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HAND-HELD PERCUSSION MACHINE AS DISCRETE NON-LINEAR CONVERTER

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The formulation is presented of a general approach to analysis and synthesis of hand-held percussion machines by using methods of non-linear dynamics and optimal control. A major objective is the analysis of the interaction of the machine with the operator and the load. An attempt is made to optimize this interaction in order to improve machine capacity and to reduce the reaction on the operator. Some general recommendations are presented on the choice of the main characteristics of such machines.

1. INTRODUCTION

The development of hand-held percussion machines is an important area of mechanical engineering. Used in machines, strongly non-linear (vibro-impact) dynamic processes have a severe influence on the operator and the environment. This applies strict limitations to the operational parameters of hand-held machines and makes them an ideal choice for the development of new dynamical and design concepts.

The complexity of hand-held percussion machine dynamical behaviour has limited their analysis to the use of some special models [1–4]. Such models did not permit the discovery of the common dynamical characteristics of the machines as a whole. The ideal is to find the connection between the main output and resulting unfavourable vibrations with the parameters of the excitation. The variety of mechanisms of vibration excitation in hand-held percussion machines, in conditions of absence of the general theory, makes their comparative analysis, choice of optimal design concepts and estimation of their improvement potential difficult. It is striking that despite such widespread use, modern textbooks fail to present to students the basic theoretical principles of percussion machine dynamics.

Also, the problem of decreasing the vibration-induced load for all types of hand-held percussion machines demands the development of a general optimal approach for estimation and optimization of their excitation [5, 6]. As is shown in this paper, this can be formulated as a problem of searching for the periodic optimal control and can be transformed into a mathematical problem of moments in the proper normed linear space. An exact solution of the formulated problem is obtained in an explicit analytical form. This solution gives one the opportunity to obtain absolute estimations of the improvement potential for all modifications of similar machines.

For the simplification of optimal design, the sequence of exact estimations is developed for the series of possible quasi-optimal excitations. Some recommendations on the choice of the main characteristics of machines are presented.

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2. HAND-HELD PERCUSSION MACHINE AS A DISCRETE MODULATOR

The general structure forms the interactions of the main units which exist in every hand-held percussion machine and defines its dynamics as a whole. It consists (see Figure 1(a)) of a source of energy, a vibro-impact converter which transforms the energy of source into a vibro-impact process, and a medium or object of treatment. In order to transfer the energy of the source to the object being treated, an operator has to apply an effort P to the machine body by pressing it to the object. This effort, which can be called *feed*, reveals the main channel of permanent processing control by the operator.

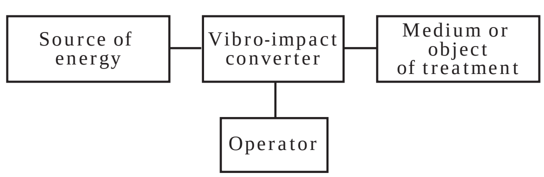
The structure of the vibro-impact converter as a vibratory machine (see Figure 1(b)) includes an exciter of oscillations 1 consisting of drive, transmission and exciting mechanism, the vibrator or oscillator 2 and the load 3 containing the mechanical transformer (intermediate striker, transmitter or wave guide), tool and medium or object of treatment.

According to mechanical laws, the force interactions between units are reciprocal ones by nature and load both machine and operator. The feed P as an outer force depends on the interaction of the machine as a whole with the object of treatment (see Figure 2). This includes both an interaction of the striker 1 with a tool 2 or intermediate transformer 3 for processing of the object or media 4 (see Figure 2(a)) and a setting of the machine 5 and the tool and transformer against the object 4 in a position of impact (see Figure 2(b)). Depending on the vibro-impact dynamics, the striker with the mass M_1 can accomplish mainly either single impact periodic motions [3] (see Figure 3(a)) or periodic motions with repeated attenuated impacts [7] (see Figure 3(b)).

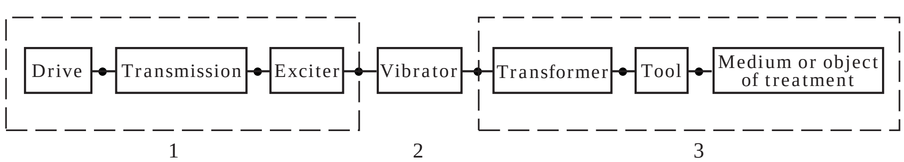
The regimes with single-impact interaction have the most intensive (resonant) nature [3]. They are used preferably for processing and demolition. During such processing, the momentum of impact interaction per period T equals $M_1(1 + R)\dot{x}_{1-}$, where $\dot{x}_{1-}$ is the velocity of striker before the impact and R is the coefficient of striker reflection after the impact. The coefficient R indicates the change of absolute velocity of the striker, and it does not always coincide with a coefficient of restitution. It has to be measured preferably for each specific impact pair in conditions of collision close to reality.

The impact momentum has to be equalized by the effort of feed P_{ef} from the operator according to the principle of impulse and momentum: $P_{ef}T = M_1(1 + R)\dot{x}_{1-}$, which gives the value of minimum feed for the accomplishment of vibro-impact periodic motion with the prescribed energy (velocity $\dot{x}_{1-}$) and frequency f of impacts:

$$P_{ef} = M_1(1 + R)\dot{x}_{1-}f. \tag{1}$$



(a)



(b)

Figure 1. General structure of a hand-held percussion machine. See text for key.

