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ICE-INDUCED NON-LINEAR VIBRATION OF AN OFFSHORE PLATFORM

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The non-linear behavior of ice-induced vibration of an offshore platform with four legs is investigated in this paper. The equations of motion of the system are derived by using the Hamiltonian Principle. The force of moving ice based on the self-excitation and locking is used to model the phenomenon of contact between the ice and the platform. By using the approach of multiple scales, the primary resonance of the ice-induced vibration of the platform is analyzed. The numerical results show that there exist several kinds of combination resonances, including self-excited vibration and locking vibration. These results coincide with those observed from an offshore platform in the North China Sea, and hence enable one to gain insight into the ice-induced vibration of offshore platforms.

1. INTRODUCTION

Offshore structures in ocean engineering are subject to a variety of loads such as ice, wave, earthquake and wind. It is important to predict the dynamics of an offshore structure on the basis of simplified mechanical models in the design phase. In practice, a complex offshore platform is usually modelled as a beam with a number of lumped masses and equivalent elastic components. Such a model has a great number of degrees of freedom and costs a lot in the computation of design phase. If any non-linearity is taken into account, even more efforts have to be made in the dynamic analysis. To reduce the computational cost in this case, it is essential to establish a simple, but proper model.

Before analyzing the interaction between ice and a structure, one must have a clear idea of the physical and mechanical properties of ice. Most of the early studies in this field were based on the work of Croasdale [1] and Michel and Toussaint [2]. The current theory can be used to estimate the maximal ice force in design. Although the computation of static ice force needs further study, the main concern is the dynamic effect of ice on the structure, namely, the relationship between the natural modes of the structure and the ice crack. Korzhavin [3] was the first to undertake the complex research on the ice strength under dynamic load. Peyton [4] found from the tests of pile-based platforms that the time history of the ice force looked like a saw and pointed out that during the resonance of ice-induced vibration of a structure its crash frequency was close to the resonant frequency of the structure. Matlock *et al*. [5] used a group of cantilever beams to simulate the evolution of ice crack. Blenkarn [6], according to Peyton's non-linear relations between the ice strength and loading velocity, established the theory of self-excited vibration. Toyama *et al*. [7, 8] observed the "locking" vibration and developed the mechanical model applicable to self-excited vibration of column-shaped structure. Haldar *et al*. [9] by taking into account the interaction among ice, water, soil, and an offshore structure, analyzed the

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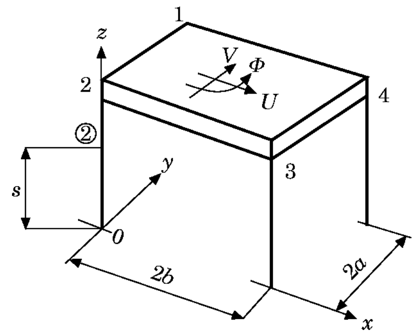

Figure 1. The model of platform.

tower frame using the finite element method and the stochastic approach, where the soil was described by the so-called p–y curve. Shi [10] investigated the relationship between the period of ice force and structural damping from the characteristics of ice force. Xu and Wang [11] carried out a theoretical and experimental study on ice-structure interaction.

The primary aim of this paper is to establish a simple, but useful mechanical model to describe the offshore platform with four legs and an ice load, and to make an analysis of the primary resonance, as well as other non-linear phenomena of the ice-induced vibration of the platform.

2. MODEL OF ICE–PLATFORM SYSTEM

As shown in Figure 1, the offshore platform of concern consists of a rigid plate and four elastic legs or piles. The plate at an arbitrary moment when the piles undergo deformation is illustrated in Figure 2. The displacements of the i-th pile at the joint with the plate in the x and y directions yield

$$U_i = U - r_{iv}\,\Phi, \qquad V_i = V + r_{iu}\,\Phi \qquad i = 1, 2, 3, 4, \tag{1}$$

where r_{iu} and r_{iv} are the co-ordinates of x and y at the ith joint.

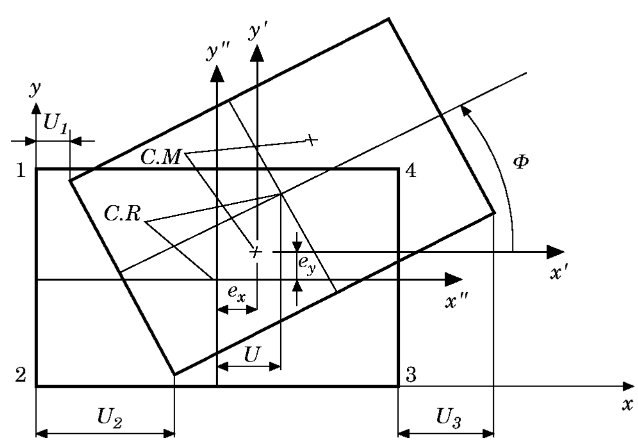

Figure 2. The position of the platform at an arbitrary instant, where CM is the center of mass, CR is the geometric center.

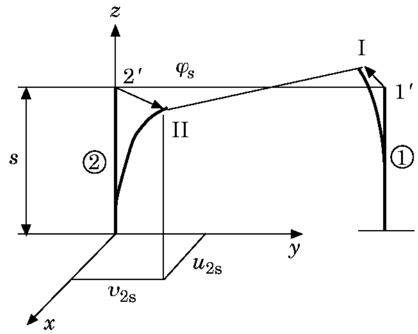

Figure 3. The deformation of piles ① and ②.

Figure 3 shows the geometric relations of piles ① and ② before and after deformation. According to equation (1), one has the displacement relations of all piles

$$u_{1,4} = u + (r_{2v} - r_{1v})\Phi, \qquad v_1 = v, \qquad u_3 = u, \qquad v_{3,4} = v + (r_{4u} - r_{1u})\Phi, \tag{2}$$

where $u_2 = u$ and $v_2 = v$. The local deformation of pile ② is illustrated in Figure 4. The local co-ordinates before and after deformation are $M\hat{x}\hat{y}\hat{z}$ and $M^*\hat{\xi}\hat{\eta}\hat{\zeta}$, respectively. M and M^* are the center of mass of the cross-section.

