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SELF-TUNING ADAPTIVE CONTROL OF FORCED VIBRATION IN ROTOR SYSTEMS USING AN ACTIVE JOURNAL BEARING

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A multivariable self-tuning adaptive controller was developed to control forced vibration of rotor systems incorporating a new type of active journal bearing. The adaptive control algorithm enables the controller to cope with non-linearity, parameter variation with time and parameter uncertainty in rotor-bearing systems. The control algorithm requires no pre-knowledge of system parameters and imbalance distribution. This is especially significant in applications with complex rotor systems supported on fluid-film bearings. The controller enables the system to retain a desired equilibrium position and at the same time reduces vibration about the equilibrium position. A pre-identification procedure was introduced before the self-tuning loop to determine the order of the controller, the time delay of the system, and to accelerate the controller tuning process. The control algorithm is presented quite generally. It may be used as a general procedure in the applications of active vibration control in rotating machinery.

1. INTRODUCTION

Forced vibration in a rotor system is usually excited by centrifugal forces due to residual mass imbalance in the rotor. The imbalance can be introduced in the manufacturing process of parts, or produced at the assembly stage. Even if the rotor is well balanced, the balance may deteriorate with use. To reduce excessive high amplitude of forced vibration is one of the main tasks in the vibration control of rotating machinery. Various balancing techniques are used to reduce the residual imbalance. They can be considered as a passive control of vibrations. Active control, by implementation of various kinds of active bearings or actuators, provides a more efficient and flexible way of on-line attenuation of the forced vibration.

There are numerous methods and concepts in designing feedback control systems for vibration control. Linearized models of rotor-bearing systems are normally adopted in controller design. Mathematical models of the systems can be described either by a state space expression or by an input–output expression.

Based on the state space expression, two control strategies may be identified. One is state feedback control where the control vector is a linear function of the system state vector. The other is output feedback control where the control vector is only a function of the measurement vector. In the state feedback control, the following methods have been used: (a) optimal state feedback to minimize a quadratic integral cost function where all the state

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variables and the control action are included; (b) eigenvalue assignments (or pole assignments) to improve system stability and forced vibration by properly choosing eigenvalues of a closed-loop system. Some algorithms have been developed specially for rotor systems to partially assign eigenstructures which are easy to implement. Some modal controllers can be used to shift one or more eigenvalues of a closed-loop system if, for whatever reason, one or more eigenvalues have positive real parts, or the system runs in the vicinity of resonance.

Full state measurement for a complicated system is usually impossible. State observers are required to reconstruct the full states in order to utilize the full state feedback control. Zhu *et al*. [1] presented a state feedback optimal control to reduce the forced vibration. A quadratic integral criterion was adopted. An extended state observers was used to reconstruct the full states and the imbalance distribution. By simulation, they showed that the optimal control could achieve almost complete vibration cancellation. Stanway and Burrows [2] and Stanway and O'Reilly [3] presented eigenvalue assignment methods by a state feedback control approach to control the system stability. Firoozian and Stanway [4] used a state estimator for their modal controller. Fürst and Ulbrich [5] also investigated the optimal state feedback and modal controllers.

Because of some difficulties in realising a full-state feedback for a flexible rotor-bearing system, optimal output feedback strategies are often used. A quadratic integral cost function that only contains output vector and control signals is most often adopted. Fan *et al*. [6] developed a ''LQR based least-squares output feedback'' procedure for vibration control. They used reduced-order models for general linear asymmetric rotor systems. The output feedback procedure eliminated the requirement of an observer for the use of the LQ regulator. Kim and Lee [7] also developed an output feedback scheme based on a reduced model of rotor systems. The vibration due to the periodical disturbances was reduced by using a disturbance observer and a feed-forward compensator.

Except the state space expression of the control system, there are some other methods which are based on input–output expressions of control systems, e.g. lead-lag compensators [8], PID controllers [9], etc.

All the control methods mentioned above have adopted an assumption that the rotor-bearing–actuator system can be considered linear, and the parameters of the mathematical model are available and time invariant. In the case of rotor systems supported by multi-fluid-film bearings, the assumption is contradictory to the reality because of some special difficulties which involve, for example (a) strong non-linear behaviour due to non-linear properties of the fluid-films; (b) unknown parameters, e.g. the unknown parameters of the operating environment which is too complicated to model, etc.; (c) parameters of the system vary with time, e.g. the operating conditions or the system alignment change with time; (d) change of imbalance distribution.

Because of the above problems, a conventional fixed parameter control strategy may experience inadequate performance. In case of strong non-linearity and/or parameter variation, a fixed parameter control may completely fail to operate. It is also often true that, for a complex rotor-bearing system, some parameters of the system are actually not known or the system is too complicated to model mathematically.

While conventional control schemes fail to provide adequate performance because of the difficulties stated above, an adaptive control strategy could be adopted. The authors have chosen a multivariable self-tuning controller to control adaptively forced vibration of a rotor system incorporating a newly developed active journal bearing [10]. The self-tuning controller features easy implementation and is applicable to complex processes with a wide variety of characteristics involving unknown parameters, time varying process dynamics, presence of non-linearity of plants, and stochastic disturbances.

Self-tuning controller represent an important class of adaptive controllers. The objective of self-tuning is to control systems with unknown constant or slowly varying parameters. In practical applications, special interest derives from its potential as a simple controller commissioning tool for controlling time-varying or non-linear plant over a range of operating points. The original idea of self-tuning was given by Kalman [11]. The major breakthrough came with the self-tuning regulator (STR) of Åström and Wittenmark [12]. Multivariable self-tuning controllers were first considered by Peterka and Åström [13]. Some applications of self-tuning controller in industry include the control of an ore crusher [14], a paper machine [15], ship steering [16], control of raw material blending in a cement plant [17], etc.

