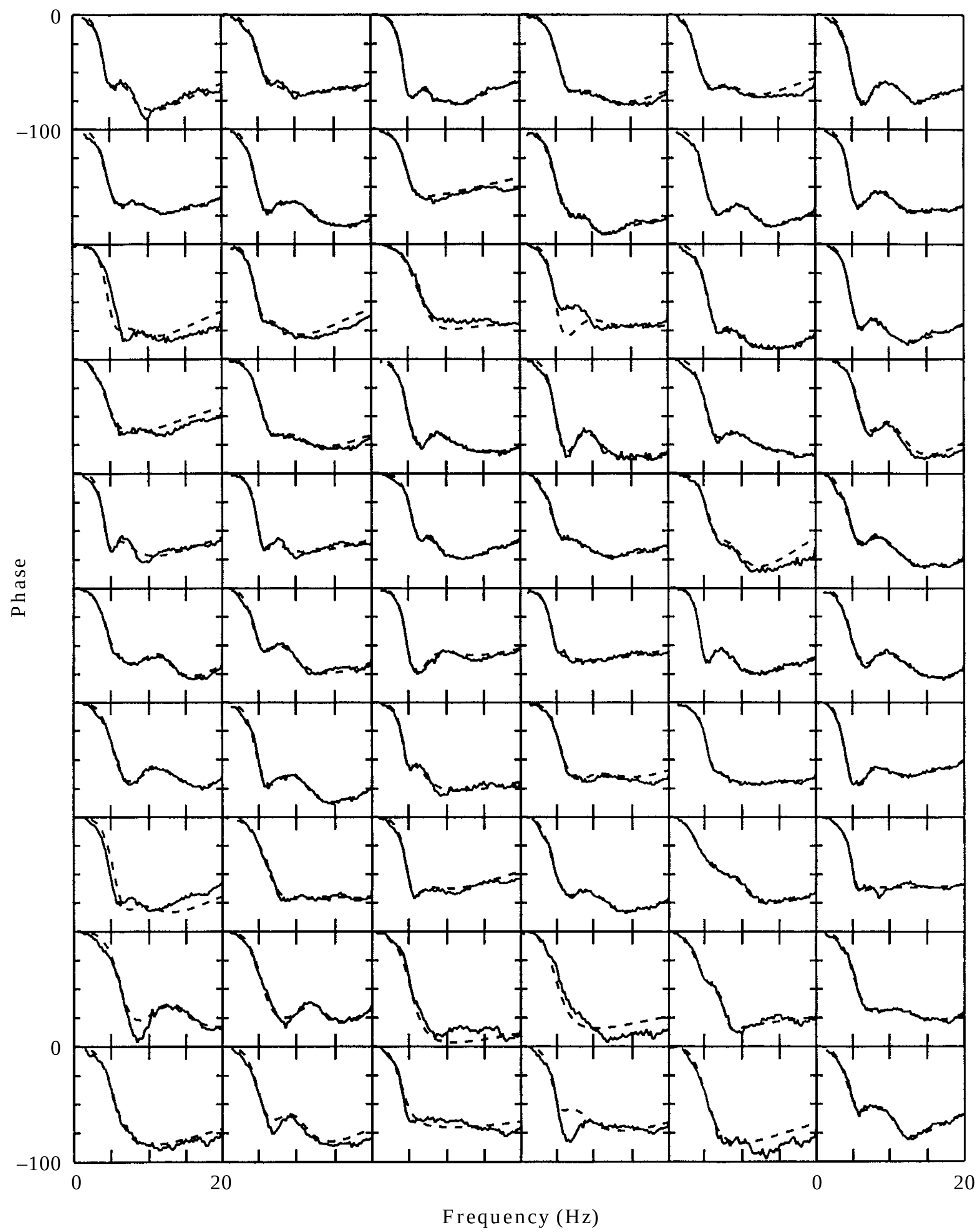

Figure 9.(a). *Caption on opposite page*.