If the axial displacement of a pile is neglected, the Hamiltonian quantity of the ith pile reads

$$l_i = \tfrac{1}{2} m_i (\dot{u}_i^2 + \dot{v}_i^2) + \tfrac{1}{2} (j_{\xi i}\, \omega_{\xi i}^2 + j_{\eta i}\, \omega_{\eta i}^2 + j_{\zeta i}\, \omega_{\zeta i}^2) - \tfrac{1}{2} (D_{\xi i}\, \rho_{\xi i}^2 + D_{\xi i}\, \rho_{\xi i}^2 + D_{\xi i}\, \rho_{\xi i}^2), \tag{3}$$

where m_i is the mass per unit length of the ith pile. $j_{\xi i}, j_{\eta i}, j_{\zeta i}$ and $D_{\xi i}$, $D_{\eta i}$, $D_{\zeta i}$ are the inertial moments and the torsional stiffness coefficients around $\hat{\xi}$, $\hat{\eta}$, $\hat{\zeta}$, respectively. $\omega_{\xi i}$, $\omega_{\eta i}$, $\omega_{\zeta i}$ and $\rho_{\xi i}$, $\rho_{\eta i}$, $\rho_{\zeta i}$ are three angular velocity components and the components of curvature, respectively [12]. Then, it is easy to write out the total Hamiltonian quantity of the system

$$l = \int_{s=0}^{L} \sum_{i=1}^{4} l_i \, \mathrm{d}s + \frac{M}{2} (\dot{U}_{C,M}^2 + \dot{V}_{C,M}^2) + \frac{J_c}{2} \dot{\Phi}^2. \tag{4}$$

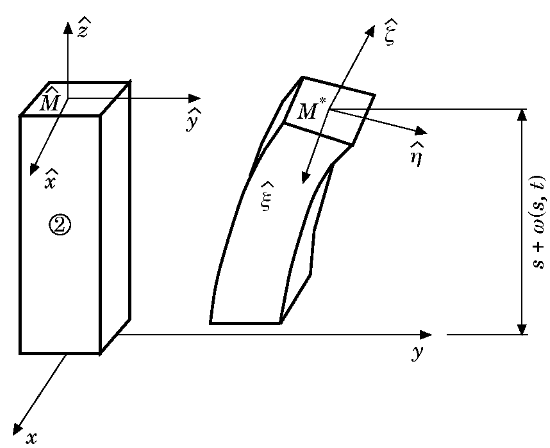

Figure 4. The local deformation of ② pile.

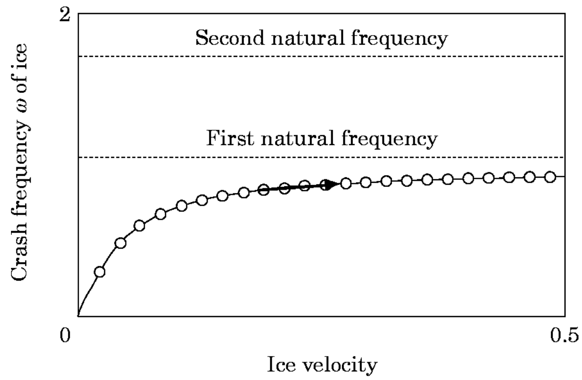

Figure 5. Crash frequency of ice locked to the first natural frequency.

It is assumed that the force of moving ice is in the y direction and acts at a point p on pile ②. From the Hamiltonian Principle, one has

$$\delta I = \int_{t_1}^{t_2} \left\{ \delta l + \sum_{i=1}^{4} \left[\int_0^L (Q_{ui}\,\delta u_i + Q_{vi}\,\delta v_i + Q_{\varphi i}\,\delta\varphi_i)\,\mathrm{d}s + \delta W_{Bi} \right] + Q_{ice}\,\delta v_p \right\} \mathrm{d}t = 0, \quad (5)$$

where M represents the mass of the platform, J_c the inertial moment around CR, $Q_{\alpha i}$ ($\alpha = u, v, \varphi$) the generalized force of the pile corresponding to virtual displacement $\delta\alpha$, W_{Bi} the virtual work of the reactive forces at the boundary ($s = 0, L$), v_p the displacements of point p, $U_{C,M}$ and $V_{C,M}$ the displacements of the platform center, respectively.

The force of moving ice yields [13]

$$Q_{ice} = B(a_1\,\dot{\epsilon} + a_2\,\dot{\epsilon}^2 + a_3\,\dot{\epsilon}^3)\sin\omega t, \quad (6)$$

where $\dot{\epsilon} = \overline{V}_0/4D$, D is the diameter of the pile section where the ice force acts. $\overline{V}_0$ is the relative velocity of ice with respect to the platform. a_1, a_2, a_3 are the optimal fitting coefficients and B is the test constant of ice force. ω is the crash frequency of ice and is a non-linear function of ice velocity as shown in Figure 5. Usually the crash frequency is locked to the natural frequency of translation vibration.

In the North China Sea, ice is the predominant load on an offshore platform. The ice-induced vibration of the platform severely affects the work environment on the platform and even threatens its safety. From a practical viewpoint, two assumptions shall be used throughout this paper as follows: (a) Each pile is a cylindrical column with the same size, and made of the same material. (b) The piles are so thin that the inertial moments of all piles can be neglected.

Let

$$u = (\tilde{u} - r\varphi)/2, \qquad v = (\tilde{v} - R\varphi)/2, \quad (7)$$

where $r = r_{2v} - r_{1v}$, $R = r_{4u} - r_{1u}$. By substituting equation (7) into equation (5), after a lengthy algebraic manipulation, one obtains a set of non-linear partial differential equations in unknowns $\tilde{u}$, $\tilde{v}$ and φ. Then, using u, v and φ to express $\tilde{u}$, $\tilde{v}$ and φ, one can derive the following equations

$$\begin{aligned}(M\delta(s-L)/4 + m)\ddot{u} - M\delta(s-L)\,(r - 2r_{ev})/4\ddot{\varphi} + C_u\,\dot{u} + D_\eta\,u''''\\ = \{-D_\varsigma\,(\varphi' + u''v'/4)v'' - (D_\eta/4)\,(u''^2 + v''^2 + R_0^2\,\varphi''^2)u'\\ - \varphi'\varphi''u''(D_\eta\,r^2/2 + D_\varsigma\,R^2/4)\}', \quad (8)\end{aligned}$$