The use of a self-tuning controller involves a two-stage process. The first consideration is to choose an appropriate control strategy to resolve what control performance is desirable if the process parameters were known, bringing in engineering judgement and some prior knowledge of the controlled plant characteristics. The self-tuning algorithm is then used to ensure this performance, despite the lack of detailed knowledge of model parameters and their variation with time. The vibration control problem in rotating machinery is generally a multivariable problem. The control strategy proposed by Clarke and Gawthrop [18, 19], and Koivo [20] was adopted in this research. The control parameters are estimated by a standard recursive least-squares algorithm in the manner indicated by Borison [21]. The procedures presented are quite general and may be applicable to a large class of active vibration control problems involving dynamics of rotors.

2. DESCRIPTION OF THE ACTIVE JOURNAL BEARING

The flexible sleeve can be considered as a new feature of the proposed active journal bearings as shown in Figure 1 [10]. The sleeve is activated by the chamber pressure p_c, which is controlled by the servo valve in the hydraulic system. The oil film of the bearing and the pressure chamber is separated by flexible sealant. Therefore, the chamber pressure will not influence the boundary conditions of the oil film.

3. OPTIMAL CONTROL LAW FOR MULTIVARIABLE ROTOR SYSTEMS

As self-tuning controllers are implemented digitally, a controlled process (rotor-bearing–actuator system) is represented by the following vector difference equation in the case of multi-inputs/multi-outputs.

$$\mathbf{r}(t) = \sum_{i=1}^{n} \mathbf{A}_i \cdot \mathbf{r}(t-i) + \sum_{i=0}^{n} \mathbf{B}_i \cdot \mathbf{u}(t-i-k) + \sum_{i=0}^{n} \mathbf{C}_i \cdot \xi(t-i) \tag{1}$$

where $\mathbf{r}(t)$ is the output vector (vibration at the stations of rotor) and $\mathbf{u}(t)$ the input vector (the chamber pressure p_c) at sample instant t; ξ is a sequence of independent, equally distributed random vectors with zero mean value; $\mathbf{A}_i$, $\mathbf{B}_i$ and $\mathbf{C}_i$ are the parameter matrices; k is the time delay ($k \geqslant 1$); n is the order of the difference equation; The dimension of output vector $\mathbf{r}(t)$ is p and the number of control inputs is m. It follows that the dimension of $\mathbf{A}_i$ is $p \times p$, while the dimension of $\mathbf{B}_i$ is $p \times m$. The cost function to be considered is of the form,

$$J = E\{\|\mathbf{W} \cdot [\mathbf{r}(t+k) - \mathbf{r}_0(\mathbf{t})]\|^2 + \|\mathbf{Q} \cdot \mathbf{u}(t)\|^2\} \tag{2}$$

where $\mathbf{W}$ is a positively semi-definite weighting matrix and $\mathbf{Q}$ a positively definite weighting matrix. Most commonly, diagonal matrices are taken as the weighting matrices. $\mathbf{r}_0(t)$ is a known reference signal. In the case of a rotor-bearing system, it can be defined as a constant desired equilibrium position as it is in this paper. $E\{\cdot\}$ is the expectation operator.

The cost function consists of two parts. The first part penalizes the deviation of the rotor displacement about an equilibrium position, i.e. penalizes the vibration of the rotor. The second part penalizes the variations in control signal to reduce fluctuation and peaking of the control signal. The weighting matrices provide a flexibility in choosing control performance.

Criterion 2 is minimized over all the admissible strategies. A control strategy is admissible, if the value of the control signal $\mathbf{u}(t)$ at time t is a function of all the observed outputs up to time t, i.e. $\mathbf{r}(t)$, $\mathbf{r}(t-1)$, $\mathbf{r}(t-2), \ldots$, and all the previously applied control signals $\mathbf{u}(t-1)$, $\mathbf{u}(t-2)$, $\mathbf{u}(t-3), \ldots$ An optimal strategy is an admissible strategy that minimizes Criterion 2.

An optimal control law is deduced by using an optimal predictor [20]. Define

$$\tilde{\mathbf{r}}(t+k|t) = \mathbf{r}(t+k) - \mathbf{e}(t+k) \tag{3}$$

to be the optimal predictor of $\mathbf{r}(t+k)$ at time t using samples up to and including time t, $\{\mathbf{r}(t), \mathbf{r}(t-1), \ldots; \mathbf{u}(t), \mathbf{u}(t-1), \ldots\}$. The prediction error $\mathbf{e}(t+k)$ is not correlated with $\mathbf{r}(t-i)$, $\mathbf{u}(t-i)$ for $i \geqslant 0$, and hence with $\tilde{\mathbf{r}}(t+k|t)$ itself.

Let the optimal k-step-ahead predictor be obtained from the following prediction model,

$$\tilde{\mathbf{r}}(t+k|t) = \sum_{i=0}^{np} \mathscr{A}_i \cdot \mathbf{r}(t-i) + \sum_{i=0}^{np} \mathscr{B}_i \cdot \mathbf{u}(t-i) \tag{4}$$

where $\mathscr{A}_i$ and $\mathscr{B}_i$ are the parameter matrices of the predictor. np is the order of the prediction model.