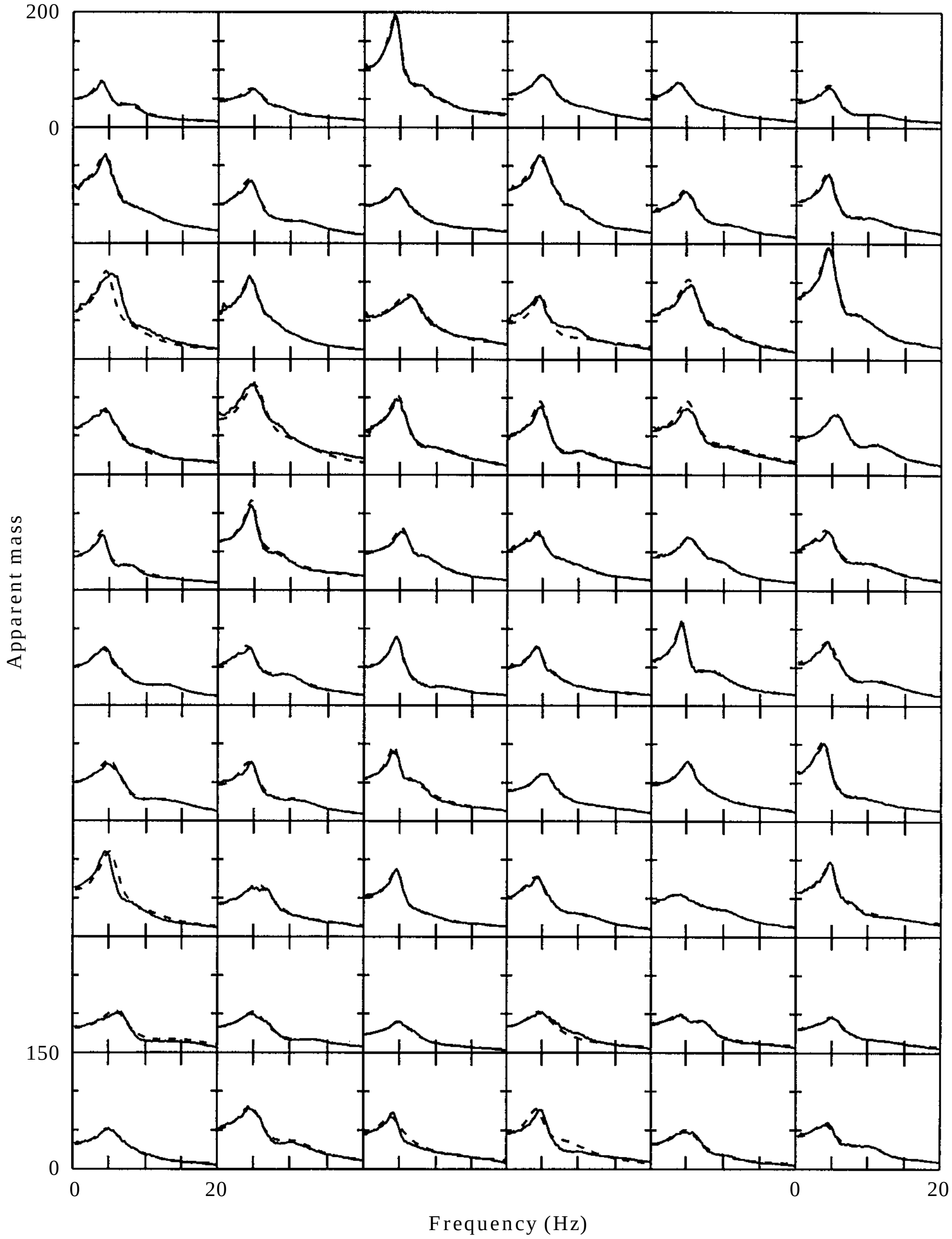

Figure 9(b)

Figure 9. Comparison of the measured modulus and phase of apparent mass compared with values fitted by using model 2b (——, experimental curve: --------, fitted curves).

By using a similar method of fitting based on the phase of the apparent mass, somewhat different models were obtained with responses as shown in Figure 7.

From Figures 6 and 7 it was concluded that models 1b and 2b obtained with phase fitting provided the best fits to the mean of the measured data. The results in Figure 7 suggest that the apparent mass is dominated by a single mode. However, this is really two modes combined together: the two natural frequencies of the two-degree-of-freedom systems are