$$(M\delta(s-L)/4+m)\ddot{v}-M\delta(s-L)\,(R+2r_{eu})/4\ddot{\varphi}+C_v\,\dot{v}+D_\eta\,v''''$$
$$=\{-D_\varsigma\,\varphi'u''+[(r^2D_\varsigma-R_0^2\,D_\eta)/4]\varphi''^2v'+[(D_\varsigma-D_\eta)/4]u''^2v'$$
$$-D_\eta\,(v'v''/4+R^2\varphi'\varphi''/2)v''\}'+B_0\sin\omega t, \tag{9}$$

$$(J_0\,\delta(s-L)+mR_0^2)\ddot{\varphi}+C_\gamma\,\dot{\varphi}+D_\eta\,R_0^2\,\varphi''''=M[(r-2_{ev})\ddot{u}+(R+2r_{eu})\ddot{v}]\delta(s-L)/4$$
$$+\{D_\varsigma\,(4\varphi'+u''v'-r^2v'v''\varphi''/2)-(D_\eta\,/2)\,[(r^2u'u''+R^2v'v'')\varphi''$$
$$+(R_0^2\,\varphi'/2)\,(v''^2+R_0^2\,\varphi''^2)]+(\varphi'u''^2/4)\,(R^2D_\varsigma-R_0^2\,D_\eta)\}'+B_0\,(2r_{px}-R)\sin\omega t, \tag{10}$$

where

$$B_0=(B/2)\,[a_1+a_2\,\dot{\epsilon}+a_2\,\dot{\epsilon}^2]\dot{\epsilon}\delta(s-L),\qquad r_{eu}=r_{2u}-e_x,\qquad r_{ev}=r_{2v}-e_y,$$
$$r_{px}=r_{pu}-r_{2u},\qquad J_0=J_c+MR_0^2\,/4+M(r_{eu}^2+r_{ev}^2+Rr_{eu}-rr_{ev}),$$
$$C_\gamma=4C_\varphi+C_u\,r^2+C_v\,R^2,\qquad R_0=\sqrt{R^2+r^2}.$$

$\delta(s)$ is the Dirac function. $\dot{v}_p$ is the velocity of the platform at the ice acting position p and r_{pu} is the co-ordinate of point p in the x'' direction away from CR. The non-linear equations (8), (9) and (10) describe the dynamics of the complex ice-platform system.

3. FREE VIBRATION

In equations (8), (9) and (10), u and v are coupled. So are the translation modes, whose dimensionless expression reads

$$F_{u,v}=C_1\,(1-\cos\pi s). \tag{11}$$

In what follows, the torsional modes will be determined. One first defines a set of dimensionless parameters $s^*=s/L$, $u^*=u/L$, $v^*=v/L$, $t^*=\omega_u\,t$, where, $\omega_u=\sqrt{\alpha_0\,D_\eta\,/m_u\,L^3}$, $m_u=\frac{1}{4}\,F_u^2\,(1)M+mL$. Then, one has a dimensionless equation of free vibration from equation (10)

$$[J_0\,\delta(s-L)/L+mR_0^2]\ddot{\varphi}+R_0^2\,D_\eta\,/L^4\varphi''''-4D_\varsigma\,/L^2\varphi''=0. \tag{12}$$

Assuming $\varphi(s,t)=F_\varphi\,(s)\exp\,(\mathrm{i}\omega_\gamma\,t)$ and substituting into equation (12), one obtains

$$F_\varphi''''-[\alpha_0\,L\omega_\gamma^2+k_2\,\delta(s-L)]F_\varphi-k_3^2\,F_\varphi''=0, \tag{13}$$

where $k_2=\alpha_0\,J_0\,\omega_\gamma^2\,/mR_0^2\,D_\eta$, $k_3=\sqrt{4D_\varsigma\,L^2/R_0^2\,D_\eta}$, and $F_\varphi\,(s)$ is the torsional mode shape of the pile. By using the boundary conditions $F_\varphi\,(0)=F_\varphi''\,(0)=0$, one obtains the Laplace transform of equation (13). It can be proved that the inverse Laplace transform yields

$$F_\varphi\,(s)=\frac{F_\varphi'\,(0)}{a^2+b^2}\left(\frac{b^2-k_3^2}{b}\sinh bs+\frac{a^2+k_3^2}{a}\sin as\right)+\frac{F_\varphi'''\,(0)}{a^2+b^2}\left(\frac{1}{b}\sinh bs-\frac{1}{a}\sin as\right)$$
$$+\frac{k_2\,F_\varphi\,(1)}{a^2+b^2}\left[\frac{1}{b}\sinh b(s-1)-\frac{1}{a}\sin a(s-1)\right]u(s-1), \tag{14}$$

where $u(s-1)$ is the unit step function and $a=\sqrt{\alpha_0\,L\omega_\gamma^2+k_3^4\,/4-k_3^2\,/2}$, $b=\sqrt{\alpha_0\,L\omega_\gamma^2+k_3^4\,/4+k_3^2\,/2}$.

According to the boundary conditions, one has $F'''_\varphi(1) = 0$ and $F'_\varphi(1) = 0$ if $J_0 = 0$. Therefore, two homogeneous equations are obtained expressed in $F'_\varphi(0)$ and $F'''_\varphi(0)$. From the condition of non-zero solution one has a characteristic equation, that is, the equation of natural frequency

$$k_2(\text{th}\, b/b - \text{tg}\, a/a) + (a^2 + b^2) = 0. \tag{15}$$

The torsional mode shapes can be evaluated from equation (14) with the solved natural frequencies.

4. PERTURBATION ANALYSIS

A set of dimensionless variable and parameters is first defined:

$$\omega_v = \sqrt{D_v/m_v L^3}, \qquad C^*_u = C_u L/m_u\omega_u, \qquad C^*_v = C_v L/m_v\omega_v, \qquad C^*_\gamma = C_\gamma L/J\omega_u,$$
$$\Omega = \omega/\omega_u,$$

where

$$D_v = \alpha_0^- D_\eta, \; m_v = 1/4F_v^2(1)M + mL, \qquad J = F_\varphi^2(1)/LJ_0 + mR_0^2.$$