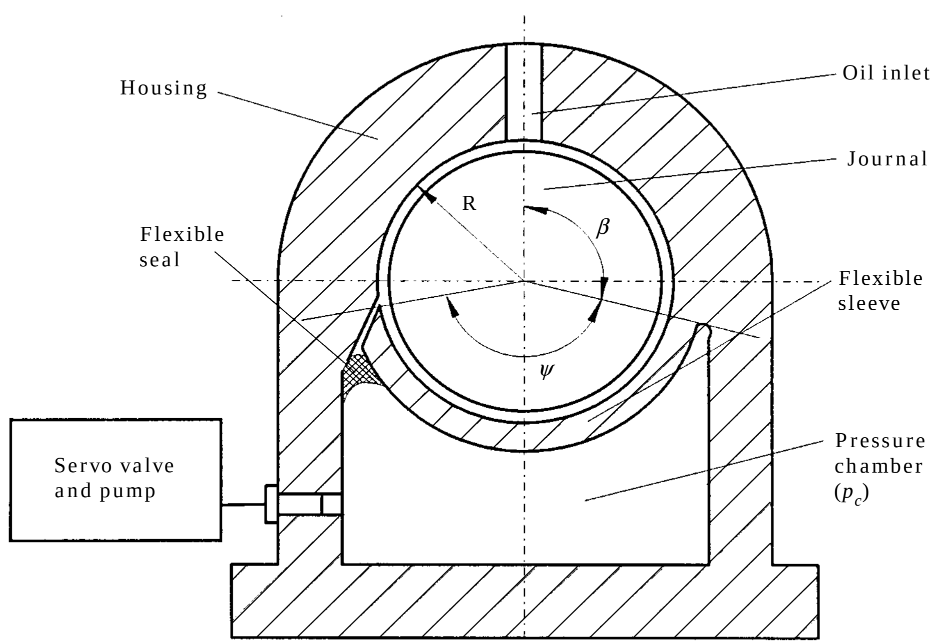

Figure 1. Schematic of the active journal bearing.

Substituting equations (3) and (4) into the cost function (2), we have

$$J = E\left\{\left\|\mathbf{W}\cdot\left[\sum_{i=0}^{np}\mathscr{A}_i\cdot\mathbf{r}(t-i)+\sum_{i=0}^{np}\mathscr{B}_i\cdot\mathbf{u}(t-i)+\mathbf{e}(t+k)-\mathbf{r}_0(t)\right]\right\|^2+\|\mathbf{Q}\cdot\mathbf{u}(t)\|^2\right\} \quad (5)$$

and because $\mathbf{e}(t+k)$ is not correlated with $\mathbf{r}(t-i)$, $\mathbf{u}(t-i)$, $\mathbf{r}_0(t)$ for $i \geqslant 0$, the cost function can be written as

$$J = \left\|\mathbf{W}\cdot\left[\sum_{i=0}^{np}\mathscr{A}_i\cdot\mathbf{r}(t-i)+\sum_{i=0}^{np}\mathscr{B}_i\cdot\mathbf{u}(t-i)-\mathbf{r}_0(t)\right]\right\|^2+\|\mathbf{Q}\cdot\mathbf{u}(t)\|^2+E\{\|\mathbf{e}(t+k)\|^2\}. \quad (6)$$

The cost function is minimized by choosing $\mathbf{u}(t)$ such that:

$$\frac{\partial J}{\partial\mathbf{u}(t)} = 2\mathscr{B}_0^T\mathbf{W}^T\left[\mathbf{W}\left(\sum_{i=0}^{np}\mathscr{A}_i\cdot\mathbf{r}(t-i)+\sum_{i=0}^{np}\mathscr{B}_i\cdot\mathbf{u}(t-i)-\mathbf{r}_0(t)\right)\right]+2\mathbf{Q}^T\mathbf{Q}\cdot\mathbf{u}(t)=\mathbf{0}. \quad (7)$$

Then the optimal control law is found as

$$\mathbf{u}(t) = -((\mathbf{W}\mathscr{B}_0)^T\mathbf{W}\mathscr{B}_0+\mathbf{Q}^T\mathbf{Q})^{-1}(\mathbf{W}\mathscr{B}_0)^T\mathbf{W} \times\left(\sum_{i=0}^{np}\mathscr{A}_i\cdot\mathbf{r}(t-i)+\sum_{i=1}^{np}\mathscr{B}_i\cdot\mathbf{u}(t-i)-\mathbf{r}_0(t)\right). \quad (8)$$

4. SELF-TUNING CONTROLLER

The optimal control law is given by equation (8). Consider now the control of the system presented by equation (1), when the system parameters are unknown and/or time varying. A recursive parameter estimator is introduced in the self-tuning controller. The self-tuning algorithm performs an identification of parameters $\mathscr{A}_i$ and $\mathscr{B}_i$ ($i = 0, 1, \ldots, np$) recursively at each sampling interval. The parameters obtained are then used to compute the control signal using equation (8). Many recursive estimation schemes can be used for the parameter identification. The recursive least squares method is the most commonly used one in parameter identifications. It is used here in the manner indicated by Borison [21].

In view of the prediction model (4), the parameter identification is based on the following estimation model,

$$\mathbf{r}(t) = \sum_{i=0}^{np}\mathscr{A}_i\cdot\mathbf{r}(t-k-i)+\sum_{i=0}^{np}\mathscr{B}_i\cdot\mathbf{u}(t-k-i)+\mathbf{e}(t) \quad (9)$$

where k is the time delay of the system, and $\mathscr{A}_i$ and $\mathscr{B}_i$ ($i = 0, 1, \ldots, np$) are the parameter matrices to be identified.