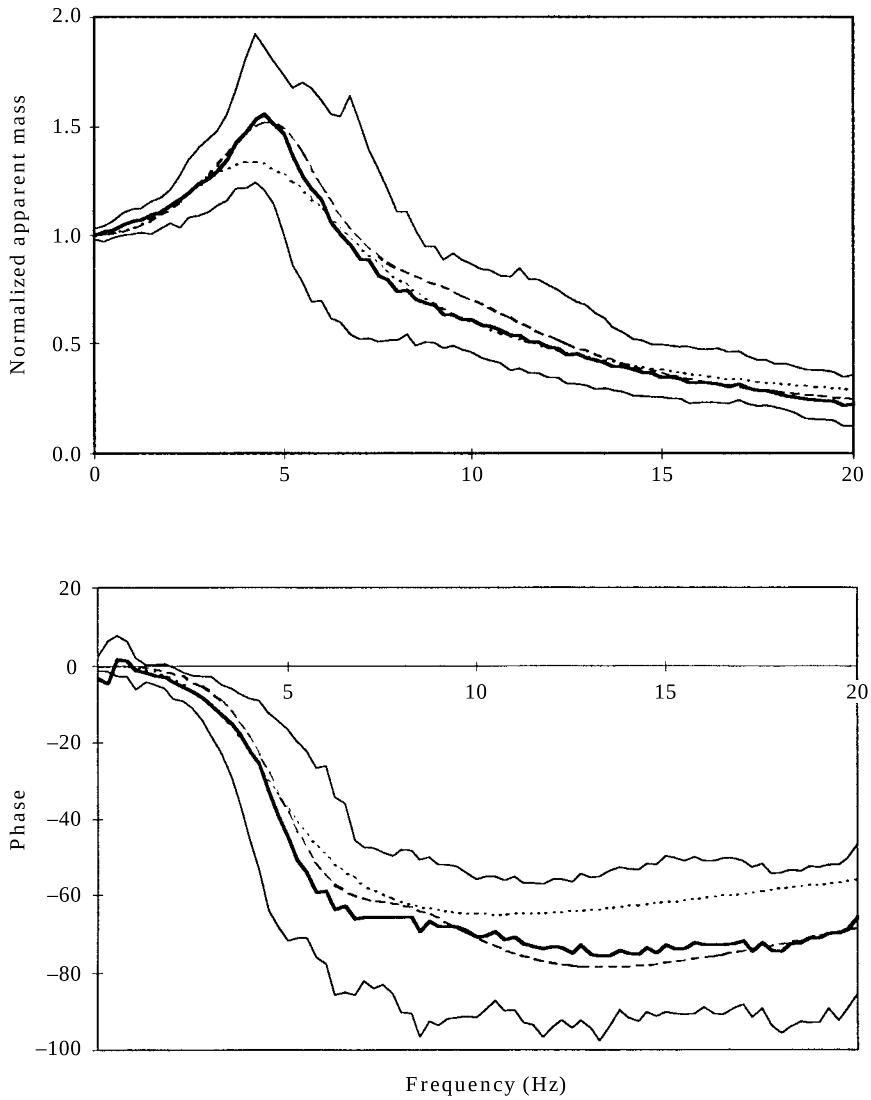

Figure 10. Mean and range of the measured normalized apparent masses of the 60 subjects compared with the fitted curves obtained by using the mean values of mass, stiffness and damping derived for each of the 60 subjects by using model 1b and model 2b (▬▬, mean experimental values; —, maximum and minimum of experimental values; · · ·, model 1b fitted curve; ---, model 2b fitted curve).

very close. Due to the high damping, the two-degree-of-freedom synthesized system shows only one peak. Where a single mode dominates, the response of such a system can be approximated by a single mode with a constant term added to represent the effect of other modes that are above the frequency range of interest. The support mass, m, may represent these higher modes and so the model may not be applied at frequencies higher than those studied here.

4.2. FITTING TO THE INDIVIDUAL RESPONSES

The forms of models 1b and 2b were used to obtain the best fits to the measured apparent masses of each of the 60 subjects who participated in the experiment. The models were fitted by minimizing the difference in phase between the measured and predicted responses. The values for each mass, stiffness and damping in both models calculated for each of the subjects are listed in Tables 1 and 2. For these tables the subjects are separated into the three groups (men, women and children). Within each part of the table, the mean values are the mean values of mass, stiffness and damping within the group of men, women or children. At the foot of the table, the mean values are the mean values of mass, stiffness and damping over the group of 60 subjects. Also shown are the values of mass, stiffness and damping obtained by an optimum fit to the mean curve given by the arithmetic average of the 60 curves from the 24 men, 24 women and 12 children. The damping ratios for model 2b, which can be calculated from the parameters in Table 2, are very high (in the range 0·25–0·4, so the effects of adjacent modes were combined. This is why increasing the number of modes in the model improved the curve fitting for a system which was dominated by a single mode.

The individual measured normalized apparent masses are compared with the predicted apparent masses in Figure 8 for model 1b and in Figure 9 for model 2b.

Figure 10 shows the mean and range of the measured normalized apparent masses of the 60 subjects compared with the fitted curves obtained using the overall mean values of mass, stiffness and damping of the 60 subjects for models 1b and 2b. The relative values of the various masses and springs affect the frequency response. When K_1 increases and m_1 decreases, the first resonance frequency rises. If m_2 decreases, the second peak will appear in the response as the resonance frequency for this mode increases well above the principal resonance. The interactions among the masses and springs is complex, but a useful discussion in the context of human response to vibration requires greater knowledge of the body movements occurring during vibration.

