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RANDOM VIBRATION OF MULTI-SPAN TIMOSHENKO BEAM DUE TO A MOVING LOAD

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The method of modal analysis is presented to investigate the random vibration of a multi-span Timoshenko beam due to a load moving at a constant velocity. The load is considered to be a stationary process with a constant mean value and a variance. The effects of both velocity and statistical characteristics of the load and the span number of the beam on both the mean value and the variance of the deflection and the moment of the structure are investigated. Moreover, the results obtained from a multi-span Timoshenko beam are compared with those from a multi-span Bernoulli–Euler beam.

1. INTRODUCTION

The vibration of a load moving on structures has attracted engineers' interest since railway bridges were erected at the beginning of the nineteenth century. Usually, the beam structures on which loads move are periodically supported. This kind of beam is called a multi-span beam. A typical example of a multi-span beam is a continuous guideway. Due to technological advancement, the speed and weight of vehicles on guideways are increasingly complex. Under these circumstances, the vibration of beams due to moving loads has become a critical consideration in structural design. Previous literature regarding the vibration of a load moving on multi-span beams has been abundant in recent years [1–3]. The maximum deflection and the maximum moment of a multi-span beam induced by a moving load are always greater than those induced by the same static load. Furthermore, a critical velocity exists at which beam structures seriously deform. Moreover, higher span numbers produce greater values of absolute maximum deflection, absolute maximum moment and critical velocity. The upper bound of the critical velocity is the lowest phase velocity of bending wave in the structure [4].

In the aforementioned work, the magnitude of the loads is considered to remain constant. In most cases, both magnitude and velocity of a moving load, e.g., traffic flow on an elevated highway and turbulence to wings, cannot be described deterministically. The response of structures due to such a kind of load is, consequently, unpredictable. Even worse, those loads may cause enormous disasters. Fortunately, the statistical characteristics of moving loads can be estimated. Numerous studies have been undertaken regarding the random vibration of one-span beams to moving loads [5–10]. In actual situations, a multi-span beam is more widely encountered in structures than a single-span beam. However, the random vibration of multi-span beams due to moving loads has never been investigated.

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The wave velocity in the Bernoulli–Euler beam becomes unreasonable within the high frequency range. This is the reason why the Bernoulli–Euler beam theory leads to erroneous results for both a large ratio of thickness to length and high modes. Such an error for high modes indicates that the theory cannot predict accurately the response of a beam subjected to a rapidly travelling load. Therefore, the Timoshenko beam theory is considered herein to investigate the random vibration of beams. A multi-span beam is considered to be homogeneous and isotropic with Young's modulus E, shear modulus G, Poisson's ratio μ, shear coefficient κ, cross-sectional area A, density ρ, second moment of area I and the radius of gyration of cross section η. A concentrated load moving on the multi-span beam at a constant velocity is adopted to investigate the fundamental response phenomena of the entire beam. The load is assumed here to be a stationary random process with a constant mean value and a variance. Four kinds of variances of the load are considered herein: a white noise process, an exponential process, an exponential cosine process and a cosine process. In addition, the effects of both velocity and variance type of the load and the effect of spans number of the multi-span beam on both the mean value and the variance of responses of the entire beam are investigated here. These results are compared with those of a multi-span Bernoulli–Euler beam.

2. THE EQUATIONS OF MOTION

A distributed load $\bar{P}(\bar{x}, \bar{t})$ on an n-span Timoshenko beam is depicted in Figure 1. Each span has the same length L. The transverse shear force $\bar{q}$ and moment $\bar{m}$ are [11]

$$\bar{q} = \kappa GA(\partial\bar{w}/\partial\bar{x} - \bar{\psi}), \qquad \bar{m} = -EI\partial\bar{\psi}/\partial\bar{x}, \tag{1a, b}$$

in which $\bar{x}$ is the axial co-ordinate, $\bar{w}$ is the transverse deflection and $\bar{\psi}$ is the rotatory angle. The equations of motion of the entire beam are

$$\partial\bar{q}/\partial\bar{x} + \bar{P}(\bar{x}, \bar{t}) = \rho A\partial^2\bar{w}/\partial\bar{t}^2, \qquad \bar{q} - \partial\bar{m}/\partial\bar{x} = \rho I\partial^2\bar{\psi}/\partial\bar{t}^2, \tag{2a, b}$$

where $\bar{t}$ is time.

According to the orthogonality of two distinct sets of mode shape functions [4], the transverse deflection, rotatory angle, transverse shear force and moment of the beam can be expressed in terms of the superposition of mode shape functions as

$$\{\bar{w}\bar{\psi}\bar{q}\bar{m}\}(\bar{x}, \bar{t}) = \sum_{j=1} a_j(\bar{t})\{\bar{W}_j\bar{\Psi}_j\bar{Q}_j\bar{M}_j\}(\bar{x}), \tag{3}$$

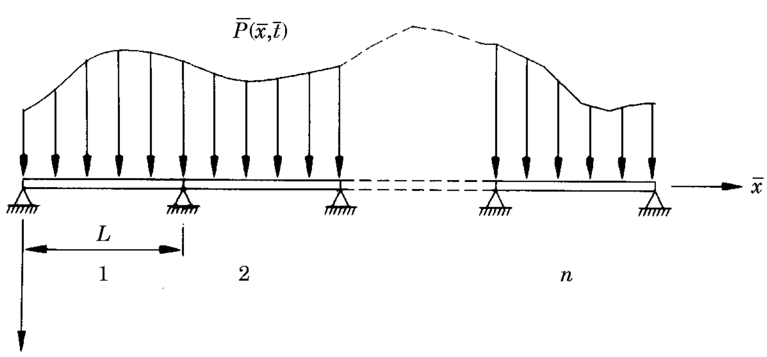

Figure 1. A distributed load $\bar{P}(\bar{x}, \bar{t})$ on an n-span Timoshenko beam.