Defining a data vector $\mathbf{X}(t-k)$ as

$$\mathbf{X}(t-k) = [\mathbf{r}^T(t-k), \mathbf{r}^T(t-k-1), \ldots, \mathbf{r}^T(t-k-np), \mathbf{u}^T(t-k), \mathbf{u}^T(t-k-1), \ldots, \mathbf{u}^T(t-k-np)], \quad (10)$$

and a parameter matrix Θ as

$$\Theta = [\theta_1, \theta_2, \ldots, \theta_p] = [\mathscr{A}_0, \mathscr{A}_1, \ldots, \mathscr{A}_{np}, \mathscr{B}_0, \mathscr{B}_1, \ldots, \mathscr{B}_{np}]^T. \quad (11)$$

Equation (9) can be written componentwise as

$$r_i(t) = \mathbf{X}(t-k) \cdot \theta_i + e_i(t), \qquad (i = 1, 2, \ldots, p). \tag{12}$$

The least squares algorithm estimates the parameters in a way such that the estimation error, up to time N, is minimum in the sense of least squares, i.e.

$$V_i = \frac{1}{N} \sum_{t=1}^{N} e_i^2(t) \to \min, \qquad (i = 1, 2, \ldots, p). \tag{13}$$

In the algorithm, the control parameters Θ are estimated one vector, θ_i, at a time by the standard recursive least-squares algorithm as follows [21],

$$\begin{aligned} \theta_i(t) &= \theta_i(t-1) + \mathbf{K}(t-1)[r_i(t) - \mathbf{X}(t-k)\theta_i(t-1)] \\ \mathbf{K}(t-1) &= \mathbf{P}(t-1)\mathbf{X}^T(t-k)[1 + \mathbf{X}(t-k)\mathbf{P}(t-1)\mathbf{X}^T(t-k)]^{-1} \\ \mathbf{P}(t) &= \mathbf{P}(t-1) - \mathbf{K}(t-1)[1 + \mathbf{X}(t-k)\mathbf{P}(t-1)\mathbf{X}^T(t-k)]\mathbf{K}^T(t-1). \end{aligned} \tag{14}$$

It can be seen that the estimation [equation (14)] is performed p times at each sampling interval and the control parameters Θ are updated. The initial values of $\mathbf{P}(t)$ need to be assumed at the first step of the estimation.

When the estimate of the system parameters is updated at each sampling interval, it is substituted into equation (8) to calculate the control signal using the current measurement of the output as well as the past output and control signals.

The above stated self-tuning control algorithm can be briefly summarized as follows:

Step 1. Read new output $\mathbf{r}(t)$.
Step 2. Form the data vector [equation (10)].
Step 3. Update Θ by the recursive least squares algorithm [equation (14)].
Step 4. Calculate new control $\mathbf{u}(t)$ using equation (8).
Step 5. Set $t = t + 1$ and go to Step 1.

The self-tuning controller algorithm presented here is called "implicit self-tuner" algorithm. It can be noticed that the unknown parameters of the plant are not estimated directly—instead, the parameters of the prediction model are used directly to compute the control signal. In the algorithm, separate steps of identification and control are avoided. The controller parameters are incorporated into the identification procedure.

It should be pointed out that rotor imbalance excitation (which are normally treated as a periodical disturbance) and its response have been included implicitly in the model parameters $\mathbf{A}_i$, $\mathbf{B}_i$ and $\mathbf{C}_i$ in equation (1), and in the estimation model parameters $\mathscr{A}_i$ and $\mathscr{B}_i$ in equation (9). They are on-line identified in the self-tuning algorithm as stated above.

As implied in the above derivation, self-tuning controllers are applicable to a system where the structure of the estimation model is known but the parameters are unknown or time varying. That is to say, the order of the difference equations of the estimation model and the value of time delay are required by the method. The coefficients of the equations can be unknown or varying with time. Of course, sufficient estimation accuracy may be obtained by assuming a very high order of estimation model. But the higher the order, the more computing time is required in the recursive estimation procedure at each sampling interval.

Another important practical problem that is related to a higher order model is the numerical stability of its solution. A higher model tends to induce numerical instability, even if the problem of computing time does not need to be considered. In practical applications, improving numerical stability and reducing computation time are often critical. The use of excessively high order of the difference equations is not necessary and

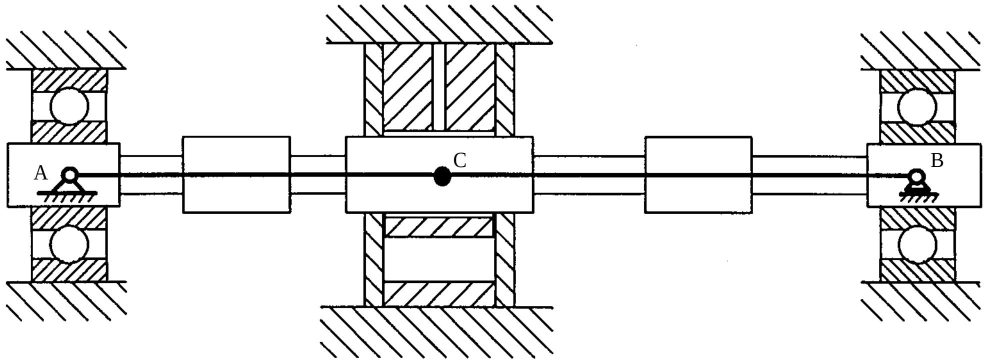

Figure 2. A test rig for a three-bearing rotor system.

should be avoided. Therefore choosing an appropriate order of a physical system is of practical importance. It can also be noted that the initial values of controller parameter vector $\theta_i(0)$, and matrix $\mathbf{P}(0)$ need to be specified in the parameter estimation given by equation (14). Good estimates of these values can reduce the transient period of the recursive estimation significantly, while poor estimates may result in a lengthy tuning of the controller parameters.

Due to the above considerations, and also, in order to assess the validity of the mathematical model described by equation (9) as an estimate of a rotor system incorporating the active journal bearing while the system is subjected to an imbalance excitation, a pre-identification procedure was considered in the research. The pseudo random binary sequence (PRBS) were used as the excitation signal for the identification. They were added to a constant chamber pressure p_{c0} as an alternative component. The mathematical model (9) and the recursive least squares identification procedure, equation (14), were used for the pre-identification of the minimum value of np in equation (9), time delay k, and the initial values of vector $\theta_i(0)$ and matrix $\mathbf{P}(0)$.