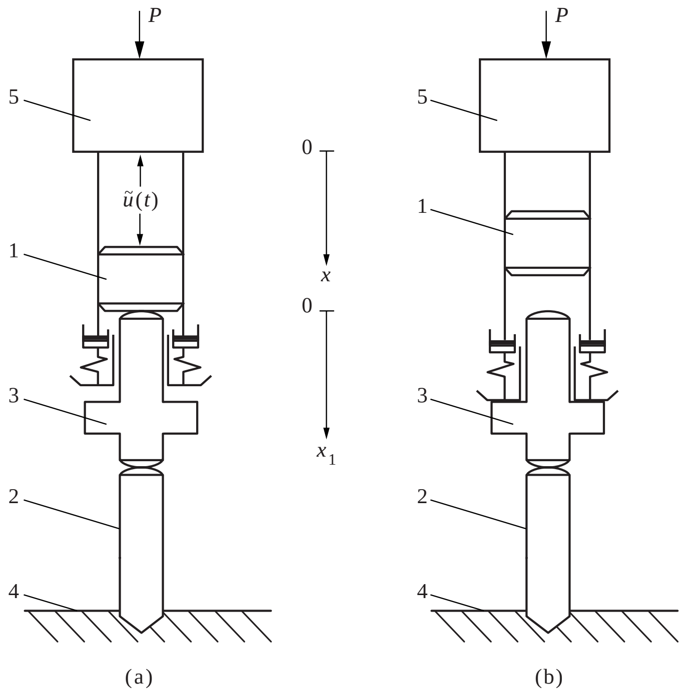


Figure 2. Dynamical model of percussion machine and its interaction with tool and operator. See text for key.

It follows from formula (1) that an increase in the processing throughput, either by the rise of impact energy or frequency, can be achieved due to the intensification of P_{ef}.

Any additional interactions between the machine and the object of treatment lead to P exceeding P_{ef}. If, for example, the interaction represents an unlimited series of attenuated impacts during the period T, then for the same velocity $\dot{x}_{1-}$ of the first impact, it follows that

$$P = M_1(1 + R)\dot{x}_{1-}f(1 + R + R^2 + \dots) = \frac{M_1(1 + R)\dot{x}_{1-}f}{1 - R} \geqslant P_{ef}. \tag{2}$$

Further increase in feed occurs if this series of impacts dies out after time $\tau < T$ or through the interaction of the machine body with the tool or intermediate striker during the setting before impact or due to rebound of the tool after impact against the object of treatment. If real feed P exceeds P_{ef}, it imposes an additional stress upon the operator. The validity

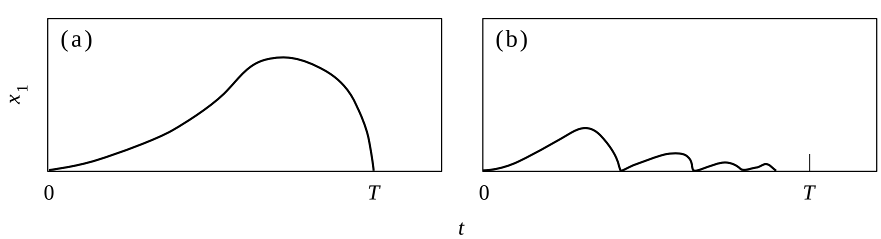


Figure 3. Regimes of vibro-impact interaction.

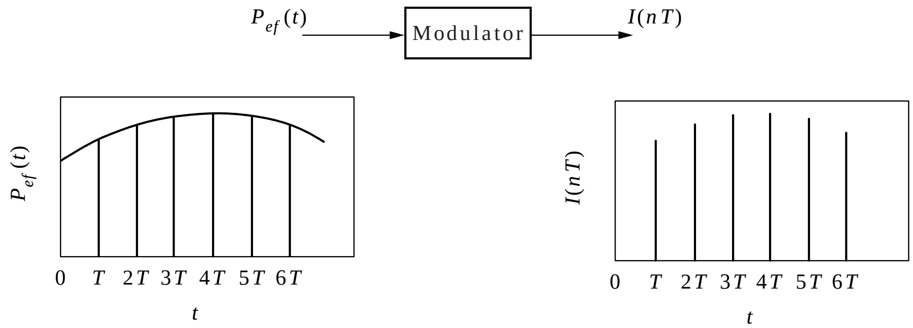


Figure 4. Percussion machine as a discrete modulator.

of the hand-held machine, from this point of view, can be estimated with the *coefficient of feed effectiveness*:

$$\lambda = P_{ef}/P, \qquad 0 \leqslant \lambda \leqslant 1. \tag{3}$$

The ideal hand-held percussion machine, which is capable of producing a periodic vibro-impact process under arbitrary feed with a single impact during the period, can be treated as a discrete modulator (see Figure 4) which converts the permanent feed P_{ef} into a series of impulses with period T:

$$I(nT) = M_1(1 + R_n)\dot{x}_{1n-} \sum_n \delta(t - nT). \tag{4}$$

Here $\delta(t)$ is Dirac's function, n is the current number of impacts, $\dot{x}_{1n-}$ is the velocity of striker before nth impact, and R_n is the proper slowly changing coefficient of striker reflection. The value of $\dot{x}_{1n-}$ is

$$\dot{x}_{1n-} \approx \int_{(n-1)T}^{nT} P_{ef}(t)\,\mathrm{d}t/M_1(1 + R_n). \tag{5}$$

Small movements of the centre of machine mass have been neglected here.

The output of the machine is defined usually as the energy of the striker before the impact $E = 0{\cdot}5M_1\dot{x}_{1-}^2$. With the help of expression (1) this yields the characteristic for modulator conversion,

$$E = \gamma P_{ef}^2, \tag{6}$$

where

$$\gamma = 1/2(1 + R)^2 M_1 f^2.$$

Let the machine, for example, process concrete with the displacement (x)–force (F) diagram of demolition during one impact as shown in Figure 5. The shaded area represents the work of demolition. Upon supposing that the whole energy of impact $E_i = 0{\cdot}5(1 - R^2)M_1\dot{x}_{1-}^2$ loads the concrete $E_i = 0{\cdot}5c_1a^2$, it is possible to calculate the efficiency of demolition with the formula $\eta = 1 - c_1/c_2$ and the capacity of chiselling

$$v = (a - b)f = 2f\left(1 - \frac{c_1}{c_2}\right)\sqrt{\frac{E_-(1 - R^2)}{2c_1}} = \left(1 - \frac{c_1}{c_2}\right)\sqrt{\frac{1 - R}{(1 + R)c_1M_1}}\,P_{ef}. \tag{7}$$

The machine will be able to accomplish this capacity if under any feed P_{ef} its exciting mechanism develops a steady state vibro-impact process with single impact during the period. It can be achieved either by the development of robust dynamical exciting systems or with the use of control facilities. In the last case it is necessary to realize an autoresonant vibro-impact structure [8].