where $\overline{W}_j$ and $\overline{\Psi}_j$ are the jth set of mode shape functions of displacement and rotatory angle, and $\overline{Q}_j$ and $\overline{M}_j$ are their corresponding transverse shear force and moment. Substituting equation (3) into equations (2a) and (2b) yields

$$-\sum_{j=1} a_j \frac{\mathrm{d}\overline{Q}_j}{\mathrm{d}x} + \sum_{j=1} \rho A \overline{W}_j \frac{\mathrm{d}^2 a_j}{\mathrm{d}\bar{t}^2} = \overline{P}(\bar{x}, \bar{t}), \tag{4a}$$

$$-\sum_{j=1} a_j \left(\overline{Q}_j - \frac{\mathrm{d}\overline{M}_j}{\mathrm{d}\bar{x}} \right) + \sum_{j=1} \rho I \overline{\Psi}_j \frac{\mathrm{d}^2 a_j}{\mathrm{d}\bar{t}^2} = 0. \tag{4b}$$

Multiplying equation (4a) by $\overline{W}_k$ and equation (4b) by $\overline{\Psi}_k$, then integrating their summation from $\bar{x} = 0$ to $\bar{x} = nL$ yield the governing equation of the kth modal amplitude a_k as

$$\frac{\mathrm{d}^2 a_k}{\mathrm{d}\bar{t}^2} + \bar{\omega}_k^2 a_k = g_k(\bar{t}), \tag{5}$$

where the modal frequency $\bar{\omega}_k$ and the modal excitation $g_k(\bar{t})$ are

$$\bar{\omega}_k^2 = s_k / m_k, \qquad g_k(\bar{t}) = \int_0^{nL} \overline{P}(\bar{x}, \bar{t}) \overline{W}_k(\bar{x}) \, \mathrm{d}\bar{x} / m_k, \tag{6a, b}$$

in which the corresponding modal mass m_k and modal stiffness s_k are

$$m_k = \int_0^{nL} (\rho A \overline{W}_k^2 + \rho I \overline{\Psi}_k^2) \, \mathrm{d}\bar{x}, \tag{7a}$$

$$s_k = -\int_0^{nL} \left\{ \overline{W}_k \frac{\mathrm{d}\overline{Q}_k}{\mathrm{d}\bar{x}} + \overline{\Psi}_k \left(\overline{Q}_k - \frac{\mathrm{d}\overline{M}_k}{\mathrm{d}\bar{x}} \right) \right\} \mathrm{d}\bar{x}. \tag{7b}$$

The initial conditions of the beam are set at zero. The response history of the kth modal amplitude $a_k(\bar{t})$ is

$$a_k(\bar{t}) = \int_0^{\bar{t}} h_k(\bar{t} - \bar{\tau}) g_k(\bar{\tau}) \, \mathrm{d}\bar{\tau}, \tag{8}$$

in which the kth modal impulse response $h_k(\bar{t})$ is

$$h_k(\bar{t}) = \left\{ \begin{matrix} \sin(\bar{\omega}_k \bar{t}) / \bar{\omega}_k & (0 < \bar{t}) \\ 0 & (\bar{t} \leqslant 0) \end{matrix} \right\}, \tag{9}$$

Equation (8) can be expressed as the form

$$a_k(\bar{t}) = \int_{-\infty}^{\infty} h_k(\bar{\tau}) g_k(\bar{t} - \bar{\tau}) \, \mathrm{d}\bar{t}. \tag{10}$$

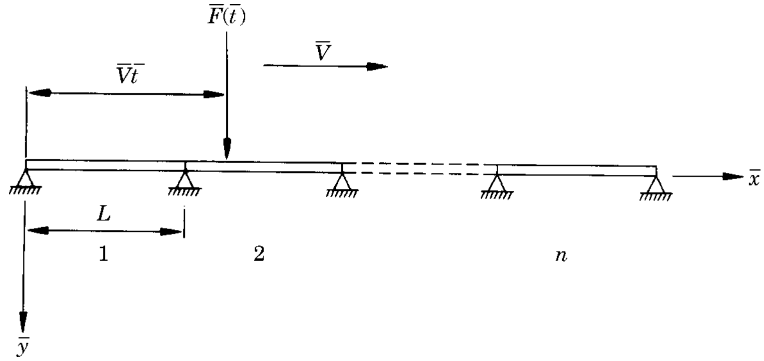

Figure 2. A random load moves on the n-span Timoshenko beam.

3. MOTION OF A RANDOM LOAD ON THE BEAM

A random load $\bar{F}(\bar{t})$ moving on the beam at a constant velocity. $\bar{v}$ is depicted in Figure 2. This random load is considered to be a stationary process with a constant mean value $\bar{F}_0(=\langle \bar{F}(\bar{t})\rangle)$ and a centred deviation $\bar{f}(\bar{t})$. The covariance $C_{\bar{f}}(\bar{t}_1, \bar{t}_2)$ between two deviations $\bar{f}(\bar{t}_1)$ and $\bar{f}(\bar{t}_2)$ is

$$C_{\bar{f}}(\bar{t}_1, \bar{t}_2) = \langle \bar{f}(\bar{t}_1)\bar{f}(\bar{t}_2)\rangle, \tag{11}$$

in which $\langle\rangle$ is the mean value operator. The respective histories of the kth modal excitation $g_k(\bar{t})$ and its corresponding mean value $\langle g_k(\bar{t})\rangle$, and the covariance $C_{g_jg_k}(\bar{t}_1, \bar{t}_2)$ between $g_j(\bar{t}_1)$ and $g_k(\bar{t}_2)$ are

(1) $0 \leqslant \bar{t}, \bar{t}_1, \bar{t}_2 \leqslant nL/\bar{v}$

$$g_k(\bar{t}) = \bar{F}(\bar{t})\bar{W}_k(\overline{vt})/m_k, \qquad \langle g_k(\bar{t})\rangle = \bar{F}_0\bar{W}_k(\overline{vt})/m_k, \tag{12a, b}$$

$$C_{g_jg_k}(\bar{t}_1, \bar{t}_2) = C_{\bar{f}\bar{f}}(\bar{t}_1, \bar{t}_2)\ \bar{W}_j(\overline{vt_1})\bar{W}_k(\overline{vt_2})/m_jm_k; \tag{12c}$$