5. NUMERICAL SIMULATIONS

Non-linear numerical simulations had been conducted on a laboratory installation of a three-bearing rotor system as illustrated in Figure 2 [10]. The shaft is 2 m long and weighs 11·24 kg. The first three bending natural frequencies of the shaft supported on the two ball bearings are 10·43, 43·04 and 106·38 Hz respectively. The active journal bearing is located at station **C** which is 663 mm from the left ball bearing and 1037 mm from the right one.

5.1. MATHEMATICAL MODEL OF THE ROTOR-BEARING–ACTUATOR SYSTEM

Finite element methods (FEMs) were used for modelling of the shaft and the flexible sleeve of the active journal bearing. Guyan condensation technique was used to reduce the dimensions of the inertia and stiffness matrices obtained from the FEMs. The equations of motion of the rotor and the flexible sleeve can be expressed in the following forms:

$$\mathbf{M}_r \cdot \ddot{\mathbf{q}} + \mathbf{K}_r \cdot \mathbf{q} = \mathbf{H}_r(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{s}, \dot{\mathbf{s}}) - \mathbf{K}_r \cdot \mathbf{a} + \mathbf{F}_r + \mathbf{Q}_r \tag{15}$$

$$\mathbf{M}_s \cdot \ddot{\mathbf{s}} + \mathbf{K}_s \cdot \mathbf{s} = \mathbf{H}_s(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{s}, \dot{\mathbf{s}}) + \mathbf{C}_s \tag{16}$$

where $\mathbf{M}_r$, $\mathbf{K}_r$, $\mathbf{M}_s$ and $\mathbf{K}_s$ stand for the inertia and stiffness matrices of the rotor and the flexible sleeve respectively; $\mathbf{q}$ represents the relative motion of the rotor with respect to the bearing; $\mathbf{s}$ is the motion of the flexible sleeve; $\mathbf{a}$ denotes configuration of the rotor bearing system (the relative position of bearings); $\mathbf{F}_r$ and $\mathbf{Q}_r$ stand for the external excitation caused by rotor imbalance and gravity force respectively. The hydrodynamic forces acting on the rotor and the flexible sleeve caused by oil film pressure p are denoted by $\mathbf{H}_r$ and $\mathbf{H}_s$ respectively; $\mathbf{C}_s$ stands for the hydraulic force caused by control pressure p_c.

Distribution of the oil film pressure p was modelled by means of the Reynolds equations given in the following form:

$$\frac{1}{R^2}\frac{\partial}{\partial \varphi}\left(\frac{h^3}{\eta}\frac{\partial p}{\partial \varphi}\right) + \frac{\partial}{\partial z}\left(\frac{h^3}{\eta}\frac{\partial p}{\partial z}\right) = 6\Omega\frac{\partial h}{\partial \varphi} + 12\frac{\partial h}{\partial t} \tag{17}$$

where h is the instantaneous thickness of the oil film; η the oil viscosity; R the radius of the journal; φ the angular co-ordinates of the journal bearing. Equations (15), (16) and (17) form a simultaneous set of non-linear differential equations which was used to simulate the real motion of the rotor bearing system.

More details on modelling of the rotor, the flexible sleeve and the oil film can be found in [10].

5.2. RESULTS OF SIMULATIONS

In the numerical simulations based on the non-linear mathematical model of the rotor-bearing system, only the vibration of the rotor at the journal bearing position is of interest. Therefore, the dimension of the output vector in equation (1) is two. Because only one pressure chamber was used in the active bearing of the test rig, the number of the control input is one. The order of the equation and the time delay of the system were determined first by the pre-identification procedure indicated before. Then the model structure obtained (order and time delay) was introduced to the self-tuning algorithm to simulate the performance of the self-tuning controller on the vibration control. The rotating speed of the rotor was fixed at 3000 RPM, the system configuration parameters were set to zero ($\mathbf{a} = \mathbf{0}$), and the constant part of the chamber pressure p_{c0} was set to 0·1 MPa for all the simulations.

In the pre-identification, the time interval of the PRBS was 2·5 ms so that the spectrum of the PRBS well covers the response of the rotor system of interest. The length of the PRBS is chosen as 127 (317·5 ms). The amplitude of the PRBS is 0·03 MPa. Figure 3(a) and (b) show the time domain response of the journal in both the horizontal (x) and vertical (y) directions before and after the PRBS was applied (non-dimensional form of the response was adopted by dividing the real response by the nominal bearing clearance). The identification began with the time delay $k = 1$ and the order of the prediction model $np = 1$. Then np was gradually increased until the cost function of the least squares method did not change much and the predicted results approximated the real response in an accepted accuracy. It was found that when $np = 3$, the predicted response was almost identical to the real response as shown in Figure 3(c) and (d). It reveals that the prediction model (9) can predict the imbalance response of the rotor system very well. This establishes a foundation to the success of the self-tuning vibration control approach. The spectrum of the response displayed in Figure 3(a) and the chamber pressure with the PRBS are shown in Figure 3(e) and (f) respectively.

Before the self-tuning control was applied, it was found that the equilibrium position of the journal is ($-0{\cdot}042$, $-0{\cdot}034$). In order not to alter the equilibrium position by the feedback controller, this equilibrium position was taken as the reference signal $\mathbf{r}_0$ in the cost function (2). The response of the feedback system with the self-tuning controller was simulated under different parameters of the diagonal weighting matrices $\mathbf{W}$ and $\mathbf{Q}$ in the cost function. Because the dimension of the output vector in equation (9) is two and the number of control signal is one, the dimension of $\mathbf{W}$ is 2×2 of which the diagonal elements are denoted as w_x and w_y corresponding to the motion of the journal in the X and Y directions respectively, and matrix $\mathbf{Q}$ becomes a scalar number whose value is denoted as Q. The sampling period of the self-tuning controller was chosen as 2·5 ms. It should be noted that a saturation non-linearity, $p_c \geqslant 0$, was introduced in the simulations to reflect reality.