3. EXCITATION OF HAND-HELD PERCUSSION MACHINE AS A PROBLEM OF OPTIMAL CONTROL

Let us investigate the optimal conditions of machine excitation with single impact during the period. It is supposed that under the influence of excitation the periodic force acting on the striker is described by an unknown periodic function $\tilde{u}(t)$ (see Figure 2). This can be due to the variation of pressure in the gas chamber of the hammer drill, hydraulic pressure in the cylinder of the breaker, compression of some spring elements of the striker suspension or electromagnetic interaction of the striker as an element of a linear motor. In all cases, this force has a simultaneous opposite action on the body of the machine and through it on the operator. The problem of optimization of excitation of $\tilde{u}(t)$ can be formulated as follows.

The differential equation of motion of the striker can be written as

$$\ddot{x}_1 = u(t), \tag{8}$$

where x_1 is the co-ordinate of the striker measured from the point of its impact, when $x_1 = 0$; $u(t) = \tilde{u}(t)/M_1$. The positive direction of x_1 is shown in Figure 2.

In order to realize the periodic motion of the striker with the prescribed energy E or velocity of impact $\dot{x}_{1-}$ and frequency f, the solution of equation (8) has to satisfy the periodic conditions

$$\begin{aligned} t = 0: &\qquad x_1 = 0, \qquad \dot{x}_1 = \dot{x}_{1-}, \\ t = T: &\qquad x_1 = 0, \qquad \dot{x}_1 = \dot{x}_{1+}, \end{aligned} \tag{9}$$

where $T = 2\pi/\omega = 1/f$ is the period of motion and $\dot{x}_{1+} = -R\dot{x}_{1-}$.

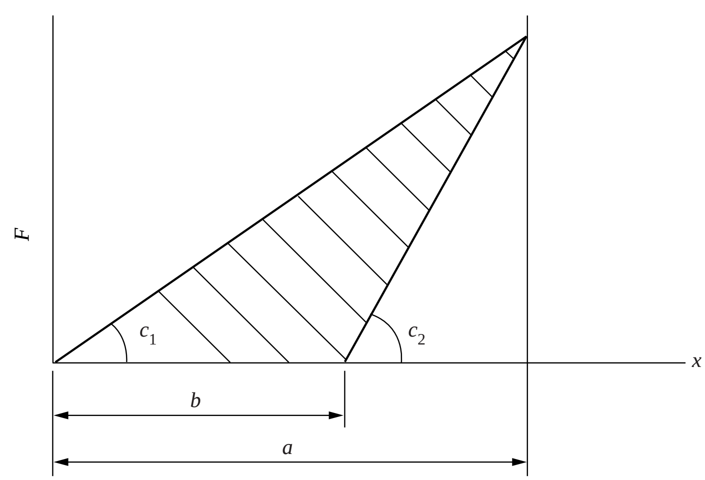


Figure 5. Force–displacement diagram of concrete demolition.

Rewriting equation (8) in Cauchy form [9] gives

$$\dot{x}_1(t) = \dot{x}_{1+} + \int_0^t u(\tau)\,\mathrm{d}\tau, \qquad x_1(t) = x_1(0) + t\dot{x}_{1+} + \int_0^t (t-\tau)u(\tau)\,\mathrm{d}\tau. \tag{10}$$

Applying the periodic conditions (9) to equations (10) yields

$$(1+R)\dot{x}_{1-} = \int_0^T u(\tau)\,\mathrm{d}\tau, \qquad RT\dot{x}_{1-} = \int_0^T (T-\tau)u(\tau)\,\mathrm{d}\tau. \tag{11}$$

On the left sides of equations (11) there are known numbers. The right hand parts of equations (11) represent the scalar products of the unknown function $u(t)$ with prescribed linear independent basic functions $h_i(t)$ $(i = 1, 2)$, where $h_1(t) = 1$, $h_2(t) = T - t$ and all functions are defined in segment $t \in [0, T]$. The problem of finding the unknown function $u(t)$ under such conditions is known in mathematics as the *moment problem* [10].

For reduction of influence on the operator, the excitation $u(t)$ has to be limited in some technical sense. Two main limitations will be analyzed and compared: $|u(t)|$ and $\sqrt{(1/T)\int_0^T u^2(t)\,\mathrm{d}t}$, which represent the important characteristics influencing vibration of the machine body. As a result, the *optimal solution* of the moment problem for periodic excitation with the prescribed period T has to be minimized either as

$$\min_u \max_{t\in[0,\,T]} |u(t)|, \tag{12}$$

or as

$$\min_u \sqrt{\int_0^T u^2(t)\,\mathrm{d}t}. \tag{13}$$

The basic mathematical foundations leading to the solution of the formulated optimal problems are given in the Appendix.

4. OPTIMAL EXCITATION WITH MINIMAL AMPLITUDE

As shown in the Appendix, the optimal excitation $u^0(t)$ satisfying equations (11) and minimizing (12) can be found in the form (see equation (A11) in the Appendix)

$$u^0(t) = \frac{\operatorname{sgn}[n_1^0(T-t) + n_2^0]}{\int_0^T |n_1^0(T-t) + n_2^0|\,\mathrm{d}t} = U_0 \operatorname{sgn}[n_1^0(T-t) + n_2^0], \tag{14}$$

where numbers n_1^0 and n_2^0 are the solution of the problem (see expression (A10) in the Appendix) of finding

$$\min_{n_1,n_2} \int_0^T |n_1(T-t) + n_2|\,\mathrm{d}t = \frac{1}{U_0} \quad \text{when} \quad n_1 RT\dot{x}_{1-} + n_2(1+R)\dot{x}_{1-} = 1.$$

In this case it is not necessary to follow the entire regular procedure, because the solution can be found by a simpler approach.

According to equation (14), the optimal excitation $u^0(t)$ is a piecewise constant function with no more than one switch during the interval of $t \in (0, T)$. This is because of the linearity of the function under the signum sign. In our case it can only be a single possible version

$$u^0(t) = \begin{Bmatrix} -U_0, & \text{if} \quad t \in [0, t_1) \\ +U_0, & \text{if} \quad t \in [t_1, T) \end{Bmatrix}, \tag{15}$$

where t_1 is an unknown value. Another combination of signs contradicts the nature of the mechanical problem.