Then $u_t(s, t)$, $v_t(s, t)$ and $\varphi_t(s, t)$ is approximated by $u^* = F_u(s)u_t(t)$, $v^* = F_v(s)v_t(t)$ and $\varphi^* = F_\varphi(s)\varphi_t(t)$ according to the Galerkin approach. Substituting them into equations (8), (9) and (10), multiplying the results by the corresponding mode shapes and integrating them from 0 to 1, one obtains the desired ordinary differential equations:

$$\ddot{u}_t + C_u\dot{u}_t + u_t = \alpha_{u1}\ddot{\varphi}_t + \alpha_{u2}\varphi_t v_t + \alpha_{u3}u_t^3 + \alpha_{u4}u_t v_t^2 + \alpha_{u5}u_t\varphi_t^2,$$
$$\ddot{v}_t + C_v\dot{v}_t + \omega_p^2 v_t = \alpha_{v1}\ddot{\varphi}_t + \alpha_{v2}\varphi_t u_t + \alpha_{v3}v_t^3 + \alpha_{v4}v_t u_t^2 + \alpha_{v5}v_t\varphi_t^2$$
$$+ f_v(1 + q_2\Theta + q_3\Theta^2)\Theta \sin\Omega t,$$
$$\ddot{\varphi}_t + C_\gamma\dot{\varphi}_t + \omega_n^2\varphi_t = \alpha_{\varphi 0}\ddot{u}_t + \alpha_{\varphi 1}\ddot{v}_t + \alpha_{\varphi 2}u_t v_t + \alpha_{\varphi 3}\varphi_t^3 + \alpha_{\varphi 4}\varphi_t v_t^2 + \alpha_{\varphi 5}\varphi_t u_t^2$$
$$+ f_\varphi(1 + q_2\Theta + q_3\Theta^2)\Theta \sin\Omega t, \tag{16}$$

where

$$\Theta = 1 - q_v\dot{v}_t - q_\varphi\dot{\varphi}_t, \qquad \omega_p = \omega_v/\omega_u, \qquad \omega_n = \bar{\omega}_\gamma/\omega_u, \qquad f_v = a_1 BL^2\lambda V_0 F_v(1)/8DD_u,$$
$$q_v = \omega_u LF_v^2(1)/2V_0, \qquad f_\varphi = [a_1 BL(2r_{px} - R)\bar{\lambda}V_0 F_\varphi(1)]/8DD_u,$$
$$q_\varphi = [\omega_u[2r_{px}F_\varphi(1) - RF_v(1)]F_\varphi(1)]/2V_0, \qquad \lambda = m_u/m_v, \qquad \bar{\lambda} = m_u L/J,$$
$$q_2 = a_2 V_0/4a_1 D, \qquad q_3 = a_3 V_0^2/16a_1 D.$$

V_0 is the velocity of ice. The Galerkin coefficients $\alpha_{\beta i}$ ($\beta = u, v, \varphi$, $i = 1, 2, \ldots, 5$) are given in Appendix A.

To analyze the motion governed by the above coupled non-linear ordinary differential equations, the approach of multiples scales will be used. One defines three time scales $T_0 = t$, $T_1 = \epsilon t$, $T_2 = \epsilon^2 t$, and expands u_t, v_t and φ_t in terms of ϵ as

$$\alpha_t(T_0, T_1, T_2) = \alpha_{t1}(T_0, T_1, T_2) + \epsilon\alpha_{t2}(T_0, T_1, T_2) + \epsilon^2\alpha_{t3}(T_0, T_1, T_2) + \cdots. \tag{17}$$

By substituting equation (17) into equation (16) and equating the same powers of ϵ, one has

$$\mathrm{D}_0^2 u_{t1} = u_{t1} = 0, \qquad u_{t1} = A_u\cos[T_0 + B_u(T_1, T_2)] = A_u\cos\varphi_u,$$
$$\mathrm{D}_0^2 v_{t1} + \omega_p^2 v_{t1} = 0, \qquad v_{t1} = A_v\cos[\omega_p T_0 + B_v(T_1, T_2)] = A_v\cos\varphi_v,$$
$$\mathrm{D}_0^2 \varphi_{t1} + \omega_n^2\varphi_{t1} = 0, \qquad \varphi_{t1} = A_\gamma\cos[\omega_n T_0 + B_\gamma(T_1, T_2)] = A_\gamma\cos\varphi_\gamma, \tag{18}$$

$$D_0^2 u_{t2} + u_{t2} = 2[(D_1 A_u) \sin \varphi_u + A_u (D_1 B_u) \cos \varphi_u] + \alpha_{u2} A_v A_\gamma \cos \varphi_\gamma \cos \varphi_v,$$

$$D_0^2 v_{t2} + \omega_p^2 v_{t2} = 2\omega_p [(D_1 A_v) \sin \varphi_v + A_v (D_1 B_v) \cos \varphi_v] + \alpha_{v2} A_u A_\gamma \cos \varphi_\gamma \cos \varphi_u,$$

$$D_0^2 \varphi_{t2} + \omega_n^2 \varphi_{t2} = 2\omega_n [(D_1 A_\gamma) \sin \varphi_\gamma + A_\gamma (D_1 B_\gamma) \cos \varphi_\gamma] + \alpha_{\varphi 2} A_u A_v \cos \varphi_u \cos \varphi_v, \quad (19)$$

where $D_i = \partial / T_i$ $(i = 0, 1, 2)$ are the differential operators. The solutions of equation (19) are:

$$D_1 A_\alpha = 0 \qquad D_1 B_\alpha = 0 \ (\alpha = u, v, \gamma),$$

$$u_{t2} = (\alpha_{u2}/2) [\cos (\varphi_\gamma + \varphi_v)/[1 - (\omega_n + \omega_p)^2] + \cos (\varphi_\gamma - \varphi_v)/[1 - (\omega_n - \omega_p)^2]] A_v A_\gamma,$$

$$v_{t2} = (\alpha_{v2}/2) [\cos (\varphi_\gamma + \varphi_u)/[\omega_p^2 - (1 - \omega_n)^2] + \cos (\varphi_\gamma - \varphi_u)/[\omega_p^2 - (1 - \omega_n)^2]] A_u A_\gamma,$$

$$\varphi_{t2} = (\alpha_{\varphi 2}/2) [\cos (\varphi_u + \varphi_v)/[\omega_n^2 - (1 + \omega_p)^2] + \cos (\varphi_u - \varphi_v)/[\omega_n^2 - (1 - \omega_p)^2]] A_u A_v. \quad (20)$$