Figure 4(a), (b) and (c) show the response of the journal and its spectrum in both the X and Y directions, and the control signal, in steady states respectivelywhen $w_x = w_y = 1$, and $Q^2 = 0$. The percentage figures in the spectrum graph refer to the reduction rate in the synchronous vibration with respect to that without the self-tuning controller. Because the control action is not penalized in the cost function ($Q^2 = 0$), although the reduction of the vibration is great, there is sharp fluctuation in the control signal, and the spectra of the response become very complex. The change of the equilibrium position can also be noticed from Figure 4(a). It reveals that the proposed self-tuning algorithm does not guarantee that the output always tracks the reference signal without bias under whatever conditions. Clarke and Gawthorp [18], and Koivo [20] discussed such a phenomenon.

Figure 5 shows the steady state response, its spectrum, and the control signal, when $w_x = w_y = 1$, and $Q^2 = 0{\cdot}1$. Because the control action is penalized in the cost function, the peaking and fluctuation are greatly reduced in the control signal. The main component in the control signal is the one which synchronizes the rotating speed. The synchronous

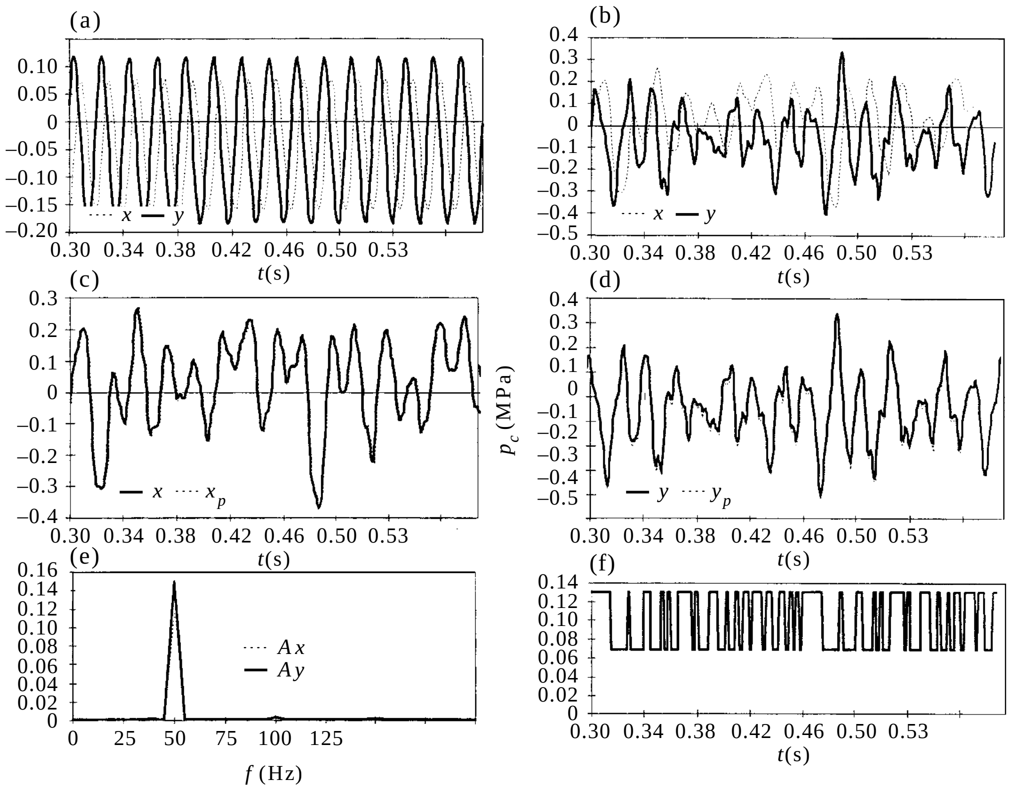

Figure 3. Response of the journal in pre-identification: (a) before the PRBS excitation, (b) after the PRBS excitation, (c) and (d) comparison of actual response and the predicted output, x, y and x_p, y_p denote the real response and the predicted output respectively, (e) spectrum corresponding to the response shown in (a), and (f) chamber pressure with the PRBS.