(2) $nL/\bar{v} < \bar{t}, \bar{t}_1$ (or $\bar{t}_2$)

$$g_k(\bar{t}) = 0, \qquad \langle g_k(\bar{t})\rangle = 0, \qquad C_{g_jg_k}(\bar{t}_1, \bar{t}_2) = 0. \tag{13a–c}$$

The mean value histories of the jth modal amplitude, transverse deflection and moment, respectively, are

$$\langle a_j(\bar{t})\rangle = \int_{-\infty}^{\infty} h_j(\bar{t} - \bar{\tau})\langle g_j(\bar{\tau})\rangle\, \mathrm{d}\bar{\tau}, \tag{14a}$$

$$\langle \bar{w}(\bar{x}, \bar{t})\rangle = \sum_{j=1} \langle a_j(\bar{t})\rangle \bar{W}_j(\bar{x}), \qquad \langle \bar{m}(\bar{x}, \bar{t})\rangle = \sum_{j=1} \langle a_j(\bar{t})\rangle \bar{M}_j(\bar{x}). \tag{14b, c}$$

The covariance $C_{a_ja_l}(\bar{t}_1, \bar{t}_2)$ between $a_j(\bar{t}_1)$ and $a_l(\bar{t}_2)$ is

$$C_{a_ja_l}(\bar{t}_1, \bar{t}_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_j(\bar{t}_1 - \bar{\tau}_1)h_l(\bar{t}_2 - \tau_2)C_{g_jg_l}(\bar{\tau}_1, \bar{\tau}_2)\, \mathrm{d}\bar{\tau}_1\, \mathrm{d}\bar{\tau}_2. \tag{14d}$$

The deflection covariance $C_{\bar{w}}(\bar{x}_1, \bar{x}_2, \bar{t}_1, \bar{t}_2)$ between $\bar{w}(\bar{x}_1, \bar{t}_1)$ and $\bar{w}(\bar{x}_2, \bar{t}_2)$ is

$$C_{\bar{w}}(\bar{x}_1, \bar{x}_2, \bar{t}_1, \bar{t}_2) = \sum_{j=1} \sum_{l=1} C_{a_j a_l}(\bar{t}_1, \bar{t}_2) \bar{W}_j(\bar{x}_1) \bar{W}_l(\bar{x}_2). \tag{15a}$$

Similarly, the moment covariance $C_{\bar{m}}(\bar{x}_1, \bar{x}_2, \bar{t}_1, \bar{t}_2)$ between $\bar{m}(\bar{x}_1, \bar{t}_1)$ and $\bar{m}(\bar{x}_2, \bar{t}_2)$ is

$$C_{\bar{m}}(\bar{x}_1, \bar{x}_2, \bar{t}_1, \bar{t}_2) = \sum_{j=1} \sum_{l=1} C_{a_j a_l}(\bar{t}_1, \bar{t}_2) \bar{M}_j(\bar{x}_1) \bar{M}_l(\bar{x}_2). \tag{15b}$$

Consider the deviation $\bar{f}(\bar{t})$ to be a stationary process, i.e.,

$$C_{\bar{f}}(\bar{t}_1, \bar{t}_2) = C_{\bar{f}}(\bar{t}_1 - \bar{t}_2). \tag{16}$$

The covariance $C_{g_j g_l}(\bar{t}_1, \bar{t}_2)$ and the covariance $C_{a_j a_l}(\bar{t}_1, \bar{t}_2)$, respectively, are

$$C_{g_j g_l}(\bar{t}_1, \bar{t}_2) = \begin{Bmatrix} C_{\bar{f}}(\bar{t}_1 - \bar{t}_2) \bar{W}^{(j)}(\overline{vt_1}) \bar{W}^{(l)}(\overline{vt_2})/(m_j m_l) & 0 \leqslant \bar{t}_1, \bar{t}_2 \leqslant nL/\bar{v} \\ 0 & nL/\bar{v} \leqslant \bar{t}_1, \bar{t}_2 \end{Bmatrix}, \tag{17a}$$

$$C_{a_j a_l}(\bar{t}_1, \bar{t}_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_j(\bar{t}_1 - \bar{\tau}_1) h_l(\bar{t}_2 - \bar{\tau}_2) C_{g_j g_l}(\bar{\tau}_1, \bar{\tau}_2)\, d\bar{\tau}_1\, d\bar{\tau}_2 / m_j m_l. \tag{17b}$$

Furthermore, the variances of deflection and moment of the entire beam are denoted as $\bar{\sigma}_{\bar{w}}^2(\bar{x}, \bar{t})$ and $\bar{\sigma}_{\bar{m}}^2(\bar{x}, \bar{t})$, respectively, which are

$$\bar{\sigma}_{\bar{w}}^2(\bar{x}, \bar{t}) = C_{\bar{w}}(\bar{x}, \bar{x}, \bar{t}, \bar{t}) = \sum_{j=1} \sum_{l=1} C_{a_j a_l}(\bar{t}, \bar{t}) \bar{W}_j(\bar{x}) \bar{W}_l(\bar{x}), \tag{18a}$$

$$\bar{\sigma}_{\bar{m}}^2(\bar{x}, \bar{t}) = C_{\bar{m}}(\bar{x}, \bar{x}, \bar{t}, \bar{t}) = \sum_{j=1} \sum_{l=1} C_{a_j a_l}(\bar{t}, \bar{t}) \bar{M}_j(\bar{x}) \bar{M}_l(\bar{x}). \tag{18b}$$

The following four types of variance with the standard deviation $\bar{\sigma}_0^2$ are considered (Figures 3(a), (b)) in the study:

(1) *white noise*

$$C_{\bar{f}}(\bar{\tau}) = \bar{\sigma}_0^2 \delta(\bar{\tau}); \tag{19a}$$

(2) *cosine*

$$C_{\bar{f}}(\bar{\tau}) = \bar{\sigma}_0^2 \cos(\bar{\omega}_0 \bar{\tau}); \tag{19b}$$

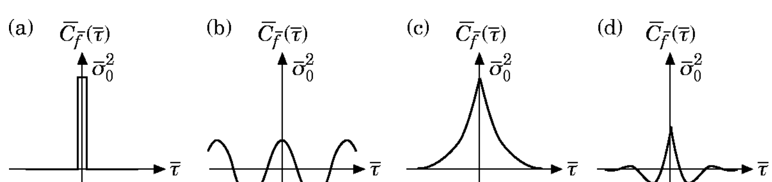

Figure 3. Four types of variances of the load: (a) white noise, (b) cosine (c) exponential and (d) exponential cosine.