It is obvious that the system may undergo the combination resonances when $1 \approx |\omega_n \pm \omega_p|$, $\omega_p \approx |\omega_n \pm 1|$ and $\omega_n \approx |\omega_p \pm 1|$. The following analysis is confined to the case when the excitation frequency Ω is near ω_p, one of the natural frequencies, and all natural frequencies are closed to each other, i.e., $\omega_n = 1$. In this case, the platform exhibits a non-linear primary resonance. Keeping the primary resonance term, one has

$$\begin{aligned} D_0^2 u_{t3} + u_{t3} = {} & -\alpha_{u1} A_\gamma \omega_n^2 \cos \varphi_\gamma + (C_u A_u + 2D_2 A_u) \sin \varphi_u + k_4 A_u A_v^2 \cos (\varphi_u - 2\varphi_v) \\ & + k_5 A_u A_\gamma^2 \cos (\varphi_u - 2\varphi_\gamma) + (2D_2 B_u + k_1 A_v^2 + k_2 A_\gamma^2 + k_3 A_u^2) A_u \cos \varphi_u, \\ D_0^2 v_{t3} + \omega_p^2 v_{t3} = {} & -\alpha_{v1} A_\gamma \omega_n^2 \cos \varphi_\gamma + \omega_p (C_v A_v + 2D_2 A_v) \sin \varphi_v + k_9 A_v A_u^2 \cos (\varphi_v - 2\varphi_u) \\ & + k_{10} A_v A_\gamma^2 \cos (\varphi_v - 2\varphi_\gamma) + (2\omega_p D_2 B_v + k_6 A_u^2 + k_7 A_\gamma^2 + k_8 A_v^2) A_v \cos \varphi_v \\ & + f_v f_g \sin \Omega T_0, \\ D_0^2 \varphi_{t3} + \omega_n^2 \varphi_{t3} = {} & -\alpha_{\varphi 0} A_u \cos \varphi_u - \alpha_{\varphi 1} A_v \omega_p^2 \cos \varphi_v + \omega_n (C_\gamma A_\gamma + 2D_2 A_\gamma) \sin \varphi_\gamma \\ & + k_{14} A_\gamma A_u^2 \cos (\varphi_\gamma - 2\varphi_u) + k_{15} A_\gamma A_v^2 \cos (\varphi_\gamma - 2\varphi_v) \\ & + (2\omega_n D_2 B_\gamma + k_{11} A_u^2 + k_{12} A_v^2 + k_{13} A_\gamma^2) A_\gamma \cos \varphi_\gamma + f_\varphi f_g \sin \Omega T_0, \end{aligned} \quad (21)$$

where

$$\begin{aligned} f_g = {} & 1 + q_2 + q_3 + [(q_2 + 3q_3)/2] (4 q_v q_\varphi A_v A_\gamma \omega_p \omega_n \sin \varphi_v \sin \varphi_\gamma - (q_v A_v \omega_p)^2 \cos 2\varphi_v \\ & - (q_\varphi A_\gamma \omega_n)^2 \cos \varphi_\gamma). \end{aligned}$$

The coefficients $k_1, k_2, \ldots, k_{15}$ are listed in Appendix B.

Now the case of no mass eccentricity in the y direction is considered, i.e., $A_u = 0$. This implies an observable fact that the actual displacement in the x direction is only concerned with φ and r. To study the steady state vibration, let $\Omega \approx \omega_p + \epsilon^2 \sigma_2$ and $\omega_p \approx \omega_n (1 + \epsilon^2 \Delta_2)$, where σ_2 and Δ_2 are detuning parameters. Then, one defines the detuning parameters as

$$\mu_u = \Delta_2 \omega_n T_2 + (B_u - B_\gamma), \qquad \mu_v = \Delta_2 \omega_n T_2 + (B_v - B_\gamma), \qquad \varphi_f = \omega_p \sigma_2 T_2 - B_v. \quad (22)$$

By assuming that the solution of a steady state vibration is $A_{\alpha e}$ and $B_{\alpha e}$ $(\alpha = v, \gamma)$. After carrying out the lengthy algebraic manipulation, one obtains the results of $\sin u_{ve}$, $\cos \mu_{ve}$

TABLE 1

Galerkin's coefficients

i	1	2	3	4	5
α_{vi}	$-0{\cdot}0267134$	-0.00174076	$0{\cdot}411246$	$0{\cdot}408051$	$0{\cdot}279233$
$\alpha_{\varphi i}$	$-1{\cdot}17533$	$-0{\cdot}0765605$	$-27{\cdot}9026$	$1{\cdot}52952$	$25{\cdot}7851$

and $\sin \varphi_{fe}$, $\cos \varphi_{fe}$. Noting that $\sin^2 u_{ve} + \cos^2 \mu_{ve} = 1$ and $\sin^2 \varphi_{fe} + \cos^2 \varphi_{fe} = 1$, one has two non-linear algebraic equations that govern the steady state solution

$$(a_{c1}\, a_{c3} - a_{c2}\, a_{c4})^2 + (a_{c1}\, a_{c5} + a_{c3}\, a_{c4})^2 - (a_{c3}^2 + a_{c2}\, a_{c5})^2 = 0,$$

$$(a_{e1}\, a_{e3} - a_{e2}\, a_{e4})^2 + (a_{e1}\, a_{e5} + a_{e3}\, a_{e4})^2 - (a_{e3}^2 + a_{e2}\, a_{e5})^2 = 0, \qquad (23)$$

where the parameters $a_{c1}, a_{c2}, \ldots, a_{e4}, a_{e5}$ are listed in Appendix C. The stability of the steady state vibration may be ascertained by perturbing it to $\bar{x}(t) = \bar{x}_e + \bar{x}_s$, where $\bar{x} = [A_v, A_\gamma, \mu_v, \varphi_f]$. The stability of perturbed motion can be determined by applying the Routh–Hurwitz criterion to the differential equation $\dot{\bar{x}}_s = A\bar{x}_s$.

5. NUMERICAL RESULTS

To study the locking behavior of ice-induced vibration of a platform, a set of parameters was taken as follows. The thickness of ice was 0·02 m and the elastic modulus was 3·0 MPa. The length of the pile was 2 m, the outside diameter of the pile cross-section was 0·04 m, the thickness of the pile cross-section was 0·0025 m, the Young's modulus was 206 GPa, the mass of platform was 72 kg, the eccentricity of the mass center in the x direction was 0·6 m, and $a = b = 0{\cdot}75$ m. The dimensionless damping coefficients were set as $C_v = 0{\cdot}61897176\text{E-}04$ and $C_\gamma = 0{\cdot}57870809\text{E-}02$. The Galerkin's coefficients calculated from these parameters are listed in Table 1.

