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DETERMINATION OF THE STEADY STATE RESPONSE OF VISCOELASTICALLY POINT-SUPPORTED RECTANGULAR ANISOTROPIC (ORTHOTROPIC) PLATES

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In the present study, the steady state response to a sinusoidally varying force applied at the centre of a point-supported anisotropic (orthotropic) elastic plate of rectangular shape is analysed. In doing this, the displacement function of the plate is approximated by using the eigenfunctions of a completely free beam. The difference between the free-end boundary conditions of the plate and the beam is compensated for by considering a differential operator in addition to the governing equation of the plate. Using Galerkin's method, the problem is reduced to the solution of a system of algebraic equations. The influence of the mechanical properties on the mode shapes and the steady state response of the viscoelastically point-supported rectangular plates is investigated numerically for a concentrated load at the centre for various values of the mechanical properties characterizing the anisotropy of the plate material. Also, the effect of the location of the point supports is studied. The problems considered are solved within the framework of the Kirchhoff–Love hypothesis.

1. INTRODUCTION

In recent years, considerable interest has been shown in the determination of the vibration characteristics of point-supported plates. Much of the interest is likely to have stemmed from the potential application of the analyses to industrial applications. The most widely studied point-supported plate problem is that of otherwise fully free plates. Such plates have been considered with four corner point supports [1–3], with supports at the mid-points of all four edges [4, 5], with multiple point supports along the edges [6, 7], with supports symmetrically located at four points on the diagonals [8–14], with supports more generally symmetrically located [15], with arbitrary numbers and locations of point supports [16–18], and with two-dimensionally periodic point supports [19]. A point-supported skew or rectangular orthotropic plate was studied by Srinivasan and Munaswamy [20], and the free vibration of thin rectangular orthotropic plates resting on point supports symmetrically located about the plate central axis was examined by Gorman [21]. An elastically point-supported plate was studied by Leuner [22] and Laura and Gutierrez [23], free vibration of a viscoelastically point-supported plate was examined by Das and Navaratna [24], and forced vibration of a viscoelastically point-supported plate was studied by Yamada *et al*. [25]. In many branches of modern industry, the structural elements, such as plates, are fabricated from composite materials. For this reason, the present investigation may be considered to be a problem of the mechanics of elements fabricated from composite materials. In the study reported here, the steady state response

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of a viscoelastically point-supported anisotropic (orthotropic) plate to a sinusoidally varying force is investigated for various values of the mechanical properties characterizing the anisotropy of the plate material. The method of investigation used in the paper [25] was developed for rectangular anisotropic (orthotropic) plates resting on a tensionless elastic foundation by Kocatürk [26]. By making the necessary developments, the method developed for orthotropic plates by Kocatürk [26] is used in the present study. The problems considered are solved within the framework of the Kirchhoff–Love hypothesis.

2. ANALYSIS

Consider a point-supported rectangular anisotropic plate under a concentrated force $F(t)$ at the centre of the plate, as shown in Figure 1. The axis of the elastic symmetry of the plate material coincides with the Ox and Oy axes. The governing equation for the displacement function $W(X, Y)$ is expressed as:

$$L(W) = D_x \frac{\partial^4 W}{\partial X^4} + 2(D_1 + 2D_{xy}) \frac{\partial^4 W}{\partial X^2 \partial Y^2} + D_y \frac{\partial^4 W}{\partial Y^4}$$

$$+ \sum_{i=1}^{I} \left(k_i W + c_i \frac{\partial W}{\partial t} \right) \delta(X - X_i)\delta(Y - Y_i) + \rho h \frac{\partial^2 W}{\partial t^2} - F(t)\delta(X)\delta(Y) = 0. \quad (1)$$

In equation (1), δ is the Dirac delta function, h is the plate thickness, k_i is the spring constant, c_i is the damping coefficient of a point support $P_i(X_i, Y_i)$. D_x, D_y, D_{xy}, and D_1 are expressed as follows:

$$D_x = \frac{E'_x h^3}{12} \qquad D_y = \frac{E'_y h^3}{12} \qquad D_{xy} = \frac{G_{xy} h^3}{12} \qquad D_1 = \frac{E'' h^3}{12}, \quad (2)$$