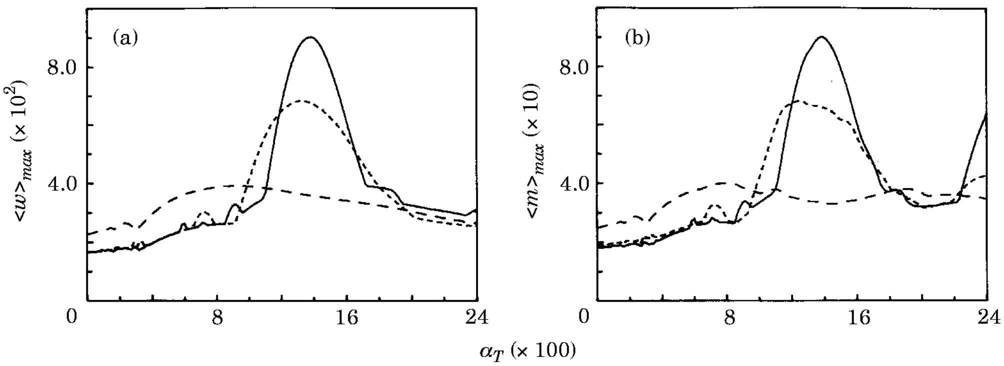

Figure 4. The span number effect on (a) the $\langle w\rangle_{max} - \alpha_T$ distribution and (b) the $\langle m\rangle_{max} - \alpha_T$ distribution of a multi-span Timoshenko beam. Key: ——, 5 spans; - - - - -, 3 spans; – – –, 1 span.

(3) *exponential*

$$C_{\bar{f}}(\bar{\tau}) = \bar{\sigma}_0^2 \, e^{-\bar{\omega}_g \bar{\tau}}; \tag{19c}$$

(4) *exponential cosine*

$$C_{\bar{f}}(\bar{\tau}) = \bar{\sigma}_0^2 \, e^{-\bar{\omega}_g \bar{\tau}}; \cos(\bar{\omega}_0 \bar{\tau}). \tag{19d}$$

4. ILLUSTRATIVE EXAMPLES AND DISCUSSION

To illustrate the numerical results of the present study, the non-dimensional variables adopted are defined as follows: $w = \bar{w}/\eta$, $\psi = \bar{\psi}L/\eta$, $x = \bar{x}/L$, $\bar{t} = (EI/\rho a L^4)^{1/2} t$, $\varepsilon = E/\kappa G$, $q = \bar{q}L^3/EI\eta$, $m = \bar{m}L^2/EI\eta$, $r = \eta/L$, $\bar{\omega} = (EI/\rho A L^4)^{-1/2}\omega$, $(\bar{F}_0, \bar{\sigma}_0) = (F_0, \sigma_0)L^4/EI\eta$, $\alpha_T (= \bar{v}/(E/\rho)^{1/2})$, in which α_T is the velocity ratio. The data $\kappa = 0{\cdot}85$, $\mu = 0{\cdot}3$ and $r = 0{\cdot}05$ are taken for the purpose of numerical analysis. It is known that the lowest fifteen modal frequencies and their corresponding sets of mode shape functions of the entire beam are sufficiently required in the method of modal analysis in the numerical computation [4]. The value of F_0 is assumed to be unity. The velocity range considered in this section is $0 \leqslant \alpha_T \leqslant 0{\cdot}24$. The following parameters are defined to illustrate the numerical results: maximum mean value of deflection during the motion of the load, $\langle w\rangle_{max}$; maximum mean value of moment during the motion of the load $\langle m\rangle_{max}$; position of maximum mean value of deflection during the motion of the load, $X_{\langle w\rangle}$; position of maximum mean value of moment during the motion of the load, $X_{\langle m\rangle}$; maximum variance of deflection during the motion of the load, $\sigma^2_{w,max}$; maximum variance of moment during the motion of the load, $\sigma^2_{m,max}$; position of maximum variance of deflection during the motion of the load, $(X_\sigma)_w$; position of maximum variance of moment during the motion of the load, $(X_\sigma)_m$; velocity ratio at which absolute maximum mean value of deflection appears, α_c; velocity ratio at which absolute maximum variance of deflection appears, α_σ.

4.1. MEAN VALUE

The effects of the span number on the $\langle w\rangle_{max} - \alpha_T$ and the $\langle m\rangle_{max} - \alpha_T$ distributions of a multi-span Timoshenko beam are displayed in Figures 4(a) and 4(b) respectively. The higher span number results in a heavier mass of the entire beam. The load can be regarded as a quasistatic load within the low velocity range $0 \leqslant \alpha_T \leqslant 0{\cdot}08$. Therefore, as the span number increases, both $\langle w\rangle_{max}$ and $\langle m\rangle_{max}$ decrease within the velocity range $0 \leqslant \alpha_T \leqslant 0{\cdot}08$. The effect of bending wave on the vibration of the beam is more apparent for higher span numbers [4]. As a result, both figures demonstrate that α_c and both absolute

