Journal of Sound and Vibration (1998) **213**(1), 189–192

LETTERS TO THE EDITOR

CHAOS IN ROCKING OSCILLATIONS OF A RIGID BODY

R. I. K. Moorthy
Reactor Engineering Division, Bhabha Atomic Research Centre, Bombay-400 085, *India*

AND

H. R. Srirangarajan
Department of Mechanical Engineering, Indian Institute of Technology, Powai, Bombay-400076, *India*

(*Received* 14 *October* 1996, *and in final form* 28 *October* 1997)

1. INTRODUCTION

Chaos is described as the apparent random or stochastic response of non-linear systems to purely deterministic excitations and/or boundary conditions. Chaotic solutions are bounded, aperiodic, random looking which have a wide band spectrum. These are associated with strange attractors. These solutions are extremely sensitive to initial conditions. Trajectories in the phase space which differ slightly in initial conditions drastically differ in their behaviour after some time. This makes the prediction of the long time behaviour of systems impossible.

This study considers chaotic behaviour of the planar rocking motion of a rigid body on a rigid foundation offering viscous damping under periodic base excitation. This is of considerable practical significance in understanding the behaviour of equipment subjected to seismic forces or transported in vehicles including aircraft.

2. ANALYSIS

Consider a rigid body, placed on a platform OO' which is under a horizontal acceleration $\ddot{x}/g = U_m \sin \lambda t$, Figure 1. The equation of motion for the planar rocking of the rigid body is given by,

$$I\ddot{\theta} + C\dot{\theta} + WR \operatorname{sgn}(\alpha \operatorname{sgn} \theta - \theta) = -WR(\ddot{x}/g) \cos(\alpha \operatorname{sgn} \theta - \theta) \tag{1}$$

where I is the moment of inertia about O or O', and W is the weight of the block. Energy dissipation due to impact is assumed to be approximated by the $C\dot{\theta}$ term. The natural frequency is given by $w = \sqrt{WR/I}$. Non-dimensionalising the equation in terms of

$$\tau = \frac{\omega t}{2\pi} \text{ leads to}$$

$$\ddot{\theta} + 4\eta\pi\dot{\theta} + 4\pi^2 \sin(\alpha \operatorname{sgn} \theta - \theta) + U_m \cos(\alpha \operatorname{sgn} \theta - \theta) \sin \bar{\lambda} t = 0 \tag{2}$$

where $\bar{\lambda} = \lambda/\omega$ and the dots denote derivatives with respect to τ. Equation (2) has been studied by Spanos and Koh [1] under different damping conditions. R. N. Iyengar *et al.* [2] have studied the possibility of chaotic behaviour of the system. For very small values of U_m and θ, the equation of motion is linearised and the resulting equation shows a periodic response. In the phase plane this results in a closed curve. As U_m increases, θ grows and linearization is not valid. Hence equation (2) is numerically integrated. Figure 2 shows the response time history for $U_m = 0{\cdot}01$, $\alpha = 0{\cdot}55$, $\bar{\lambda} = 1{\cdot}0$. It is observed that response

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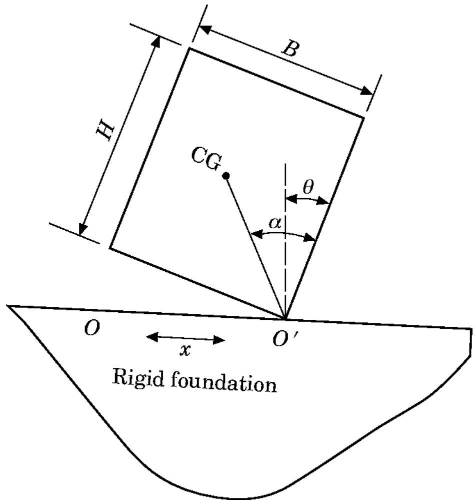

Figure 1. Rocking rigid body.

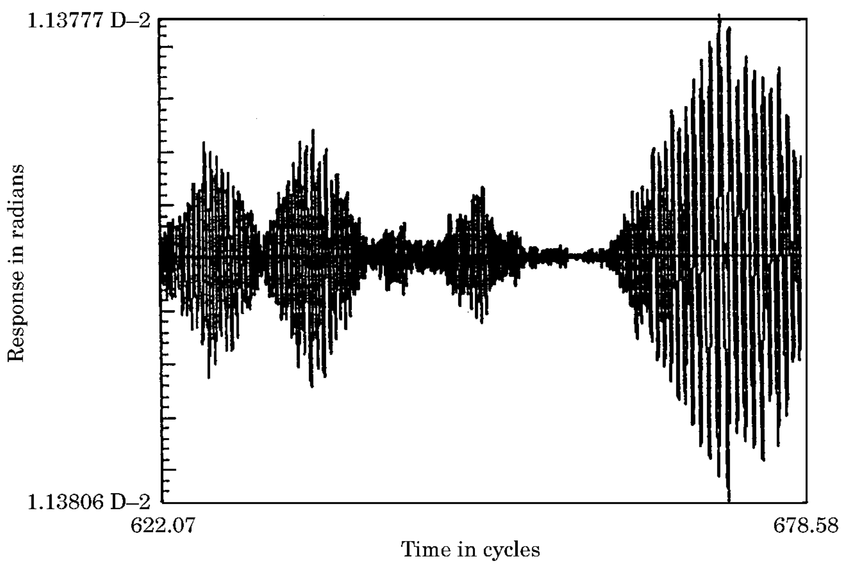

Figure 2. Response time history $\eta = 0{\cdot}05$, $\bar{\lambda} = 1{\cdot}0$, $u_m = 0{\cdot}01$.