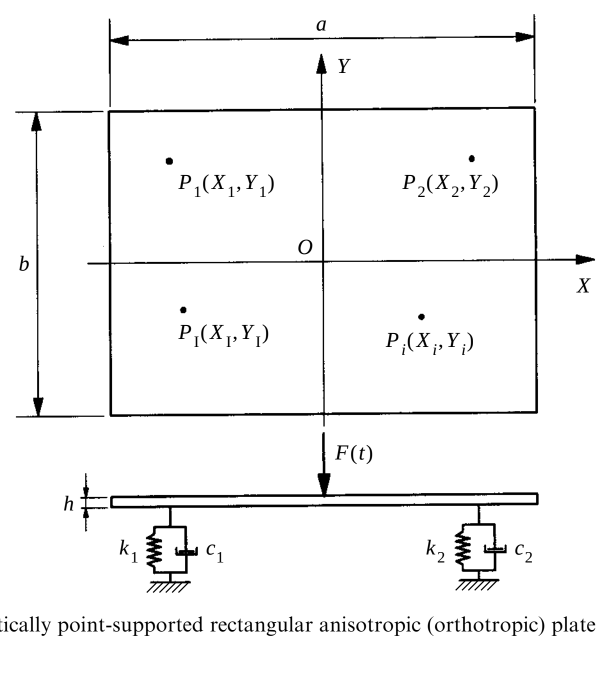

Figure 1. Viscoelastically point-supported rectangular anisotropic (orthotropic) plate subjected to an external force.

where G_{xy} is shear modulus and E'_x, E'_y, E'' are derived as follows:

$$E'_x = \frac{E_1}{1-\nu_{12}^2 e} \qquad E'_y = \frac{E_1}{(1-\nu_{12}^2 e)e} \qquad E'' = \frac{\nu_{12}E_1}{1-\nu_{12}^2 e} \qquad e = \frac{E_1}{E_2}. \tag{3}$$

Here E_1, E_2 are Young's moduli in the Ox and Oy directions, respectively, and υ_{12} is Poisson's ratio. Since the analytical solution of equation (1) is impossible, the generalized Galerkin method is employed by assuming the steady state response of the plate to a sinusoidally varying force $F(t) = Q\,\mathrm{e}^{\mathrm{j}\omega t}$ as

$$W(X, Y) = aw(x, y)\,\mathrm{e}^{\mathrm{j}\omega t} = a\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}Z_m(x)Z_n(y)\,\mathrm{e}^{\mathrm{j}\omega t} \qquad \mathrm{j} = \sqrt{-1}, \tag{4}$$

where $x = X/a$ and $y = Y/b$. In the present study, $w(x, y)$ is a complex variable containing a phase angle. The normalized eigenfunctions $Z_m(x)$ of a free beam are given by

$$Z_1(x) = 1 \qquad Z_2(x) = 2\sqrt{3}x$$

$$Z_m(x) = \sqrt{2/[\cosh^2(\beta_m/2) + \cos^2(\beta_m/2)]}[\cosh(\beta_m/2)\cos(\beta_m x) + \cos(\beta_m/2)\cosh(\beta_m x)], \qquad m = 3, 5, 7, \ldots.$$

$$Z_m(x) = \sqrt{2/[\sinh^2(\beta_m/2) - \sin^2(\beta_m/2)]}[\sinh(\beta_m/2)\sin(\beta_m x) + \sin(\beta_m/2)\sinh(\beta_m x)], \qquad m = 4, 6, 8, \ldots,$$

where the parameters β_m are the positive roots of the following equation:

$$\tan(\beta/2) \pm \tanh(\beta/2) = 0 \qquad m = \begin{Bmatrix} 3, 5, 7, \ldots\ldots \\ 4, 6, 8, \ldots\ldots \end{Bmatrix}. \tag{6}$$