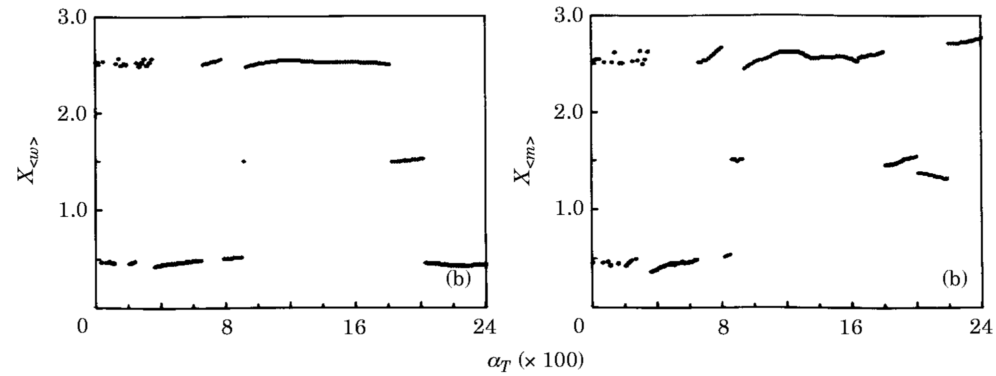

Figure 5. (a) The $X_{\langle w\rangle} - \alpha_T$ distribution and (b) the $X_{\langle m\rangle} - \alpha_T$ distribution of a three-span Timoshenko beam.

$\langle w\rangle_{max}$ and $\langle m\rangle_{max}$ increase as the span number increases. Furthermore, α_c is more obvious for the higher span number.

The $X_{\langle w\rangle} - \alpha_T$ and the $X_{\langle m\rangle} - \alpha_T$ distributions of a three-span Timoshenko beam are shown in Figures 5(a) and 5(b) respectively. There is no reaction moment at both the first support ($x = 0$) and the fourth support ($x = 3$). Therefore, $\langle w\rangle_{max}$ always appears near the mid-point of either the first span or the third span. Within a low velocity range $0 \leqslant \alpha_T \leqslant 0{\cdot}07$, $\langle m\rangle_{max}$ occurs when the load travels on the beam. Therefore, $\langle m\rangle_{max}$ appears approximately near the mid-point of either the first span or the third span within the velocity range. For a load moving at a supercritical velocity, $\langle m\rangle_{max}$ will appear after the load has left the beam. Therefore, $\langle m\rangle_{max}$ occurs within the third span for the load travelling at a supercritical velocity. Both the displacement and the resultant moment are zero at the hinge supports of the beam. As a result, both figures show that the $X_{\langle w\rangle} - \alpha_T$ and $X_{\langle m\rangle} - \alpha_T$ distributions of the beam are discontinuous.

4.2. WHITE NOISE PROCESS

The frequency range of the power spectrum of the white noise process extends from negative infinity to positive infinity. Therefore, all modal responses of the Timoshenko beam are excited by the white noise process. A slow moving load results in a long duration of forced vibration of the beam. Therefore, the beam will be in the steady state of vibration as the duration of forced vibration goes to infinite, i.e., the velocity of the load approaches zero. Under this circumstance, the beam will be in resonance. Therefore, both $\sigma^2_{w,max}$ and

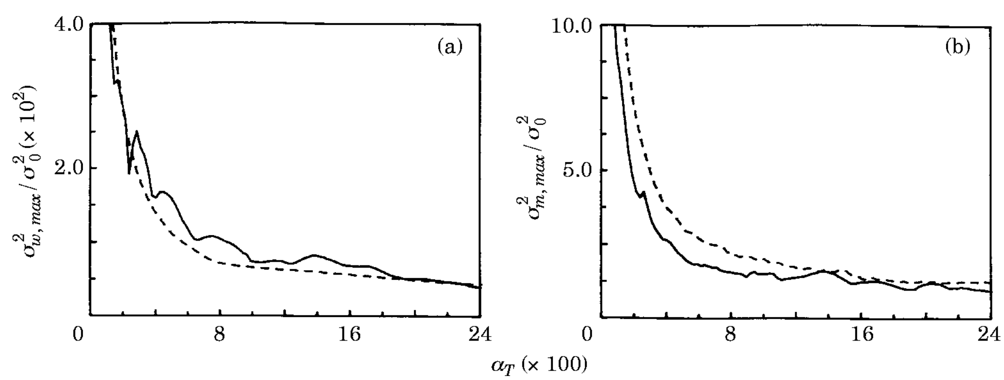

Figure 6. The span number effect on (a) the $\sigma^2_{w,max} - \alpha_T$ distribution and (b) the $\sigma^2_{m,max} - \alpha_T$ distribution of a multi-span Timoshenko beam due to a white noise process. Key: ——, 5 spans; - - - -, 1 span.

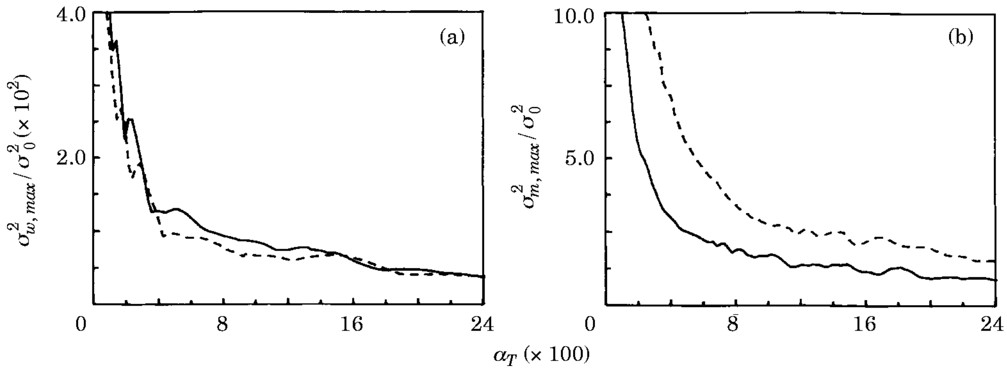

Figure 7. Comparisons of the effects of two beam theories on (a) the $\sigma^2_{w,max} - \alpha_T$ distribution and (b) the $\sigma^2_{m,max} - \alpha_T$ distribution of a three-span beam due to a white noise process. Key: ——, Timoshenko beam; - - - - , Euler beam.