Substituting expression (15) into equations (11) yields, after integrating the system of two equations with two unknown values, U_0 and t_1,

$$(1 + R)\dot{x}_{1-} = (T - 2t_1)U_0, \qquad RT\dot{x}_{1-} = [(T - 2t_1)T + t_1^2 - T^2/2]U_0. \tag{16}$$

After solution of these equations, it finally follows that

$$\frac{t_1}{T} = \frac{1}{1+R}\left(1 - \sqrt{1 - \frac{1 - R^2}{2}}\right),$$

$$U_0 = \frac{(1 + R)^2\dot{x}_{1-}}{2T(\sqrt{1 - 0{\cdot}5(1 - R^2)} - 0{\cdot}5(1 - R))}. \tag{17}$$

It is important to mention that t_1 does not depend on "power" characteristics of the machines. The amplitude of excitation U_0 is proportional to the feed P_{ef}, upon taking into account formula (1). This characterizes the dependence of the excitation energy on intensity of the drilling process.

5. OPTIMAL EXCITATION WITH MINIMAL ROOT-MEAN-SQUARE VALUE

The estimation of the root-mean-square (RMS) value of the excitation follows ISO recommendations. The proper optimal excitation has to satisfy equalities (11) under condition (13). According to the Appendix (see equation (A12)), an optimal excitation can be found as a linear combination of the basic functions $h_1(t)$ and $h_2(t)$,

$$u^0(t) = q_1^0 + q_2^0(T - t), \tag{18}$$

where q_1^0 and q_2^0 are unknown numbers.

Substituting equation (18) into equalities (11), produces after integrating the system of two equations with two unknown values q_1^0 and q_2^0:

$$(1 + R)\dot{x}_{1-} = q_1^0 T + q_2^0(T^2/2), \qquad R\dot{x}_{1-} = q_1^0(T/2) + q_2^0(T^2/3). \tag{19}$$

After solution of these equations, this finally produces, with use of expression (18)

$$u^0(t) = \frac{2\dot{x}_{1-}}{T}\left[2R - 1 + \frac{3(1 - R)t}{T}\right]. \tag{20}$$

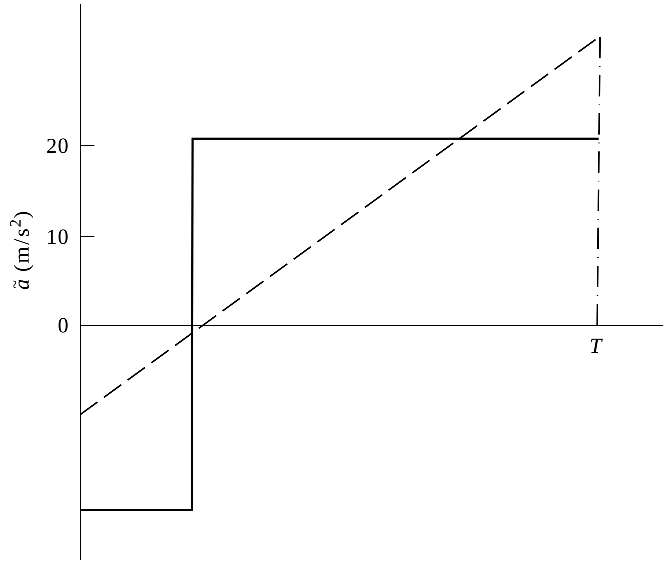


Figure 6. Comparison of the body acceleration for different optimal excitations.

After the calculation of optimal excitation, it is possible to estimate the proper variable component of body acceleration $\tilde{a}(t)$ perceived by the operator,

$$\tilde{a}(t) = (M_1/M)u^0(t), \tag{21}$$

where M is the mass of the body.

The characteristics of body acceleration for two different optimal excitations (see Figure 6) are shown in Table 1. They are calculated for the typical parameters of a heavy hammer drill: $\dot{x}_{1-} = 8{\cdot}8$ m/s ($E \sim 7$ Nm), $M_1 = 0{\cdot}2$ kg, $M = 8{\cdot}0$ kg, $f = 45$ Hz, $R = 0{\cdot}25$. These parameters will be used in all following examples.

The results for the different optimal excitations turned out to be comparatively close. Due to this, in the following analysis excitations with limited amplitude will be used as being more convenient for estimations and accomplishment.

TABLE 1

Body acceleration characteristics for two different optimal excitations

	$\min\limits_{u} \max\limits_{t\in[0,T]} \lvert u(t)\rvert$ (m/s^2)	$\min\limits_{u} \sqrt{\frac{1}{T}\int_0^T u^2(t)\,\mathrm{d}t}$ (m/s^2)
$\sqrt{\frac{1}{T}\int_0^T a^2(t)\,\mathrm{d}t}$	16·9	12·0
$\frac{1}{T}\int_0^T \lvert a(t)\rvert\,\mathrm{d}t$	14·0	10·4
$\max\limits_{t\in[0,T]} \lvert a(t)\rvert$	20·4	32·4

6. OPTIMAL LAWS OF STRIKER MOTIONS

The optimal laws of striker motion can be obtained by integration of equations (10) with respect to expressions (15) and (17). From the first of equations (10) it follows that

$$\dot{x}_1(t) = \dot{x}_{1+} + \int_0^T u(\tau)\,\mathrm{d}\tau = -R\dot{x}_{1-} - 2U_0 t_1 + U_0 t. \tag{22}$$

A maximum of striker deflection occurs, corresponding to when the velocity equals null $\dot{x}_1(t_2) = 0$, where t_2 is the moment of maximum deflection. Then it follows from equation (22) that $t_2 = 2t_1 + R\dot{x}_{1-}/U_0$, or by taking into account expression (17),

$$\frac{t_2}{T} = \frac{1-R}{2} + \frac{2R(\sqrt{1-0{\cdot}5(1-R^2)} - 0{\cdot}5(1-R))}{(1+R)^2}. \tag{23}$$

Integration of the second of equations (10) gives

$$x_1(t) = \dot{x}_{1+}t + \int_0^T (t-\tau)u(\tau)\,\mathrm{d}\tau = -R\dot{x}_{1+}t - 2U_0 t_1 t + U_0 t^2/2.$$

Then, with expressions (15) and (17) taken into account,

$$x_{1\max} = x_1(t_2) = \frac{U_0 T^2}{2}\left[\frac{1-R}{2} + \frac{2R(\sqrt{1-0{\cdot}5(1-R^2)} - 0{\cdot}5(1-R))}{(1+R)^2}\right]. \tag{24}$$

This values defines the length of working stroke which influences the size of the machine.