The eigenfunctions $Z_n(y)$ are given by expressions of the same forms as equations (5) and (6). The difference between the free-end boundary conditions of the plate and the beam is compensated for by adding

$$\begin{aligned} L_r(W) = {} & D_1\frac{\partial}{\partial X}\left\{\frac{\partial^2 W}{\partial Y^2}[\delta(X + a/2) - \delta(X - a/2)]\right\} \\ & + D_1\frac{\partial}{\partial Y}\left\{\frac{\partial^2 W}{\partial X^2}[\delta(Y + b/2) - \delta(Y - b/2)]\right\} \\ & + (D_1 + 4D_{xy})\frac{\partial^3 W}{\partial X\partial Y^2}[\delta(X + a/2) - \delta(X - a/2)] + (D_1 + 4D_{xy})\frac{\partial^3 W}{\partial X^2\partial Y} \\ & \times [\delta(Y + b/2) - \delta(Y - b/2)] \\ & + 4D_{xy}\frac{\partial^2 W}{\partial X\partial Y}[\delta(X + a/2) - \delta(X - a/2)][\delta(Y + b/2) - \delta(Y - b/2)] \end{aligned} \tag{7}$$

to the equation of the plate as it is employed in reference [26]. This operator is obtained by considering non-vanishing boundary conditions and satisfying them by applying external forces and moments that are taken into account as external loads.

For simplicity of the analysis, the following dimensionless quantities are introduced:

$$D_2 = 2(D_4 + 2D_5) \qquad D_3 = \frac{D_y}{D_x} = \frac{E_2}{E_1} \qquad D_4 = \frac{D_1}{D_x} = v_{12} \qquad D_5 = \frac{D_{xy}}{D_x} = \frac{G_{xy}}{E_1}(1 - v_{12}^2 e)$$

$$D_6 = D_4 + 4D_5 \qquad \alpha = \frac{a}{b} \qquad \kappa = \frac{k_i a^3}{bD_x} \qquad \gamma_i = \frac{c_i a}{b\sqrt{\rho h D_x}} \qquad \lambda^2 = \frac{\rho h \omega^2 a^4}{Dx} \qquad q = \frac{Q_a}{D_x}. \tag{8}$$

Thus, taking into account the above-mentioned quantities, the equilibrium equation of the plate may be written in terms of the dimensionless quantities as

$$\begin{aligned} L(w) + L_r(w) = {} & \frac{\partial^4 w}{\partial x^4} + \alpha^2 D_2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \alpha^4 D_3 \frac{\partial^4 w}{\partial y^4} + \sum_{i=1}^{I} (\kappa_i + \mathrm{j}\gamma_i \lambda) w \delta(x - x_i)\delta(y - y_i) \\ & - \lambda^2 w - \alpha q \delta(x)\delta(y) + \alpha^2 D_4 \frac{\partial}{\partial x}\left\{\frac{\partial^2 w}{\partial y^2}[\delta(x + 1/2) - \delta(x - 1/2)]\right\} \\ & + \alpha^2 D_4 \frac{\partial}{\partial y}\left\{\frac{\partial^2 w}{\partial x^2}[\delta(y + 1/2) - \delta(y - 1/2)]\right\} + \alpha^2 D_6 \frac{\partial^3 w}{\partial x \partial y^2} \\ & \times [\delta(x + 1/2) - \delta(x - 1/2)] + \alpha^2 D_6 \frac{\partial^3 w}{\partial x^2 \partial y}[\delta(y + 1/2)] \\ & - \delta(y - 1/2)] + 4\alpha^2 D_5 \frac{\partial^2 w}{\partial x \partial y}[\delta(x + 1/2) - \delta(x - 1/2)] \\ & \times [\delta(y + 1/2) - \delta(y - 1/2)] = 0. \end{aligned} \tag{9}$$

Application of the Galerkin orthogonalization process to the last equation with respect to the eigenfunctions yields the following system of algebraic equations for the unknown coefficients A_{mn}

$$\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} C_{mnkl} A_{mn} = D_{kl}, \tag{10}$$