$\sigma^2_{m,max}$ will be infinite as α_T approaches zero. Moreover, both $\sigma^2_{w,max}$ and $\sigma^2_{m,max}$ rapidly decrease as α_T increases. The first modal frequency of a multi-span beam is independent of the span number. The higher the span number of the beam, the smaller the values of modal frequencies $\omega_i (i > 1)$. This result indicates that the higher the number of the span, the more flexible the beam is. Therefore, Figure 6(a) indicates that the higher span number leads to a larger $\sigma^2_{w,max}$. However, a larger span number causes a smaller $\sigma^2_{m,max}$, as indicated in Figure 6(b).

The effects of two beam theories on the $\sigma^2_{w,max} - \alpha_T$ and the $\sigma^2_{m,max} - \alpha_T$ distributions of a three-span beam are shown in Figures 7(a) and 7(b) respectively. The effect of shear deformation causes $\sigma^2_{w,max}$ of the Timoshenko beam to be larger than that of the Bernoulli–Euler beam. However, due to the effect of rotatory inertia, $\sigma^2_{m,max}$ of the Timoshenko beam is smaller than that of the Bernoulli–Euler beam. In Figure 8(a) it is shown that $(X_\sigma)_w$ of the Timoshenko beam is always near the mid-point of either the first span or the third span. However, in Figure 8(b) it is shown that $(X_\sigma)_m$ is near the left side of one hinge support.

4.3. EXPONENTIAL PROCESS

The effects of three $\omega_g (= 0, 0{\cdot}1v, 0{\cdot}3v)$ values of an exponential process on the $\sigma^2_{w,max} - \alpha_T$ and the $\sigma^2_{m,max} - \alpha_T$ distributions of a three-span Timoshenko beam are displayed in

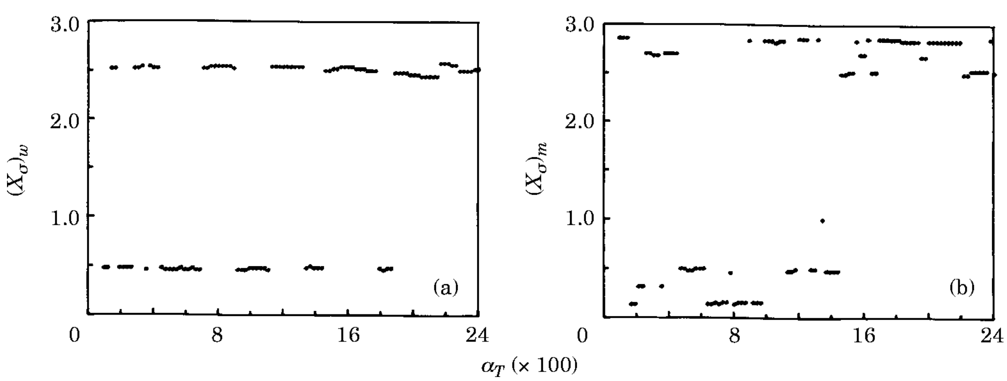

Figure 8. (a) The $(X_\sigma)_w - \alpha_T$ distribution and (b) the $(X_\sigma)_m - \alpha_T$ distribution of a three-span Timoshenko beam due to a white noise process.

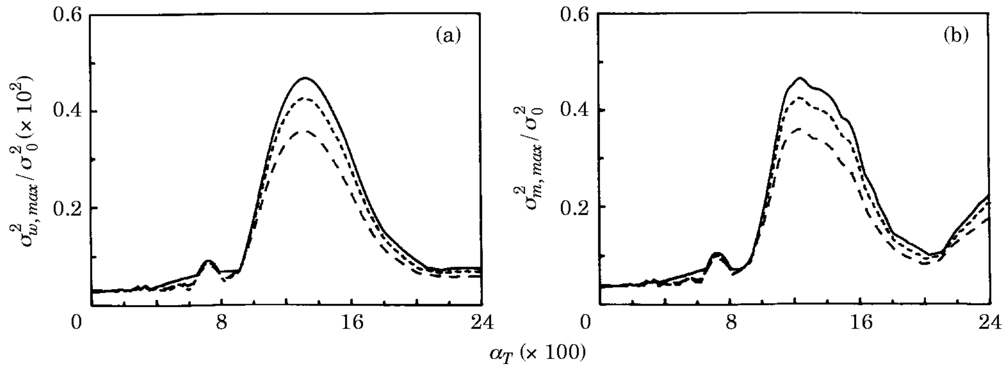

Figure 9. Comparisons of the effects of three ω_g values of an exponential process on (a) the $\sigma^2_{w,max} - \alpha_T$ distribution and (b) the $\sigma^2_{m,max} - \alpha_T$ distribution of a three-span Timoshenko beam. Key for ω_g values: ——, 0·0; - - - - -, 0·1v; – – –,0·3v.

Figures 9(a) and 9(b) respectively. Both figures show that the parameter ω_g of the process has an obvious effect on reducing both absolute $\sigma^2_{w,max}$ and $\sigma^2_{m,max}$ of the beam, especially, as the velocity ratio is near α_σ.

4.4. EXPONENTIAL COSINE PROCESS

The effects of the two kinds of exponential cosine processes ($\omega_g = 0$, $\omega_0 = 0{\cdot}5\omega_1$; $\omega_g = 0{\cdot}3v$, $\omega_0 = 0{\cdot}5\omega_1$) on the $\sigma^2_{w,max} - \alpha_T$ and the $\sigma^2_{m,max} - \alpha_T$ distributions of a three-span Timoshenko beam are displayed in Figures 10(a) and 10(b) respectively. It can be seen that the parameter ω_g has an apparent effect on reducing both absolute $\sigma^2_{w,max}$ and $\sigma^2_{m,max}$. The α_σ presented in Figure 10(a) is smaller than that in Figure 9(a). This finding suggests that α_σ is determined only by ω_0.