The optimal laws of striker motion are shown in Figure 7. The influence of R on values t_1 and U_0 are presented in Figure 8. It is important to remark that U_0 depends weakly on the changing of R.

7. ESTIMATIONS OF QUASI-OPTIMAL EXCITATIONS WITH CONDENSED ACCELERATION IMPULSE

In real mechanisms it is difficult to accomplish an optimal excitation. For example, in typical hammer drills with an air chamber and crank-slide mechanism, it is possible to achieve only a limited expansion of gas during the initial stage of excitation just after an impact. In these conditions an attempt to realize the optimal excitations (15) or (20) looks impractical. In order to bring the improvement potential to realization, it is useful to find some quasi-optimal practical excitations and their estimations.

The reduction of the initial negative impulse of excitation within the framework of limited amplitude can be achieved with the help of the following quasi-optimal excitations:

$$u(t) = \left\{\begin{matrix} -U, & \text{if} & t\in[0, t) \\ +nU, & \text{if} & t\in[t_1, T) \end{matrix}\right\}. \tag{25}$$

Here U, t_1 are unknown values and n is an approximating coefficient.

Substituting expression (25) into equalities (11), gives, after integrating the system of two equations for the unknown values U and t,

$$(1+R)\dot{x}_{1-} = -Ut_1(1+n) + nUT,$$

$$RT\dot{x}_{1-} = -UTt_1(1+n) + t_1^2\frac{U(1+n)}{2} + \frac{nU}{2}T^2. \tag{26}$$

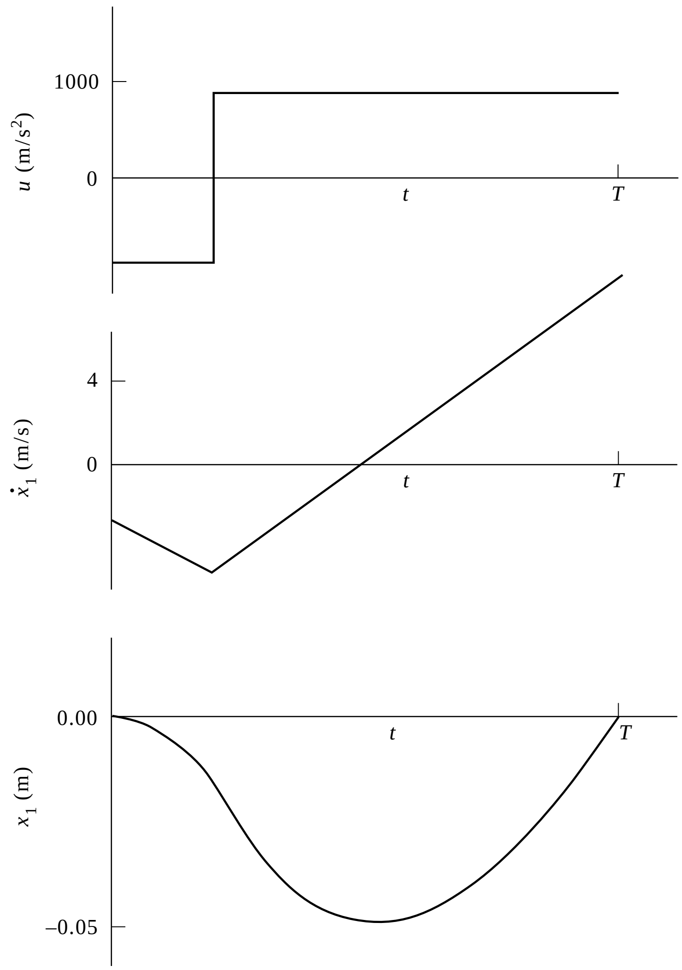


Figure 7. Optimal laws of striker motion.

The solution of these equations gives finally

$$\frac{t_1}{T} = \frac{1}{1+R}\left[1 - \sqrt{1 - \frac{n(1-R^2)}{1+n}}\right],$$

$$U = \frac{(1+R)\dot{x}_{1-} f}{n - \dfrac{1+n}{1+R}\left[1 - \sqrt{1 - \dfrac{n(1-R^2)}{1+n}}\right]}. \tag{27}$$

The quasi-optimal laws of excitation (25) and (27) for different n are presented in Figure 9. They demonstrate the tendency to reduce the initial negative acceleration and to condense positive acceleration impulse. It is seen that if $n = 100$ the exciting force (acceleration) acts practically in the positive direction only. Further estimations of possibility of these condensed positive impulses of excitation can be calculated with the help of the following approximations of the excitation:

$$u(t)=\begin{Bmatrix}0, & \text{if} & t\in[0,t_1)\\ U & \text{if} & t\in[t_1,t_1+\Delta)\\ 0, & \text{if} & t\in[t_1+\Delta,T)\end{Bmatrix}, \tag{28}$$

Here U, t_1 are unknown values, and Δ is the permissible duration of positive acceleration, $(t_1+\Delta)/T\leqslant 1$.