where

$$\begin{aligned} C_{mnkl} = {} & (\beta_m^4 + \alpha^4 \beta_n^4 D_3)\delta_{mk}\delta_{nl} + \alpha^2 D_2 a_{mk} a_{nl} + \sum_{i=1}^{I} (\kappa_i + \mathrm{j}\gamma_i\lambda) Z_m(x_i) Z_k(x_i) Z_n(y_i) Z_l(y_i) \\ & - \lambda^2 \delta_{mk}\delta_{nl} + \alpha^2 D_4 \{a_{nl}[Z_m(\tfrac{1}{2})Z_k'(\tfrac{1}{2}) - Z_m(-\tfrac{1}{2})Z_k'(-\tfrac{1}{2})] + a_{mk}[Z_n(\tfrac{1}{2})Z_l'(\tfrac{1}{2}) \\ & - Z_n(-\tfrac{1}{2})Z_l'(-\tfrac{1}{2})]\} + \alpha^2 D_6 \{a_{nl}[Z_m'(-\tfrac{1}{2})Z_k(-\tfrac{1}{2}) - Z_m'(\tfrac{1}{2})Z_k(\tfrac{1}{2})] \\ & + a_{mk}[Z_n'(-\tfrac{1}{2})Z_l(-\tfrac{1}{2}) - Z_n'(\tfrac{1}{2})Z_l(\tfrac{1}{2})]\} \\ & + 4\alpha^2 D_5 [Z_n'(-\tfrac{1}{2})Z_l(-\tfrac{1}{2})Z_m'(-\tfrac{1}{2})Z_k(-\tfrac{1}{2}) - Z_n'(-\tfrac{1}{2})Z_l(-\tfrac{1}{2})Z_m'(\tfrac{1}{2})Z_k(\tfrac{1}{2})] \\ & + 4\alpha^2 D_5 [-Z_n'(\tfrac{1}{2})Z_l(\tfrac{1}{2})Z_m'(-\tfrac{1}{2})Z_k(-\tfrac{1}{2}) + Z_n'(\tfrac{1}{2})Z_l(\tfrac{1}{2})Z_m'(\tfrac{1}{2})Z_k(\tfrac{1}{2})], \end{aligned}$$

$$D_{kl} = +\alpha q Z_k(0) Z_l(0) \qquad a_{mk} = \int_{-l/2}^{l/2} Z_m''(x) Z_k(x)\,\mathrm{d}x \qquad \mathrm{j} = \sqrt{-1} \tag{11}$$

Equation (10) is a set of linear equations in the unknown coefficients A_{mn}. The transverse deflection of the plate is determined from equation (4) by calculating the unknown coefficients. The magnitude of the reaction force $F_T\,\mathrm{e}^{\mathrm{j}\omega t}$ at the point supports is given by

$$F_T = \sum_{i=1}^{I} (k_i + \mathrm{j}c_i\omega)aw(x_i, y_i), \tag{12}$$

and therefore the force transmissibility at the supports is determined by

$$T_R = \frac{1}{q}\sum_{m=1}^{M}\sum_{n=1}^{N} A_{mn}\sum_{i=1}^{I} (\kappa_i + \mathrm{j}\gamma_i\lambda)Z_m(x_i)Z_n(y_i). \tag{13}$$

3. NUMERICAL RESULTS AND DISCUSSION

In this section, the steady state response to a point force q acting at the centre of the plate is calculated numerically for an orthotropic square plate viscoelastically supported at four points symmetrically located at the corners or on the two diagonals, where the parameters κ_i and λ_i are taken to have the same respective values at all the supports denoted by $\kappa_i = \kappa_s$ and $\gamma_i = \gamma_s$. Because of the structural symmetry and symmetry of the external force, only symmetrical vibrations arise in the plate. In this case, the natural frequencies and the dynamic responses can be calculated by taking only the odd terms in the series solution. The symbol SS represents symmetrical vibration with respect to centrelines.

For a better understanding of the responses presented, a short investigation of the free vibration of an elastically point-supported plate is necessary. The natural frequencies (the frequency parameters) of the plate are determined by calculating the eigenvalues λ of the frequency equation obtained by taking the damping parameter of the supports as $\gamma_s = 0$ and the force $q = 0$ in equation (10), and the mode shapes of vibration can be determined from equation (4) by calculating the eigenvectors corresponding to the eigenvalues. In all of the numerical calculations, the value of the G_{xy}/E_1 raito is taken as 0·3846. In Figure 2(a), the values on the ordinates at $\kappa_s = 0$ and ∞, respectively, represent the