4.5. COSINE PROCESS

The effects of two $\omega_0(=\omega_1, 0{\cdot}5\omega_1)$ values of the cosine process on the $\sigma^2_{w,max} - \alpha_T$ and the $\sigma^2_{m,max} - \alpha_T$ distributions of a three-span Timoshenko beam are displayed in Figures 11(a) and 11(b) respectively. The beam is subjected to a quasi-steady state loading as the velocity of load approaches zero. Both $\sigma^2_{w,max}$ and $\sigma^2_{m,max}$ will, consequently, be infinite due to the beam being in resonance at $\omega_0 = \omega_1$ and $\alpha_T = 0$. However, both $\sigma^2_{w,max}$ and $\sigma^2_{m,max}$ will be finite for any α_T value except for the case of $\omega_0 = \omega_1$. A rapidly moving load implies

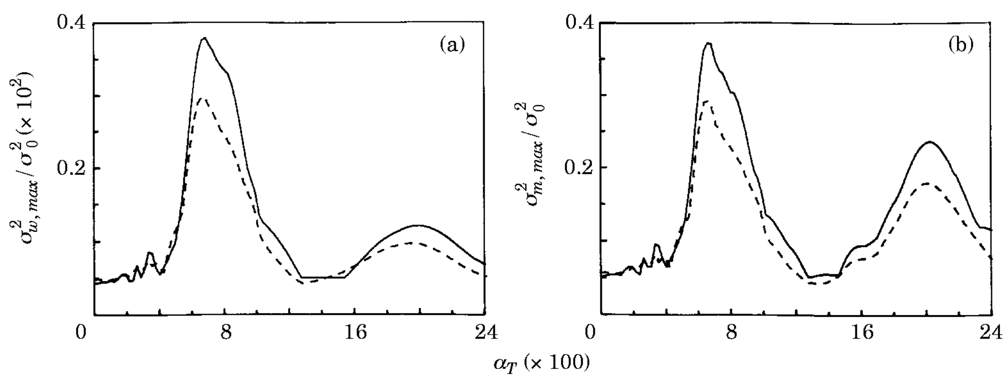

Figure 10. Comparisons of the effects two ω_g values of an exponential cosine process ($\omega_0 = 0{\cdot}5\omega_1$) on (a) the $\sigma^2_{w,max} - \alpha_T$ distribution and (b) the $\sigma^2_{m,max} - \alpha_T$ distribution of a three-span Timoshenko beam. Key: ——, $\omega_g = 0{\cdot}0$, $\omega_0 = 0{\cdot}5\omega_1$; - - - , $\omega_g = 0{\cdot}3v$, $\omega_0 = 0{\cdot}5\omega_1$.

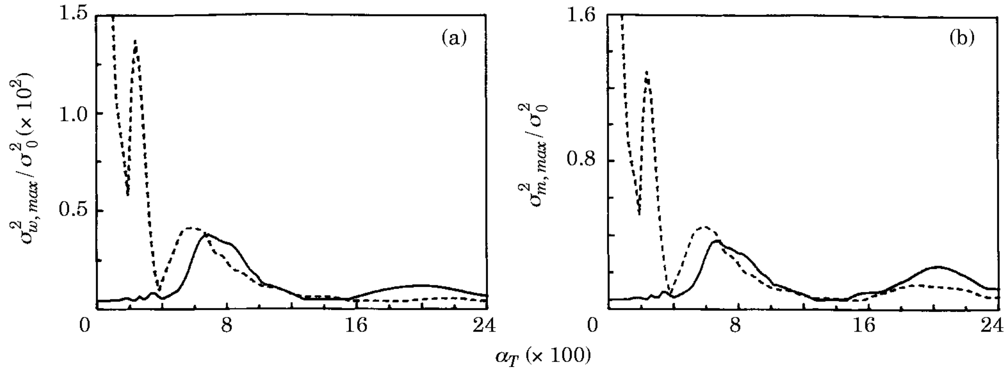

Figure 11. Comparison of the effects of two ω_0 values of a cosine process on (a) the $\sigma^2_{w,max} - \alpha_T$ distribution and (b) the $\sigma^2_{m,max} - \alpha_T$ distribution of a three-span Timoshenko beam. Key: $\omega_0 = 0{\cdot}5\omega_1$; - - - -, $\omega_0 = 1{\cdot}0\omega_1$.

a short duration of the forced vibration of the beam. The value of cosine function does not abruptly change as the loading time is short. Therefore, $\sigma^2_{w,max}$ approaches a constant value for the load moving at a supercritical speed. Moreover, a greater ω_0 ($\leqslant\omega_1$) value of the process requires a longer duration of a load moving on the beam to cause the extreme values of both $\sigma^2_{w,max}$ and $\sigma^2_{m,max}$. The above phenomena indicate that the greater ω_0 ($\leqslant\omega_1$) implies smaller α_σ.

The effects of the span number on the $\sigma^2_{w,max} - \alpha_T$ and the $\sigma^2_{m,max} - \alpha_T$ distributions of a multi-span Timoshenko beam due to a moving load with a variance of cosine process ($\omega_0 = 0{\cdot}5\omega_1$) are displayed in Figures 12(a) and 12(b) respectively. The frequency ω_0 ($=0{\cdot}5\omega_1$) is smaller than all modal frequencies of the beam. Therefore, this cosine process can be regarded as a constant variance process. Consequently, both tendencies of the $\sigma^2_{w,max} - \alpha_T$ (or $\sigma^2_{m,max} - \alpha_T$) and the $\langle w\rangle_{max} - \alpha_T$ (or $\langle m\rangle_{max} - \alpha_T$) distributions are very similar.

The comparisons of two beam theories on the $\sigma^2_{w,max} - \alpha_T$ and the $\sigma^2_{m,max} - \alpha_T$ distributions of a three-span beam due to the moving load with a variance of cosine process ($\omega_0 = 0{\cdot}5\omega_1$) are shown in Figures 13(a) and 13(b) respectively. The first modal frequency of the Timoshenko beam is smaller than that of a Bernoulli–Euler beam. The frequency ω_0 is closer to the first modal frequency of the Timoshenko beam than that of the Bernoulli–Euler beam. Accordingly, both $\sigma^2_{w,max}$ and $\sigma^2_{m,max}$ of the Timoshenko beam due to the load moving at a low speed are greater than those of the Bernoulli–Euler beam.