Substituting expression (28) into equalities (11) and then integrating brings once again a system of two equations for the unknown values U and t_1. The solution of these equations gives finally

$$\frac{t_1}{T}=1-\frac{1}{2}\left(\frac{\Delta}{T}\right)-\frac{R}{1+R},\qquad U=\frac{(1+R)\dot{x}_{1-}f}{\Delta/T}. \tag{29}$$

Figure 10 demonstrates the different possible excitations of the striker with the aid of positive impulses of different duration Δ/T. This continues the sequence of quasi-optimal excitations of Figure 9. The dash-dotted line shows the position of the middle of the impulse interval dependent upon R. The shortening of the interval Δ leads naturally to the essential increasing of the exciting force (acceleration) amplitude. The worst case when $\Delta/T=0{\cdot}1$ corresponds approximately to the excitation of modern hammer drills.

For vibration protection of operators of hand-held vibro-impact machines, the initial harmonics of excitation play the most important role. That is why, for the estimation of the improvement possibilities of different quasi-optimal excitations, it is reasonable to

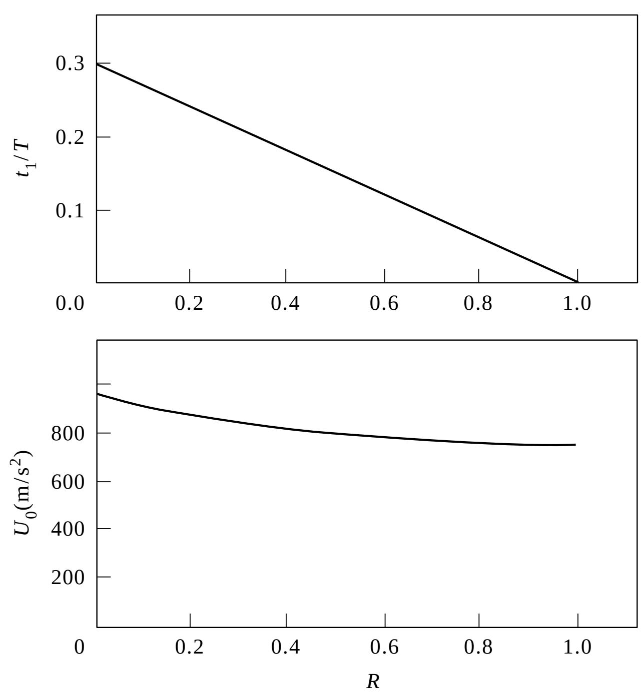


Figure 8. Dependence of optimal excitation parameters on R.

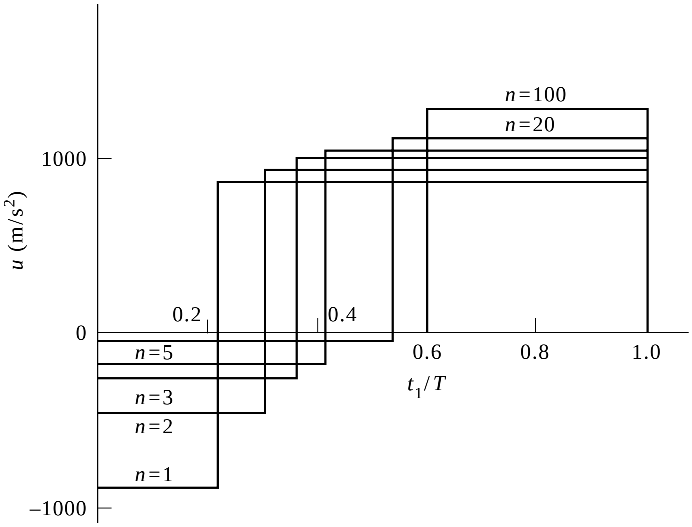


Figure 9. Quasi-optimal excitations with reduced rebound of striker.

compare the amplitudes of these harmonics. The Fourier series for the excitations (25) will be

$$u(t) = nU\left[1 - \frac{t_1}{T}\left(1 + \frac{1}{n}\right)\right] + \sum_{k=1}^{\infty} \frac{\sqrt{2}(1+n)U}{\pi k} \sqrt{1 - \cos 2\pi k \frac{t_1}{T}} \cos(k\omega t + \vartheta_k), \quad (30)$$

where U and t_1 are defined by expressions (27).

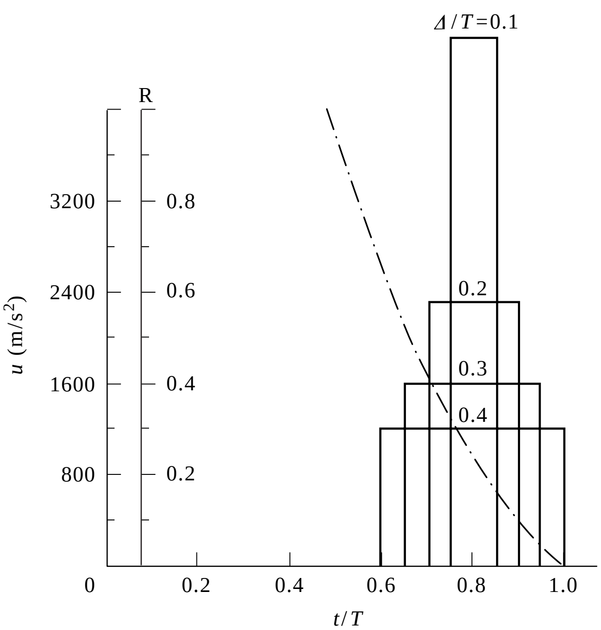


Figure 10. Quasi-optimal excitations with condensed acceleration impulses.

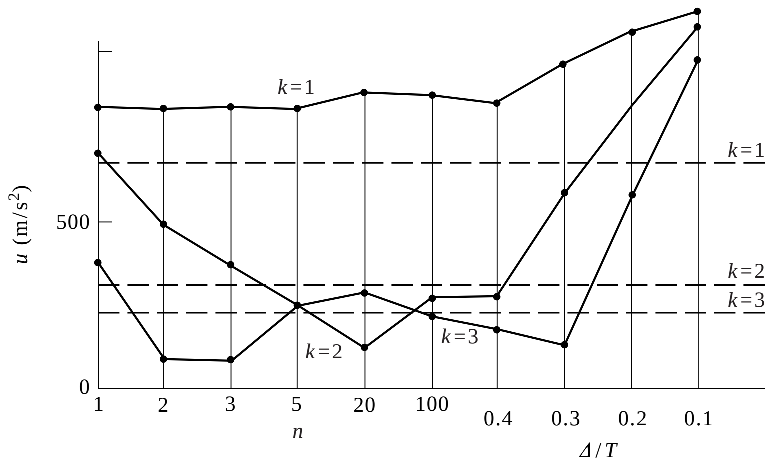


Figure 11. Vibration action of quasi-optimal excitations.