In the computation of ice-induced vibration, the ice force was supposed to act at point p on a pile in the y direction. The x and y co-ordinates of point p were $r_{pu} = -0{\cdot}75$ m and $r_{pv} = -0{\cdot}75$ respectively. The computed relationship between the natural frequencies of platform and the ice velocity is shown in Figure 6. One can see from Figure 6 that when ice velocity V_0 increases, the crash frequency of ice and the first natural frequency ω_p of platform come to the state of "locking". In this case, the platform undergoes primary resonance.

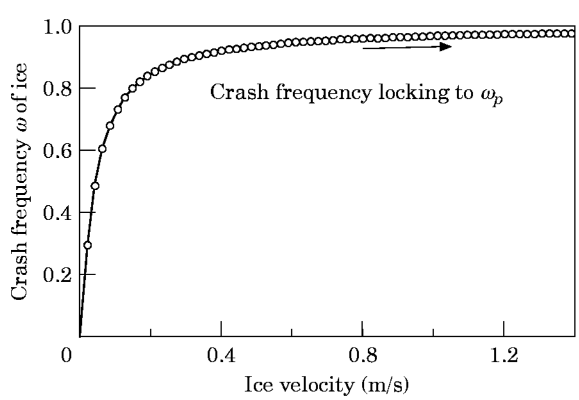

Figure 6. The locking response of platform.

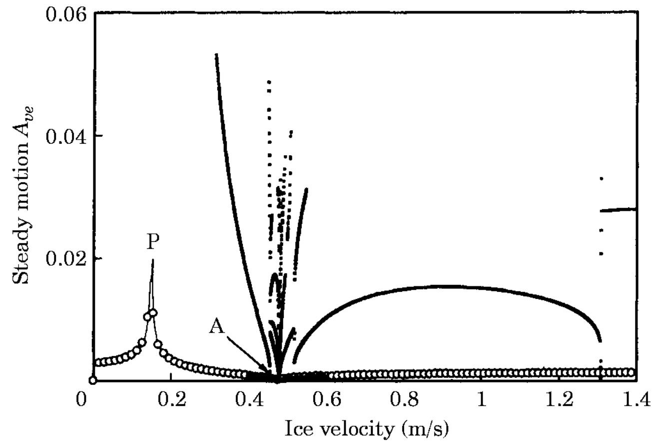

Figure 7. Translational response of ice-induced vibration. Key: · · ·, stable; —○—, unstable.

Figures 7 and 8 show that if the ice velocity was within 60 cm/s, the translational and rotational vibrations of the platform would undergo a jump from point A to B in the figures. As reported by Shang *et al.* [14], the SP-62C deep water jacket platform began to vibrate at the ice velocity of 10 cm/s, and then vibrated intensely. One can see from Figures 7 and 8 that the ice velocity corresponding to the maximal response (point P) was about 15 cm/s. Hence, the computational results coincide well with the experimental ones. As the translational and torsional displacements are coupled, the platform would jump twice, namely, from point P to A and from point A to B with the increase of ice velocity. The numerical results accord with the observed phenomenon of the ice-induced vibration of platform in the North China Sea. As shown in Figure 8, a jump occurred at point C when ice velocity decreased.

Figure 9 shows the vibration amplitude and locking frequency versus the ice velocity. It is obvious that when the vibration reached a peak with an increase in ice velocity, so did the curvature of the locking frequency curve. It can be deduced that the platform underwent a transition from self-excited vibration to frequency-locked vibration at a lower ice velocity. At a higher ice velocity, the platform vibration did not undergo any jump.

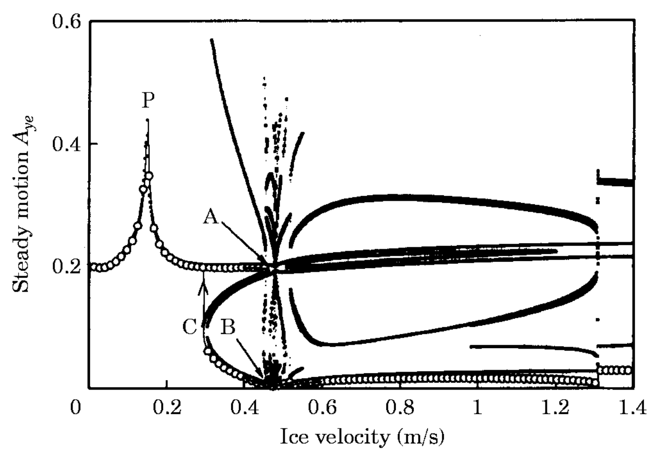

Figure 8. Rotational response of ice-induced vibration. Key as for Figure 7.

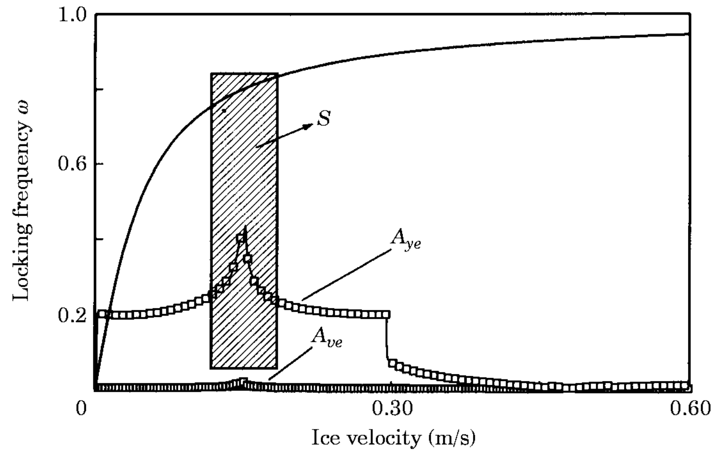

Figure 9. Vibration amplitude and crash frequency of ice versus ice velocity.

6. CONCLUSIONS

An offshore platform and its surrounding environment constitutes a complex non-linear system. To gain insight into the complicated dynamics of the system, a simple, but useful mechanical model is presented in this paper. The model shows that the non-linear behavior of ice-induced vibration of a platform depends not only on the ice-force parameters, but also on the parameters of the structure itself. The computation based on this model indicated that the strong vibration of the platform would occur at a specific ice velocity, which was very close to the observed dangerous velocity for SP-62C deep water jacket platform in the North China Sea.