4.3. STATISTICAL COMPARISONS

Statistical comparisons have been made to identify the causes of variations in model parameters between and within the three groups of subject.

Between the groups of men, women and children, the male and female group were not significantly different in age ($p > 0{\cdot}1$, Mann-Whitney U-test) although, of course, significantly older than the children. The fitted parameters for the men, women and children have been compared by using model 2b. The men had a total mass, a mass m_1, a stiffness k_2 and damping c_2 marginally significantly greater than the females ($p < 0{\cdot}1$; see Table 2). There were no statistically significant differences in the values of m_2, m, k_1 or c_1. All measures of mass were significantly less for the group of children than for the men and women. There was no difference in k_2, but the children had values of k_1 significantly less than the men. The value of c_2 was also significantly lower for the children than for the men ($p < 0{\cdot}01$) and marginally significantly lower than for the women ($p < 0{\cdot}1$).

Within groups of men, women and children, the correlations between subject age and the masses, stiffnesses and damping of the models fitted to the phase data were investigated by using Kendall's correlation coefficient. Except where stated, the significance level is 0·05. For both the single-degree-of-freedom (model 1b) and the two-degree-of-freedom model (model 2b) there was a significant positive correlation between age and total mass among the men and among the children, but not among the women. The effect was mainly due to a correlation with the mass m_2 in model 1b and due to a correlation with m_1 in the men ($p < 0{\cdot}01$) and with m and m_2 in the children when using model 2b.

The only other significant correlations with age were both among the men and were also positive: the value of k_1 in both models 1b and 2b, and the value of c_1 in model 2b ($p < 0{\cdot}001$). These correlations suggest greater mass, stiffness and damping with increased age among the 24 men.

With both models 1b and 2b, there was a highly significant positive correlation between k_1 and c_1 for the men ($p < 0{\cdot}001$). This correlation was also significant for the women but not for the children. Similarly, in model 2b there was a highly significant positive correlation between k_2 and c_2 for the men and the women ($p < 0{\cdot}001$) but not for the children. In model 2b there was a positive correlation between values of k_1 and k_2 only among the children. In model 2b there was a highly significant negative correlation between c_1 and c_2 among the women only ($p < 0{\cdot}001$).

For all three groups of subjects in model 1b there was a negative correlation between mass m_1 and damping c_1: this was highly significant for the women ($p < 0{\cdot}001$) and only marginally significant among the children ($p < 0{\cdot}1$). For the corresponding mass, m, in model 2b there was a significant negative correlation with c_1 and a significant positive correlation with c_2, but only for the women.

In model 1b there were significant positive correlations between m_2 and c_1 for all three groups and the correlation was highly significant for the men and women ($p < 0{\cdot}001$). In model 2b there were significant positive correlations between m_1 and c_1, for the men and children only; for all three groups there were significant positive correlations between m_2 and c_2 which were highly significant for the men ($p < 0{\cdot}001$) and marginally significant for the children ($p < 0{\cdot}1$).

In model 1b there was a highly significant positive correlation between k_1 and m_2 ($p < 0{\cdot}001$), but only for the men. Similarly, in model 2b, there was a highly significant positive correlation between k_1 and m_1 ($p < 0{\cdot}001$), and a significant positive correlation between k_2 and m_2, but again only for the men.

These correlations seem to suggest some differences between the groups of subjects, especially between the men and the women. Heavier men seem to exhibit increaseed stiffness whereas this is less obvious for the heavier women. However, such conclusions require care since the models are not necessarily representative of the mechanical structure and movements of the body, which is far more complex than a two-degree-of-freedom system.

5. DISCUSSION

The driving point impedance differs between subjects and so different model parameters are required to obtain the optimum impedance for each subject. Other studies show that the driving point dynamic response of the body is non-linear (see, e.g., reference [10, 13]), so different parameters will provide the optimum model at different vibration magnitudes. Alternatively, the parameters in the model should be non-linear.

Comparing the parameters of model 1b with the previous model developed by Fairley and Griffin [11], reveals no large or systematic differences between the two models. It appears that either model could be used to represent the apparent masses of people over the frequency range 0–20 Hz by a single-degree-of-freedom system.

The individual data shown in Figures 8 and 9, and the mean values shown in Figure 10, indicate that a two-degree-of-freedom mechanical model provides a better fit to the measured data than a single-degree-of-freedom model. Use of the two-degree-of-freedom model provides a better fit to the phase data at frequencies greater than about 8 Hz and an improved fit to the modulus at the frequencies around 5 Hz.