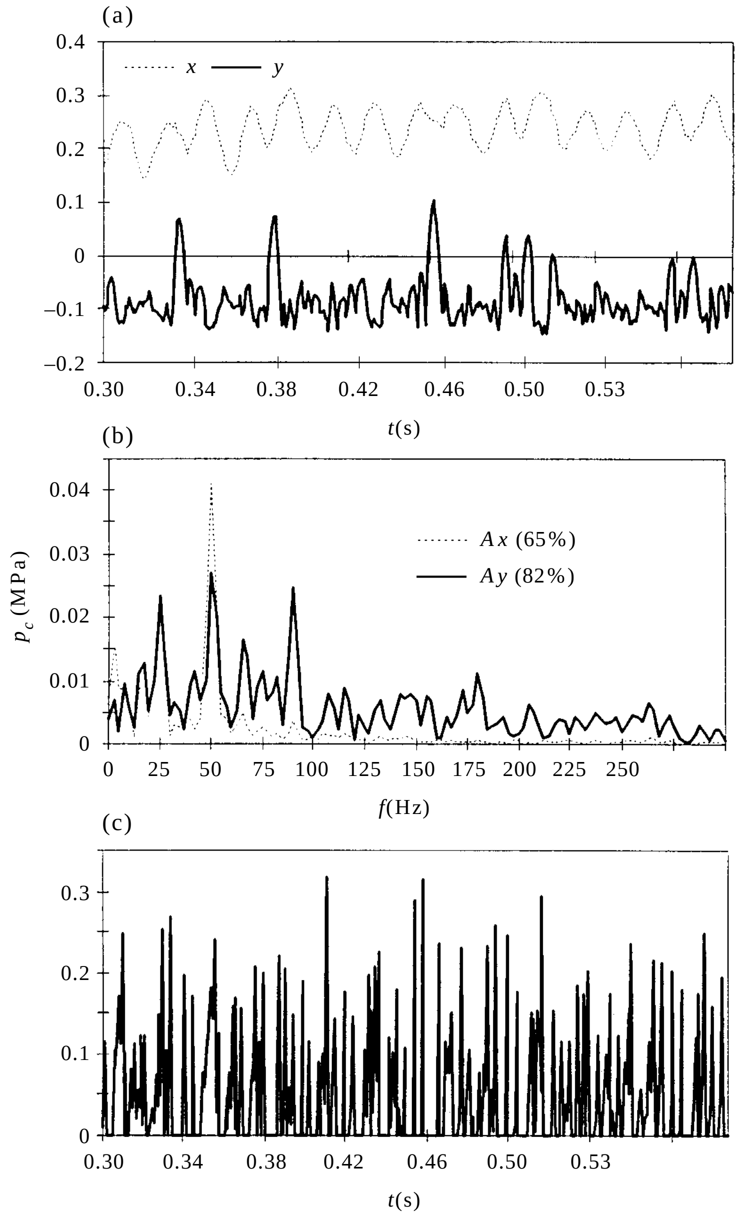

Figure 4. (a) Response of journal, (b) spectrum, and (c) control pressure ($w_x/w_y = 1$, $Q^2 = 0$).

vibration also dominates the spectra of the journal response. The equilibrium position of the journal remains unchanged. It can also be noticed that the reduction rate of the synchronous vibration in the X-direction is quite different from that in the Y direction (37% in the X direction, 80% in the Y direction). The amplitudes of the vibration in these two directions are also quite different after control. A better vibration control ability was observed in the Y direction.

After altering the coefficients of the weighting matrices, Figure 6(a), (b) and (c) show the steady state response of the journal, its spectrum and the control signal, when $w_y = 1$, $w_x = 1{\cdot}67$ and $Q^2 = 0{\cdot}025$. The equilibrium position of the journal remains the same as before the feedback control was applied. The synchronous vibration is reduced by 45 and 65% in the X and Y directions respectively compared with those without the feedback control. After such a change in the weighting matrices, the two components of the vibration are close to each other. Figure 6(d), (e) and (f) also show the response of the journal, the control signal and one of the controller parameters in the transient period. Because the results from pre-identification were used as the initial values of the controller parameters, the transient tuning period has been reduced significantly. The transient period finished after about 0·2 s which is about 10 revolutions of the rotor.

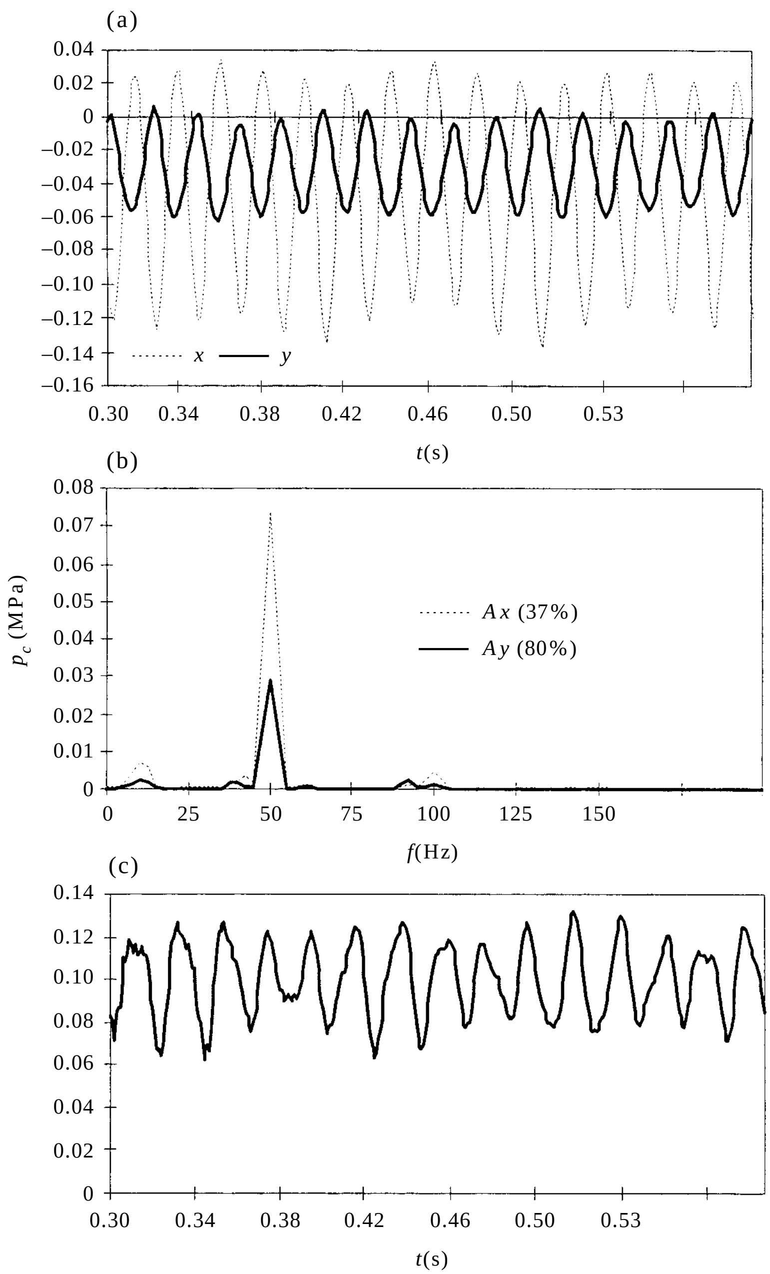

Figure 5. (a) Response of journal, (b) spectrum, and (c) control pressure ($w_x/w_y = 1$, $Q^2 = 0{\cdot}1$).