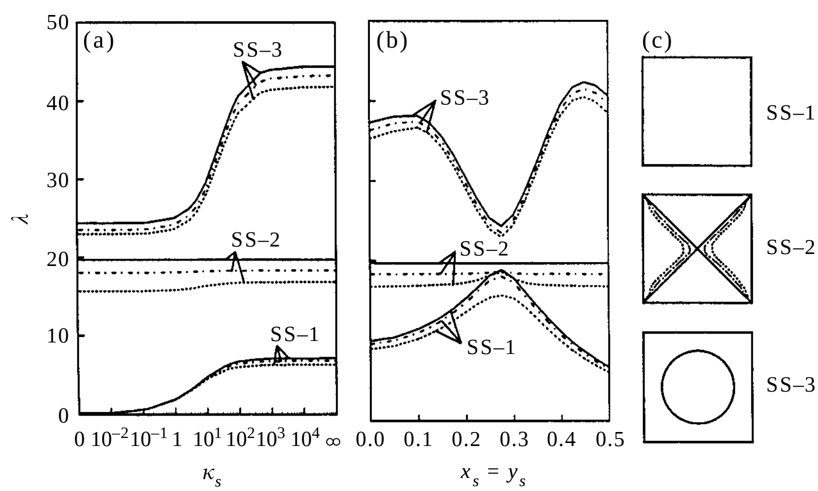

Figure 2. (a) Frequency parameters of square orthotropic plates elastically supported at the corners, $v = 0{\cdot}3$. (b) Frequency parameters of square orthotropic plates elastically supported at four points symmetrically located on the diagonals, $v = 0{\cdot}3$, $\kappa_s = 100$. (c) The nodal patterns for the SS-1, SS-2 and SS-3 vibrations. —, $E_2/E_1 = 1{\cdot}0$; –·–·, $E_2/E_1 = 0{\cdot}8$; - - - -, $E_2/E_1 = 0{\cdot}6$.

frequency parameters of an unconstrained free plate and a simply point-supported plate. With an increase in the parameters κ_s, the frequency parameters monotonically increase and ultimately become the values of a simply point-supported plate. Figure 2(b) shows the frequency parameters λ versus the location of the point supports for a square plate with undamped supports symmetrically located on the diagonals, where (x_s, y_s) denote the absolute values of the co-ordinates of all the supports. In the isotropic case, nodal lines arising in the SS-2 vibration mode coincide with the diagonals passing through the supports for $E_2/E_1 = 1$ as seen in the mode shapes (Figure 2(c)), and therefore the frequency parameter remains constant without being affected by the variation of κ_s and $x_s = y_s$ [25], as can be seen from Figures 2(a) and (b). However, in the orthotropic case, nodal lines arising in the SS-2 vibration mode do not coincide with the diagonals (Figure 2(c), SS-2 mode) and their shapes change with the variation in κ_s and $x_s = y_s$. In the SS-3 mode, the circle changes it's shape when the plate is orthotropic but this difference is not shown in Figure 2(c) for it is very small for the parameters considered. By decreasing the values of E_2/E_1, the frequency parameter λ decreases relative to the isotropic case for all of the modes, as can be seen in Figures 2(a) and (b).

Figure 3 shows the force transmissibilities for various E_2/E_1 values and support conditions for a viscoelastically point-supported plate. It is seen from Figures 3(a) and (b) that, especially for small values of $x_s = y_s$, and E_2/E_1 values different from unity, the SS-2 mode becomes more important than the other modes. Figure 3(a) shows that, for $x_s = y_s = 0{\cdot}1$, $E_2/E_1 = 0{\cdot}8$ and $E_2/E_1 = 0{\cdot}6$, the force transmisibilities in the SS-2 mode become greater than in the other modes. In Figure 3(b), it is seen that, when $x_s = y_s = 0{\cdot}2$,