It would be possible to develop mathematical models of the driving point impedance of the body having more than two degrees-of-freedom, but the results shown here suggest that this is unnecessary when representing the average response of a group of subjects to a specific vibration input. A greater number of degrees of freedom may be required to explain the movements of the body responsible for apparent mass or predict the transmission of vibration through the body.

6. CONCLUSIONS

Curve fitting has allowed the development of mathematical models which provide a good fit to measured values of the normalized apparent masses of subjects.

There are large differences in model parameters for different persons (see Tables 1 and 2), but the mean parameters of the two adult groups of subjects (men and women) are similar. This may explain why different seat transmissibilities are obtained with different seats but there are fairly small differences between the mean values given by groups of subjects with the same seat.

Four human body mathematical models have been considered. By comparing the responses of the models with the measured responses, model 1b (single-degree-of-freedom with a rigid support) and model 2b (two-degree-of-freedom with a rigid support) were selected as the most suitable models for representing the effective apparent masses of subjects exposed to vertical vibration.

The single-degree-of-freedom model and the two-degree-of-freedom model both provided results close to the measured modulus of apparent mass. However, the two-degree-of-freedom model provided a better fit to the phase and also a better fit near the principal resonance at 5 Hz. For best results a two-degree-of-freedom model is therefore recommended.

When predicting the transmissibility of seats, it is recommended that the two-degree-of-freedom model with a support mechanism (i.e., model 2b) is used.

ACKNOWLEDGMENT

The authors are pleased to acknowledge the contribution of Dr Tom Fairley in conducting the original experimental study which made this paper possible (see reference [10]).

REFERENCES

1. S. Kitazaki and M. J. Griffin 1997 *Journal of Sound and Vibration*. A modal analysis of whole-body vertical vibration, using a finite element model of the human body. **200,** 83–103
2. M. J. Griffin, C. H. Lewis, K. C. Parsons and E. M. Whitham 1979 *AGARD Conference Proceedings CP*-253. *Models and Analogues for the Evaluation of Human Biodynamic Response*, *Performance and Protection*, *Paper A*28 (H. E. von Gierke, editor) The biodynamic response of the human body and its application to standards.
3. L. H. Vogt, R. R. Coerman and H. D. Fust 1968 *Aerospace Medicine* **39**, 675–679. Mechanical impedance of the sitting human under sustained acceleration.
4. C. W. Suggs, C. F. Abrams and L. F. Stikeleather 1969 *Ergonomics* **12**, 79–90. Application of a damped spring-mass human vibration simulator in vibration testing of vehicle seats.
5. I. Kaleps, H. E. von Gierke and E. B. Weis 1971 *Aerospace Medical Research Laboratories-TR*-71-29. *Symposium on Biodynamic Models and their Applications*, *Dayton*, *OH Paper* 8, 211–231. A five-degree-of-freedom mathematical model of the body.
6. International Organization for Standardization 1981 *International Standard* 5982. Vibration and shock-mechanical driving point impedance of the human body.
7. Y. K. Liu, H. J. Cramer and D. U. von Rosenberg 1973 *AD* – 773 859, *Aerospace Medical Research Laboratory*, *Wright-Patterson Air Force Base*, *OH*, *AMRLTR*-73-65. A distributed parameter model of the inertially loaded human spine: a finite difference solution.
8. H. J. Cramer, Y. K. Liu and D. U. von Rosenberg 1976 *Journal of Biomechanics* **9**, 15–130. A distributed parameter model of the inertially loaded human spine.

9. S. P. NIGAM and M. MALIK 1987 *Journal of Biomechanical Engineering* **109**, 148–153. A study on a vibration model of a human body.
10. T. E. FAIRLEY and M. J. GRIFFIN 1989 *Journal of Biomechanics* **22**, 81–94. The apparent mass of the seated human body: vertical vibration.
11. T. E. FAIRLEY and M. J. GRIFFIN 1986 *Society of Automotive Engineers International Congress and Exposition*, *Detroit*, *SAE Paper* 860046. A test method for the prediction of seat transmissibility.
12. H. G. LEE and B. J. DOBSON 1991 *Journal of Sound and Vibration* **145**, 61–81. The direct measurement of structural mass, stiffness and damping properties.
13. N. J. MANSFIELD and M. J. GRIFFIN 1997. Non-linearities in biodynamic responses to whole-body vibration. *Awaiting publication*.
14. C. DIERCKX 1995 *Curve and Surface Fitting with Splines*. Oxford: Oxford Science Publications.