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REFERENCES

1. K. R. Croasdale 1970 *International Association of Hydraulic Research Symposium on Ice Problems. Reykjavik, Iceland.* The nutcracker ice strength tester and its operation in the Beaufort sea.
2. B. Michel and N. Toussaint 1977 *Journal of Glaciology* **19**, 285–301. Mechanisms and theory of indentation of ice plates.
3. K. N. Korzhavin 1971 *Novosibirsk, Akad. Nauka USSR* (English translation, CRREL, Hanover, N.H.), Action of ice on engineering structures.
4. H. R. Peyton 1966 *Proceedings of the Conference in Ice Pressure Against Structures. Lavial University, Quebec, November 10–11.* Sea ice forces.
5. H. Matlock, W. P. Dawkins and J. J. Panak 1971 *Journal of Engineering Mechanics* **97**, 1082–1092. Analytical model for ice-structure interaction.
6. K. A. Blenkarn 1970 *Second Annual Offshore Technology Conference, Houston, Texas, April 22–24.* 365–378. Measurement and analysis of ice forces on Cook inlet structures.
7. Y. Toyama, T. Sensu, M. Minami and N. Yashima 1983 *Seventh International Conference on Port and Ocean Engineering under Arctic Conditions. Helsinki, Finland, Vol.* 3. 834–843. Model tests on ice-induced self-excited vibration of cylindrical structures.
8. Y. Toyama and N. Yashima 1985 *Ninth International Conference on Port and Ocean Engineering under Arctic Conditions. Hanover, New Hampshire.* 677–688. Dynamic response of moored conical structures to a moving ice sheet.

9. A. K. HAIDAR, A. S. J. SWAMIDAS, D. V. REDDY and M. AROCKIASAMY 1977 *Ninth Annual Offshore Technology Conference. Houston, Texas, May* 2–5, 225–234 Dynamic ice-water-soil-structure interaction of offshore towers including nonlinear soil behavior.
10. L. Y. SHIH 1991 *Applied Mathematical Modeling* **115**, 632–638. Analysis of ice-induced vibrations on a flexible structure.
11. J. Z. XU and L. Y. WAN 1986 *Proceedings of International Towing Tank Conference, Cambridge University, Mass, USA*. The ice force oscillator model for dynamic ice-structure interaction analysis.
12. M. R. M. CRESPO DA SILVA and C. C. GLYNN 1978 *Journal of Structural Mechanics* **6**, 437–448. Nonlinear flexural-flexural-torsional dynamics of in extensional beams. I: Equations of motion.
13. D. P. JIN and Y. S. CHEN 1997 *Acta Oceanologica Sinica* **19**, 107–114. Nonlinear analysis of ice-induced vibrations of structures.
14. J. Q. SHANG, L. D. YANG and W. X. SHAN 1990 *Journal of Vibration and Shock*. **33**, 22–36. Modal test and calculation of Jackt's offshore platform model.

APPENDIX A

$$\alpha_0 = \int_0^1 F_u F_u'''' \,\mathrm{d}s, \qquad \alpha_{u1} = \frac{(r-2r_{ev})F_u(1)F_\varphi(1)}{(F_u^2(1)+4mL/M)L}, \qquad \alpha_{u2} = -\beta_\varsigma \int_0^1 F_u (F_v'' F_\varphi')\,\mathrm{d}s,$$

$$\alpha_{u3} = -\frac{\beta_\eta}{4}\int_0^1 F_u (F_u' F_u''^2)'\,\mathrm{d}s, \qquad \alpha_{u4} = -\tfrac{1}{4}\left[\beta_\varsigma \int_0^1 F_u (F_u'' F_v' F_v'')'\,\mathrm{d}s + \beta_\eta \int_0^1 F_u (F_u' F_u''^2)'\,\mathrm{d}s\right],$$

$$\alpha_{u5} = -\frac{1}{2L^2}\left[\left(\beta_\eta r^2 + \beta_\varsigma \frac{R^2}{2}\right)\int_0^1 F_u (F_u'' F_\varphi' F_\varphi'')'\,\mathrm{d}s + \beta_\eta \frac{R^2+r^2}{2}\int_0^1 F_u (F_u' F_\varphi''^2)'\,\mathrm{d}s\right],$$

$$\alpha_{\bar{0}} = \int_0^1 F_v F_v'''' \,\mathrm{d}s, \qquad \alpha_{v1} = \frac{(R+2r_{eu})F_v(1)F_\varphi(1)}{(F_v^2(1)+4mL/M)L}, \qquad \alpha_{v2} = \lambda\beta_\varsigma \int_0^1 F_v (F_u'' F_\varphi')'\,\mathrm{d}s,$$

$$\alpha_{v3} = -\frac{\lambda\beta_\eta}{4}\int_0^1 F_v (F_v' F_v''^2)'\,\mathrm{d}s, \qquad \alpha_{v4} = \frac{\lambda(\beta_\varsigma - \beta_\eta)}{4}\int_0^1 F_v (F_u''^2 F_v')'\,\mathrm{d}s,$$

$$\alpha_{v5} = \frac{\lambda}{2L^2}\left[\frac{\beta_\varsigma r^2 - (R^2+r^2)\beta_\eta}{2}\int_0^1 F_v (F_v' F_\varphi''^2)'\,\mathrm{d}s - \beta_\eta R^2 \int_0^1 F_v (F_v'' F_\varphi' F_\varphi'')'\,\mathrm{d}s\right],$$

$$\alpha_{\varphi 0} = \frac{M(r-r_{ev})F_u(1)F_\varphi(1)}{2J}, \qquad \alpha_{\varphi 1} = \frac{M(R+r_{eu})F_v(1)F_\varphi(1)}{2J}$$

$$\alpha_{\varphi 2} = \bar{\lambda}\beta_\varsigma \int_0^1 F_\varphi (F_u'' F_v')'\,\mathrm{d}s, \qquad \alpha_{\varphi 3} = \frac{(R^2+r^2)^2\bar{\lambda}\beta_\eta}{4L^4}\int_0^1 F_\varphi (F_\varphi' F_\varphi''^2)'\,\mathrm{d}s,$$

$$\alpha_{\varphi 4} = -\frac{\bar{\lambda}}{4L^2}\left[(2R^2\beta_\eta + r^2\beta_\varsigma)\int_0^1 F_\varphi (F_\varphi'' F_v' F_v'')'\,\mathrm{d}s + (R^2+r^2)\beta_\eta \int_0^1 F_\varphi (F_\varphi' F_v''^2)'\,\mathrm{d}s\right],$$

$$\alpha_{\varphi 5} = \frac{\bar{\lambda}}{4L^2}\left[(R^2\beta_\varsigma - (R^2+r^2)\beta_\eta)\int_0^1 F_\varphi (F_\varphi' F_u''^2)'\,\mathrm{d}s - 2r^2\beta_\eta \int_0^1 F_\varphi (F_\varphi'' F_u' F_u'')'\,\mathrm{d}s\right],$$

where

$$\beta_\eta = 1/\alpha_0. \qquad \beta_\varsigma = D_\varsigma/\alpha_0 D_\eta.$$