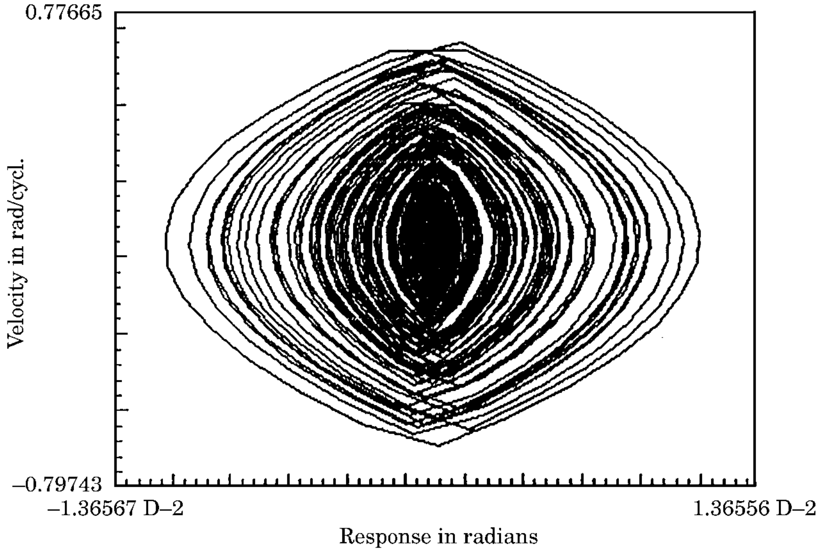

Figure 3. Phase plane plot $\eta = 0{\cdot}05$, $\bar{\lambda} = 1{\cdot}0$, $u_m = 0{\cdot}01$.

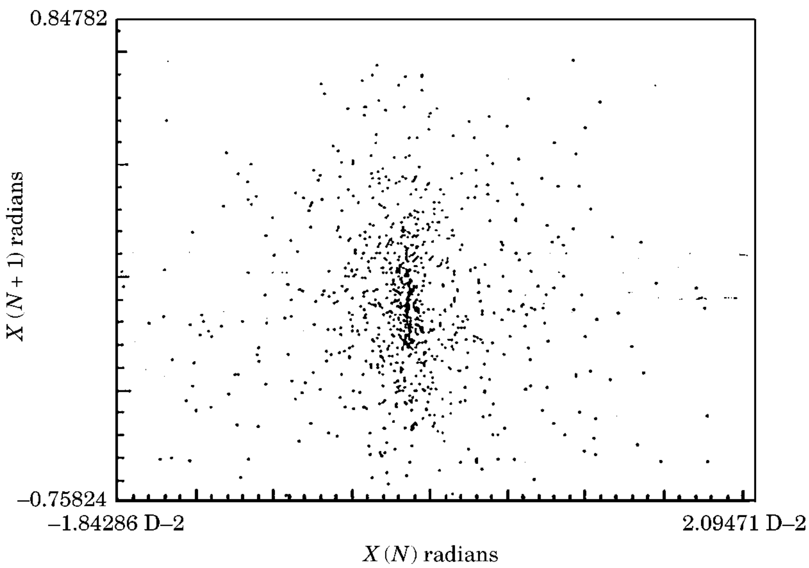

Figure 4. Poincaré map $\eta = 0{\cdot}05$, $\bar{\lambda} = 1{\cdot}0$, $u_m = 0{\cdot}01$.

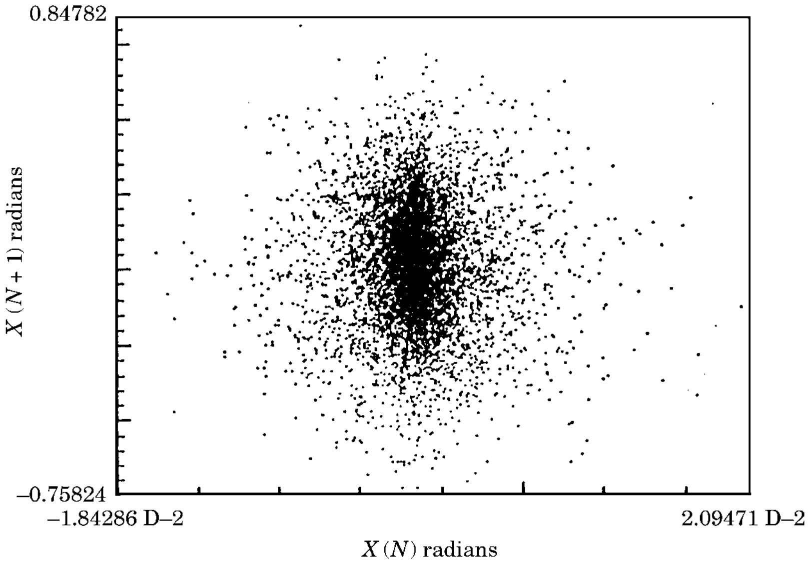

Figure 5. Poincaré map $\eta = 0{\cdot}05$, $\bar{\lambda} = 1{\cdot}0$, $u_m = 0{\cdot}01$.

occurs in bursts combining many frequencies and looks erratic and random. Figure 3 shows the trajectory in the phase plane, which fills the region. Figure 4 shows the Poincaré map, which neither contains a few discrete points nor a closed curve as the attractor. The map is quite patchy and thus represents chaotic behaviour. Figure 4 is obtained for 1000 points, whereas Figure 5 is obtained using 10,000 points which shows the structure of a chaotic attractor clearly.

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2. R. N. Iyengar, K. R. Reddy and C. S. Manohar 1989 *Journal of the Aeronautical Society of India* **41**, 1–6. Chaotic response of some simple nonlinear oscillators.