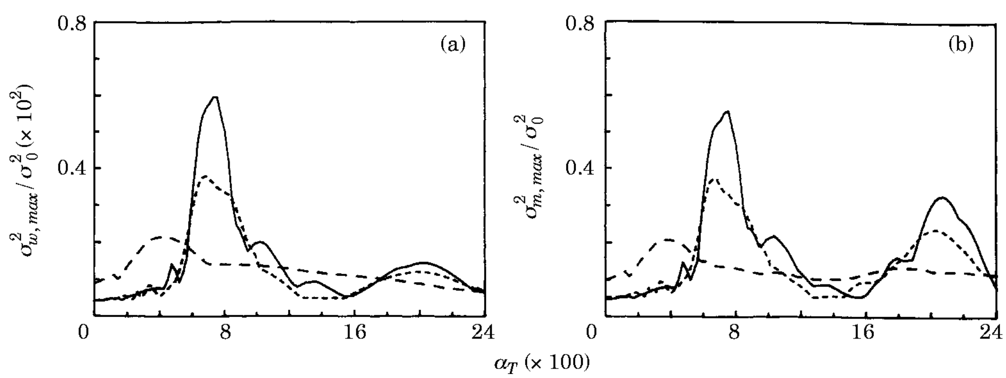

Figure 12. The span number effect on (a) the $\sigma^2_{w,max} - \alpha_T$ distribution and (b) the $\sigma^2_{m,max} - \alpha_T$ distribution of a multi-span Timoshenko beam due to a cosine process ($\omega_0 = 0{\cdot}5\omega_1$). Key as for Figure 4.

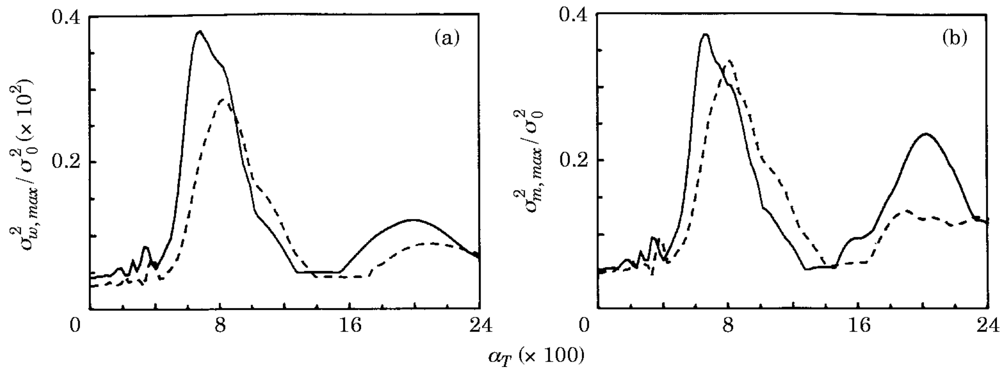

Figure 13. Comparisons of the effects of two beam theories on (a) the $\sigma^2_{w,max} - \alpha_T$ distribution and (b) the $\sigma^2_{m,max} - \alpha_T$ distribution of a three-span beam due to a cosine process ($\omega_0 = 0{\cdot}5\omega_1$). Key as for Figure 7.

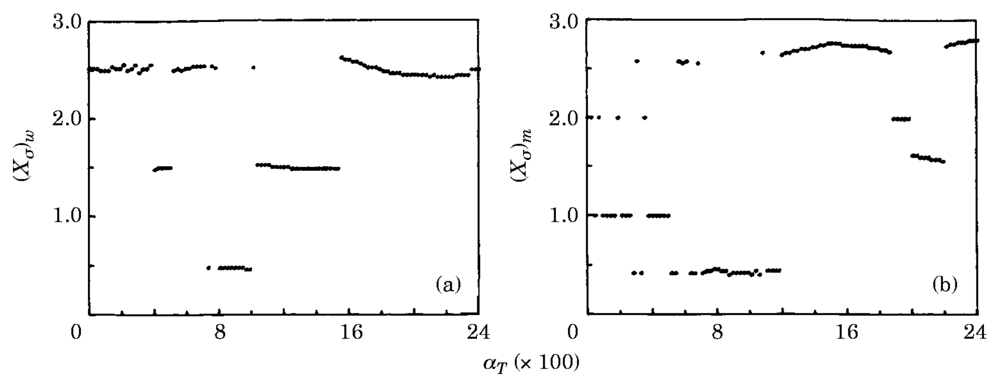

Figure 14. (a) The $(X_\sigma)_w - \alpha_T$ distribution and (b) the $(X_\sigma)_m - \alpha_T$ distribution of a three-span Timoshenko beam due to a cosine process ($\omega_0 = 0{\cdot}5\omega_1$).

However, the Timoshenko beam has a low α_σ. The periodicity of the variance of the load causes both $(X_\sigma)_w - \alpha_T$ and $(X_\sigma)_m - \alpha_T$ distributions of the beam to be different from those of a white noise process and of the mean values. The $(X_\sigma)_W - \alpha_T$ distribution of the three-span Timoshenko beam displayed in Figure 14(a) indicates that $\sigma^2_{w,max}$ always appears in close proximity to the mid-point of one span. However, $\sigma^2_{m,max}$ may occur at the mid-point of each span or on the left side of either the first support or the second support, as indicated in Figure 14(b).

5. CONCLUSIONS

The maximum mean value of transverse deflection of a multi-span Timoshenko beam due to a random load moving at a constant velocity always occurs in close proximity to the mid-point of either the first span or the last span. Both the maximum variance of transverse deflection and that of moment of the beam due to the load with the variance of white noise process decrease for an increasing velocity. The maximum variance of transverse deflection due to the white noise process always appears near the mid-point of the first span or the last span of the beam. A rapidly moving load with the variance of a cosine function will not induce significant variances of deflection and moment of the beam.

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