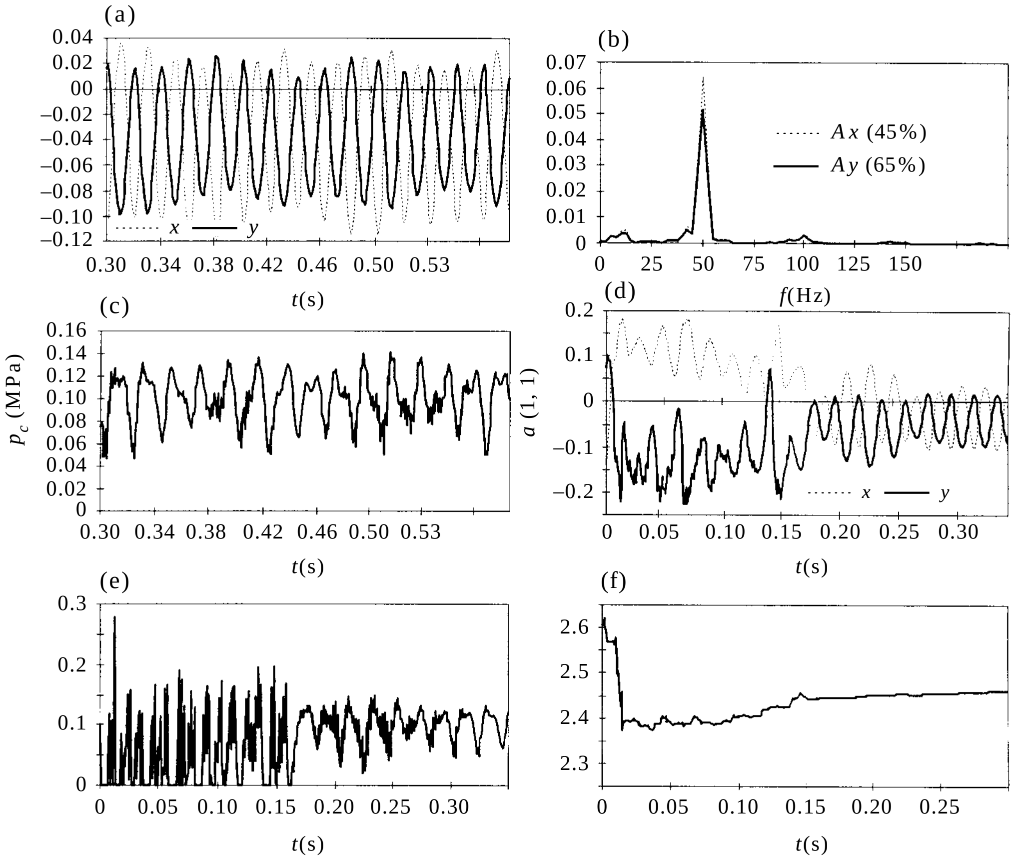

Figure 6. (a) Steady state response of journal, (b) spectrum, (c) steady state control pressure p_c, (d) transient response of journal, (e) transient control pressure, and (f) control parameter $a(1, 1)$; ($w_x/w_y = 1{\cdot}67$, $Q^2 = -0{\cdot}025$).

In all the above simulations, the predicted motion of the journal using the prediction model (4) was almost identical to the actual vibration simulated from the non-linear model of the rotor-bearing system. Good predictions of the output are essential to the success of the self-tuning controller.

It was expected and proved by the simulations that the controllability of the bearing actuator shown in Figure 1 is different in the vertical and horizontal directions. A better vibration suppression in the vertical direction was obtained. To obtain better controllability, bearing with two flexible sleeves that are controlled by two separate chambers may be used. Such modifications would allow for much more effective two input control.

6. CONCLUSIONS

The proposed self-tuning controller is suitable for the forced vibration control of the rotor system incorporating the active journal bearing. The amplitude of the synchronous vibration due to rotor imbalance can be effectively reduced. The main advantages of the self-tuning controller are: (a) no pre-knowledge of the parameters of the rotor-bearing system is required—neither is the imbalance distribution in the rotor; (b) the self-tuning controller can adjust its parameters to adapt to the changes of the characteristics of the rotor-bearing system; and (c) as the self-tuning controller is based on an input/output model, it is easy to implement.

The inclusion of the reference signal in the cost function enables the response of the rotor system to follow a desired equilibrium position; or in other words, the equilibrium position can be controlled while the vibration is reduced. By including the control signal in the cost function, excessive control action can be reduced.

The pre-identification procedure, which is introduced before the self-tuning loop, can greatly reduce the transient tuning time of the controller parameters. The procedure is quite general and may be used as a general procedure in the applications related with the active control of rotor-bearing systems and other applications when a self-tuning controller is used.

Various simulations had been conducted to prove the effectiveness of the presented control scheme under different conditions, including changes of imbalance distribution, rotating speed, lubricant properties and journal equilibrium position, etc. The rotor-bearing system model, equations (15), (16) and (17), had been verified by experiment [22]. The experimental investigation of the effectiveness of the self-tuning adaptive control algorithm on vibration attenuation of rotor system supplied with the proposed active journal bearing would be a future research.

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