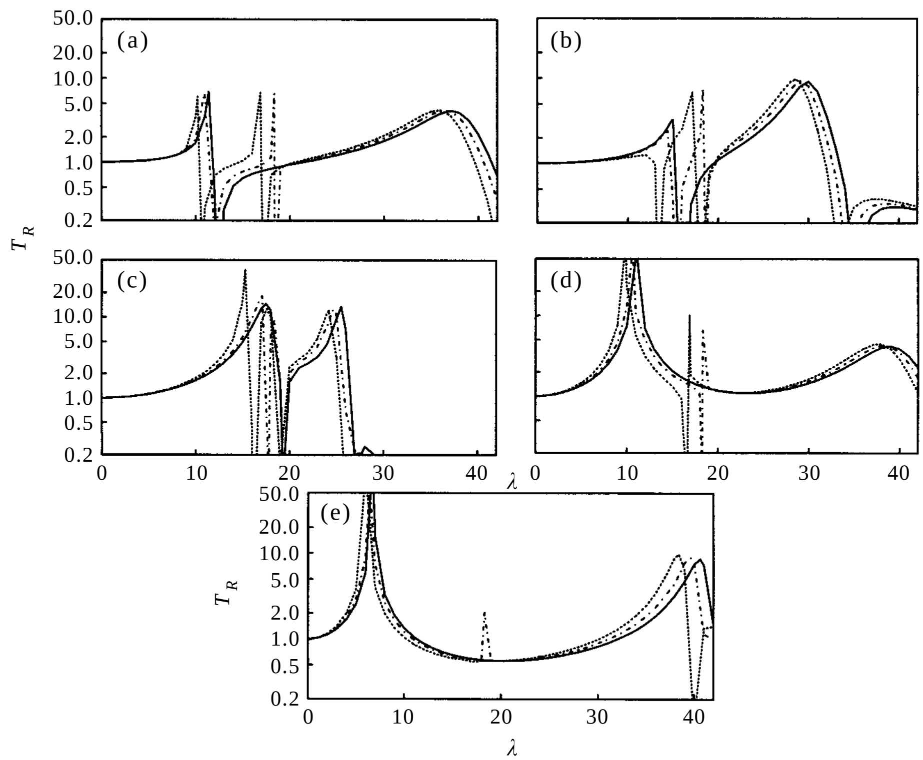

Figure 3. The force transmissibilities of square orthotropic plates for various E_2/E_1 values for: (a) $x_s = y_s = 0{\cdot}1$; (b) $x_s = y_s = 0{\cdot}2$; (c) $x_s = y_s = 0{\cdot}3$; (d) $x_s = y_s = 0{\cdot}4$; (e) $x_s = y_s = 0{\cdot}5$. $\gamma_s = 1$; $\kappa_s = 100$; $\nu = 0{\cdot}3$. —, $E_2/E_1 = 1{\cdot}0$; –·–·, $E_2/E_1 = 0{\cdot}8$; - - - -, $E_2/E_1 = 0{\cdot}6$.

TABLE 1

Frequency parameters λ for a free and simply point-supported orthotropic square plate: $\nu = 0{\cdot}3,\ \gamma_s = 0,\ x_s = y_s = 0{\cdot}5$

Vibration mode	$\kappa_s = 0$ $E_2/E_1 = 1{\cdot}0$	$\kappa_s = \infty$ $E_2/E_1 = 1{\cdot}0$	$\kappa_s = 0$ $E_2/E_1 = 0{\cdot}8$	$\kappa_s = \infty$ $E_2/E_1 = 0{\cdot}8$	$\kappa_s = 0$ $E_2/E_1 = 0{\cdot}6$	$\kappa_s = \infty$ $E_2/E_1 = 0{\cdot}6$
SS-1	0	7·139	0	6·815	0	6·326
SS-2	19·684	19·684	18·033	18·329	15·674	16·839
SS-3	24·347	44·383	23·508	43·237	22·981	41·786

the SS-3 vibration becomes important in the force transmissibility. For values of $x_s = y_s$ smaller than approximately $x_s = y_s = 0{\cdot}25$ and for E_2/E_1 values different from unity, the SS-2 vibration frequency range becomes larger than the range when $x_s = y_s$ are greater than approximately 0·25. As it is seen from Figure 2, for $E_2/E_1 = 0{\cdot}8$ and $x_s = y_s = 0{\cdot}275$, the frequency parameters, λ, for the SS-1 and SS-2 modes become 18·033 and 18·749, respectively, and as a result of this situation, the SS-1 and SS-2 vibration responses become very close to each other. This situation can be observed from Figure 3.

Table 1 shows the frequency parameters of a completely free and a corner point-supported orthotropic square plate. In the case of isotropy, where $E_2/E_1 = 1$, the obtained results are the same as that of Yamada *et al*. [25].

4. CONCLUSIONS

The steady state response to a sinusoidally varying force has been studied for a viscoelastically point-supported anisotropic (orthotropic) square plate.

The response equation is derived from the equation of vibration of the point-supported orthotropic plate by the generalized Galerkin method with the eigenfunctions of free beams used as admissible functions.

By the application of the method, the response curves to a sinusoidally varying point force acting at the centre have been calculated numerically for orthotropic square plates viscoelastically supported at four points at the corners or on the diagonals, together with the natural frequencies of undamped point-supported plates. It is seen that because of the orthotropy, the SS-2 mode occurs in the plate and results in narrower frequency ranges and small frequencies relative to the isotropic case.

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