APPENDIX B

$$k_1 = \alpha_{u4}/2 + \alpha_{u2}\,\alpha_{\varphi 2}/6, \qquad k_2 = \alpha_{u5}/2 + \alpha_{u2}\,a_{v2}/6, \qquad k_3 = \tfrac{3}{4}\alpha_{u3}, \qquad k_4 = \alpha_{u4}/4 + \alpha_{u2}\,\alpha_{\varphi 2}/4,$$

$$k_5 = \alpha_{u5}/4 + \alpha_{u2}\,\alpha_{v2}/4, \qquad k_6 = \alpha_{v4}/2 + \alpha_{v2}\,\alpha_{\varphi 2}/6, \qquad k_7 = \alpha_{v5}/2 + \alpha_{u2}\,\alpha_{v2}/6,$$

$$k_8 = \tfrac{3}{4}\alpha_{v3}, \qquad k_9 = \alpha_{v4}/4 + \alpha_{v2}\,\alpha_{\varphi 2}/4, \qquad k_{10} = \alpha_{v5}/4 + \alpha_{u2}/\alpha_{v2}/4,$$

$$k_{11} = \alpha_{\varphi 5}/2 + \alpha_{v2}\,\alpha_{\varphi 2}/6, \qquad k_{12} = \alpha_{\varphi 4}/2 + \alpha_{u2}\,\alpha_{\varphi 2}/6, \qquad k_{13} = \tfrac{3}{4}\alpha_{\varphi 3},$$

$$k_{14} = \alpha_{\varphi 5}/4 + \alpha_{v2}\,\alpha_{\varphi 2}/4, \qquad k_{15} = \alpha_{\varphi 4}/4 + \alpha_{u2}\,\alpha_{\varphi 2}/4.$$

APPENDIX C

$$a_{c1} = (a_{\gamma v} - b_v\,\beta_s)k_{15}\,A_{ve} + (b_\gamma - \alpha_v\,\beta_s)\beta_s\,k_{10}\,A_{\gamma e},$$

$$a_{c2} = (a_{\gamma v} - b_v\,\beta_s)\beta_s\,k_{10}\,A_{\gamma e} + (b_\gamma - a_v\,\beta_s)k_{15}\,A_{ve} + (k_{15}^2\,A_{ve}^2 - \beta_s^2\,k_{10}^2\,A_{\gamma e}^2)A_{ve}\,A_{\gamma e},$$

$$a_{c3} = (C_\gamma\,k_{15}\,A_{ve} + C_v\,\beta_s^2\,k_{10})A_{ve}\,A_{\gamma e}, \qquad a_{c4} = -(C_\gamma\,k_{10}\,A_{\gamma e}^2 + C_v\,k_{15}\,A_{ve}^2)\beta_s,$$

$$\alpha_{c5} = (a_{\gamma v} - b_v\,\beta_s)\beta_s\,k_{10}\,A_{\gamma e} + (b_\gamma - a_v\,\beta_s)k_{15}\,A_{ve} - (k_{15}^2\,A_{ve}^2 - \beta_s^2\,k_{10}^2\,A_{\gamma e}^2)A_{ve}\,A_{\gamma e},$$

$$a_{e1} = a_v\cos\mu_{ve} + b_v + k_{10}\,A_{ve}\,A_{\gamma e}^2\cos 2\mu_{ve}, \qquad a_{e2} = -(f_{v5}\sin 2\mu_{ve} + f_{v6}\sin\mu_{ve}),$$

$$a_{e3} = -f_{v3} - f_{v4} - f_{v5}\cos 2\mu_{ve} + f_{v6}\cos\mu_{ve},$$

$$a_{e4} = a_v\sin\mu_{ve} + C_{v2}\,A_{ve} + k_{10}\,A_{ve}\,A_{\gamma e}^2\sin 2\mu_{ve},$$

$$a_{e5} = f_{v3} + f_{v4} + f_{v5}\cos 2\mu_{ve} + 3f_{v6}\cos\mu_{ve},$$

where

$$\beta_s = f_\varphi/f_v, \qquad a_v = -\alpha_{v1}\,A_{\gamma e}, \qquad b_v = (2\omega_p\,\sigma_2 + k_7\,A_{\gamma e}^2 + k_8\,A_{ve}^2)A_{ve}, \qquad a_{\gamma v} = -\alpha_{\varphi 1}\,A_{ve},$$

$$b_\gamma = [2(\Delta_2\,\omega_n + \sigma_2\,\omega_p) + k_{11}\,A_{ve}^2 + k_{13}\,A_{\gamma e}^2]A_{\gamma e},$$

$$f_1 = 1 + q_2 + q_3 + (q_v^2\,A_v^2 + q_\varphi^2\,A_\gamma^2)\,(q_2 + 3q_3)/2,$$

$$f_{v3} = f_1\,f_v, \qquad f_{v4} = q_v^2\,A_v^2\,(q_2 + 3q_3)f_v/4, \qquad f_{v5} = (q_\varphi\,A_\gamma/q_v\,A_v)^2 f_{v4},$$

$$f_{v6} = 2(q_\varphi\,A_\gamma/q_v\,A_v)f_{v4},$$

$$f_{v6} = 2(q_\varphi\,A_\gamma/q_v\,A_v)f_{v4}, \qquad \mu_{ve} = \sin^{-1}([a_{c1}\,a_{c3} - a_{c2}\,a_{c4}]/[a_{c3}^2 + a_{c2}\,a_{c5}]).$$