For the excitations (28), the Fourier series will be

$$u(t) = \frac{\Delta U}{T} + \sum_{k=1}^{\infty} \frac{2U}{\pi k} \sin \pi k \frac{\Delta}{T} \cos (k\omega t + \beta_k), \tag{31}$$

where U is defined by expressions (29). In the last two expressions the phases of the harmonics were not specified because they will not be used in the following analysis.

It is simple to check that the constant components in both Fourier series are equal to P_{ef}/M_1, as is physically clear. As a result, from expressions (27) and (29) it follows that an increase in P_{ef}, for the achievement of a higher capacity, has to be supported by the proportional rise of U. An independent control of impact energy or frequency demands a special synchronization of frequency.

The values of the amplitudes of the three initial harmonics for different quasi-optimal excitations are shown in Figure 11 with solid lines. For comparison, the proper amplitudes of the harmonics of excitation (20) with minimum root-mean-square value of excitation are superimposed with dashed lines. The values of vibration presented are lower than the proper characteristics of modern machines.

The comparison of initial harmonics with the total solution shows that traditional estimation with the neglect of phase relations between harmonics can seriously underestimate the injury risk in the case of the use of condensed acceleration impulses.

8. CONCLUSIONS

The reorganization of hand-held percussion machine excitation in the direction of optimal excitation is an effective and conformable way of excitation improvement and reduction of unfavourable vibration. It also relieves the load on the drive. This can be achieved mainly through the extension of excitation impulse and stabilization of resonant vibro-impact excitation.

The implementation of control units can transform a one-regime vibratory machine into a multi-regime program-operated system. As a result, the machine becomes adaptable to processes, tools and operators. It permits the operator to realize his programme of work in accordance with formula (7) by variations of P_{ef}.

The technique developed gives the possibility of estimating an improvement potential at every stage of reorganization. In practice, it can be made by direct measurement of the force diagram of excitation or acceleration of the striker. The optimal cycle of excitation also permits the comparison of percussion machines with different design by calculation of their improvement potential from the point of their capacity and unfavourable vibration activity.

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APPENDIX

A.1. OPTIMAL SOLUTION OF MOMENT PROBLEM FOR EXCITATION WITH MINIMUM AMPLITUDE

There are m integral equalities

$$\alpha_i = \int_0^T h_i(t)u(t)\,\mathrm{d}t, \qquad i = 1, 2, \ldots, m, \tag{A1}$$

where α_i are known real numbers and $h_i(t)$ are the given linear independent functions described on segment $t \in [0, T]$. It is necessary to find the unknown function $u(t)$ which satisfies equalities (A1) and has minimum modulus, according to expression (12). As shown in the special mathematical literature [10, 11], the basic functions $h_i(t)$ have to be limited then in the sense of $\int_0^T |h_i(t)|\,\mathrm{d}t$ which is always satisfied hereafter. Following reference [3], here simple considerations will be given leading to the solution of the formulated problem.

Upon choosing n_i as arbitrary real numbers and multiplying every equality (A1) by n_i and summing them up, it follows, after changing the order of integration and summation, that

$$\sum_{i=1}^{m} \alpha_i n_i = \int_0^T \sum_{i=1}^{m} n_i h_i(t) u(t)\, \mathrm{d}t. \tag{A2}$$

As the numbers n_i are proposed as arbitrary ones, it is possible to choose them specifically so that

$$\sum_{i=1}^{m} \alpha_i n_i = 1. \tag{A3}$$

Then, from equation (A2),

$$1 = \int_0^T \sum_{i=1}^{m} n_i h_i(t) u(t)\, \mathrm{d}t. \tag{A4}$$

On the right side of equation (A4), the following inequalities are obvious:

$$1 = \int_0^T \sum_{i=1}^{m} n_i h_i(t) u(t)\, \mathrm{d}t \leqslant \int_0^T \left| \sum_{i=1}^{m} n_i h_i(t) u(t) \right| \mathrm{d}t \leqslant \max_{t \in [0,t]} |u(t)| \int_0^T \left| \sum_{i=1}^{m} n_i h_i(t) \right| \mathrm{d}t. \tag{A5}$$

From expression (A5), it is clear that

$$\max_{t \in [0,T]} |u(t)| \geqslant \left[\int_0^T \left| \sum_{i=1}^{m} n_i h_i(t) \right| \mathrm{d}t \right]^{-1}. \tag{A6}$$

The inequality (A6) is satisfied for the all numbers n_i from equality (A3) and therefore, for such numbers n_i^0 which maximize the right side of inequality (A6)

$$\max_{t \in [0,T]} |u(t)| \geqslant \left[\min_{n_i} \int_0^T \left| \sum_{i=1}^{m} n_i h_i(t) \right| \mathrm{d}t \right]^{-1} = \left[\int_0^T \left| \sum_{i=1}^{m} n_i^0 h_i(t) \right| \mathrm{d}t \right]^{-1}, \qquad \sum_{i=1}^{m} n_1^0 \alpha_i = 1. \tag{A7}$$

From relation (A7) it follows that

$$\min_u \max_{t \in [0,T]} |u(t)|,$$

which has to be in reality for all of n_i, is

$$U_0 = \min_u \max_{t \in [0,T]} |u(t)| = \left[\int_0^T \left| \sum_{i=1}^{m} n_i^0 h_i(t) \right| \mathrm{d}t \right]^{-1}. \tag{A8}$$

Now, it is necessary to find the function $u(t)$ in such a manner that the chain of the two inequalities in expression (A5) transforms itself into equalities. It can be shown that this takes place if and only if

$$u(t) = U \operatorname{sgn} \sum_{i=1}^{m} n_i h_i(t), \tag{A9}$$

where $U = const$.

So, the first inequality in expression (A5) transforms itself into an equality if and only if functions

$$\textstyle\sum_{i=1}^{m} n_i h_i(t)$$

and $u(t)$ have, at every moment of time, similar signs. The second inequality transforms itself into an equality if and only if $u(t)$ is a constant: $|u(t)| = U = const$ for all $t \in [0, T]$. As a result, the function $u(t)$ which satisfies the equalities (A1) has to be in the form (A9). But because it is necessary to minimize amplitude

$$U = \max_t |u(t)|,$$

the $u(t)$ has to be chosen from expression (A8), where numbers n_i^0 are found from the solution of the following problem: *to find*

$$\min_{n_i} \int_0^T \left| \sum_{i=1}^{m} n_i h_i(t) \right| \mathrm{d}t \tag{A10}$$

under condition (A3).

Therefore, the solution of the initial problem according to expressions (A8) and (A9) is as follows:

$$u^0(t) = \frac{\operatorname{sgn} \sum_{i=1}^{m} n_i^0 h_i(t)}{\int_0^T \left| \sum_{i=1}^{m} n_i^0 h_i(t) \right| \mathrm{d}t}. \tag{A11}$$

A.2. OPTIMAL SOLUTION OF MOMENT PROBLEM FOR EXCITATION WITH MINIMAL ROOT-MEAN-SQUARE VALUE

To obtain an absolute estimation of improvement potential of excitation in the sense of a root-mean-square value, it is necessary to find the unknown function $u^0(t)$ which satisfies equalities (A1) and has a minimum value as shown in expression (13). Following reference [10], it will be shown below that the optimal excitation in this case can be found as a linear combination of the basic functions $h_i(t)$,

$$u^0(t) = q_1^0 h_1(t) + \cdots + q_m^0 h_m(t), \tag{A12}$$

where the unknown numbers $q_1^0, \ldots, q_m^0$ are the solution of the set of linear equations. The basic functions $h_i(t)$ have to be limited in this case in the sense of the norm $\sqrt{\int_0^T h_i^2(t)\,\mathrm{d}t}$ which is always satisfied hereafter.

Substituting expression (A12) into equalities (A1) instead of $u(t)$, it follows, after integration of the set of equations for finding the numbers $q_1^0, \ldots, q_m^0$ is

$$\begin{gathered} l_{11}q_1^0 + \cdots + l_{1m}q_m^0 = \alpha_1, \\ \cdots\cdots\cdots \\ l_{m1}q_1^0 + \cdots + l_{mm}q_m^0 = \alpha_m, \end{gathered} \tag{A13}$$

where the coefficients l_{ij} are defined by the equalities

$$l_{ij} = \int_0^T h_i(t)h_j(t)\,\mathrm{d}t, \qquad i, j = 1, \ldots, m. \tag{A14}$$

In order to prove the existence of the non-trivial solution of equations (A13), it has to be shown that the determinant of the set of equations (A13) is not equal to zero. So, due to the linear independence of functions $h_i(t)$ for the arbitrary numbers $q_1, \ldots, q_m$,

$$h(t) = h_1(t)q_1 + \cdots + h_m(t)q_m \neq 0, \tag{A15}$$

and it follows that

$$\int_0^T h^2(t)\,\mathrm{d}t = \int_0^T [h_1(t)q_1 + \cdots + h_m(t)q_m]^2\,\mathrm{d}t = \sum_{i,j=1}^m l_{ij}q_iq_j > 0, \qquad \text{if} \quad \sum_{i=1}^m q_i^2 \neq 0.$$

With the above mentioned propositions, the quadratic form

$$\Phi(q_1, \ldots, q_m) = \sum_{i,j=1}^m l_{ij}q_1q_j$$

of variables q_i is positive definite when $q_1^2 + \cdots + q_m^2 \neq 0$.

It is known that the coefficients l_{ij} of a positive definite form satisfy the Sylvester inequalities [12]:

$$l_{11} > 0, \quad \begin{vmatrix} l_{11} & l_{12} \\ l_{21} & l_{22} \end{vmatrix} > 0, \ldots, \quad \begin{vmatrix} l_{11} \ldots l_{1m} \\ \ldots\ldots \\ l_{m1} \ldots l_{mm} \end{vmatrix} = \Lambda\{l_{ij}\} > 0,$$

This proves the assertion about the determinant Λ of the set of equations (A13).

It will be shown now that the difference $\varphi(t) = u(t) - u^0(t)$ between the arbitrary solution of equalities (A1) and the optimal one (A12) is orthogonal to the function $u^0(t)$. Indeed, because $u(t)$ and $u^0(t)$ satisfy equalities (A1), it follows that

$$\int_0^T h_i(t)[u(t) - u^0(t)]\,\mathrm{d}t = \int_0^T h_i(t)\varphi(t)\,\mathrm{d}t = 0. \tag{A16}$$

Upon multiplying the equalities (A16) consecutively by q_i^0 and summing these relations up, it follows with respect to expression (A12) that

$$\int_0^T u^0(t)\varphi(t)\,\mathrm{d}t = 0, \tag{A17}$$

which proves the orthogonality of functions $u^0(t)$ and $\varphi(t)$.

Finally, it will be proved that (A12) has a minimum norm $\|u^0(t)\| = \int_0^T u^2(t)\,\mathrm{d}t$. Let $u(t) \neq u^0(t)$; then

$$\int_0^T \varphi^2(t)\,\mathrm{d}t > 0. \tag{A18}$$

Upon taking into consideration that $u(t) = u^0(t) + \varphi(t)$, it follows that

$$\|u(t)\|^2 = \int_0^T [u^0(t) + \varphi(t)]^2\,\mathrm{d}t = \int_0^T [u^0(t)]^2\,\mathrm{d}t + \int_0^T [\varphi(t)]^2\,\mathrm{d}t + 2\int_0^T u^0(t)\varphi(t)\,\mathrm{d}t. \tag{A19}$$

The last term on the right side of equation (19) is equal to zero according to relation (17); the second one is positive due to expression (18). So, $\|u(t)\|^2 > \|u^0(t)\|^2$ or $\|u(t)\| > \|u^0(t)\|$. Q.E.D.