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BANDPASS VIBRATION ABSORBER

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A new concept for active vibration absorption is presented. A properly designed compensator in the local feedback converts the well-known passive absorber into a bandpass absorber. Such an absorber amplifies frequencies in a given frequency range, thus almost producing resonance for all these frequencies. If such an absorber is attached to a vibrating body, it absorbs vibrations at all frequencies that belong to the bandpass range. The presented compensator design guarantees the stability of the system. The proposed concept needs an additional local control of the absorber mass displacement, which has also been resolved. The paper presents the idea, the design procedure, and the simulation results that prove the relevance of the solution.

1. INTRODUCTION

Vibration absorbers have a history of almost a century [1] and the research in the field is still very productive. A common passive absorber [2, 3] is a mass–damper–spring trio (Figure 1(a)), whose purpose is to suppress the disturbances from the primary system that the absorber is attached to. Using one of the optimal designs [4] the response of the primary structure can be highly attenuated.

Further improvements in the absorption are possible with an additional active force introduced as a part of an absorber (Figure 1(b)). Such an active absorber is then controlled with different algorithms making it more responsive to primary disturbances. With the usage of modern control techniques they are becoming an alternative in new fields: structural control [5, 6], flexible space structures [7], vehicle suspension [8], super-rapid trains [9], helicopter vibrations [10], etc.

A special type of active absorber uses only a local feedback force, thus acting as a separate unit without the need for measuring the primary system state (e.g. primary displacements). Such active *resonant* absorbers [11, 12] are able to suppress discrete frequencies very efficiently.

This paper introduces a new type of active absorber with a local feedback force. The intention is to give it the ability to absorb all disturbances in a given frequency *band*. This should be done by expanding a single resonance frequency of a resonant absorber into a band of frequencies. However, it will be shown in the paper that this is not a straightforward step, since the stability of the system can be ruined. With the approach proposed in the next section, the simulations show that the bandpass absorber (BPA, patent pending) is stable and able to suppress vibrations in a given frequency band for a wanted degree of suppression.

The paper is organized as follows. In section 2 the concept for the design of the BPA is presented. A simple model is used in section 3 to present characteristics of the combined

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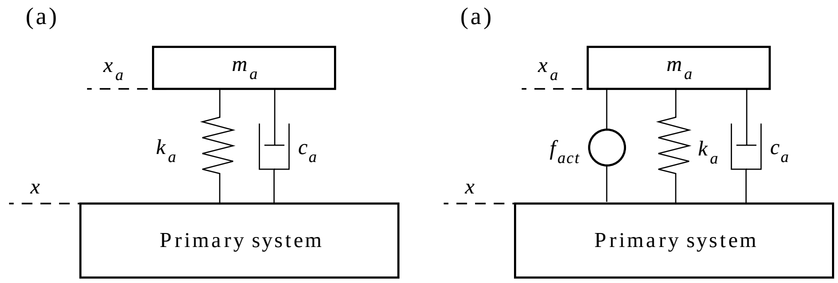

Figure 1. (a) Passive absorber; (b) active absorber.

system with the BPA. Section 4 shows simulation results and, finally, the conclusion and possible applications are considered in section 5.

2. THE CONCEPT OF THE BPA

Taking the same approach as for the design of the resonant absorber [12], the aim is to design a feedback compensator to achieve a bandpass absorber with the frequency characteristics depicted in Figure 2(a).

The global system with such a BPA would have the bandstop frequency characteristics shown in Figure 2(b), as required. However, the stability of the combined system would not be guaranteed. This drawback imposes a different approach for the design. In other words, during the design the primary system characteristics should be taken into consideration and, thus, the feedback compensator would also depend on the system. Therefore, the design is changed.

The following design procedure is proposed: the primary system transfer function $G_p(s)$ is modified only in a given absorption frequency range leaving $G_p(s)$ untouched outside that absorption bandwidth. This can be achieved by multiplying $G_p(s)$ with a bandstop filter transfer function $F_{bs}(s)$, so obtaining the global transfer function

$$G(s) = F_{bs}(s)G_p(s). \tag{1}$$

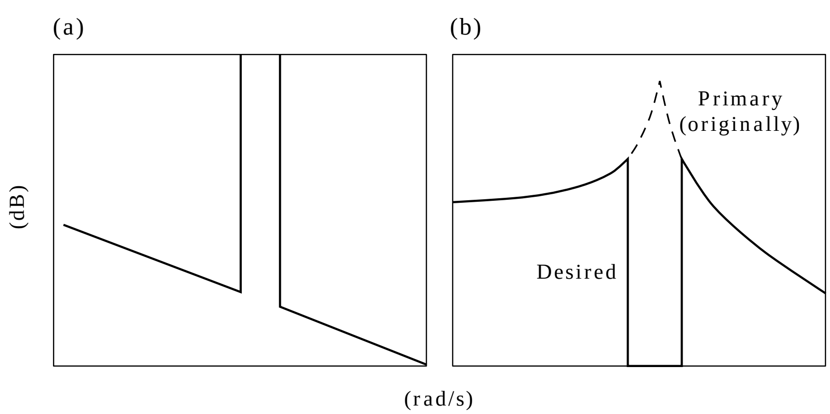

Figure 2. Frequency characteristics of an ideal (a) bandpass absorber, (b) system with the BPA.

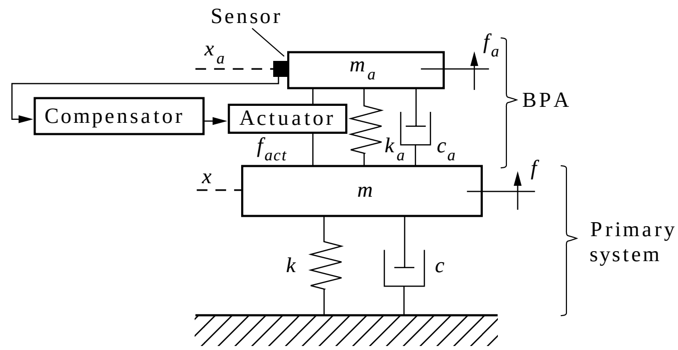

Figure 3. Single-d.o.f. primary system with the vibration absorber.

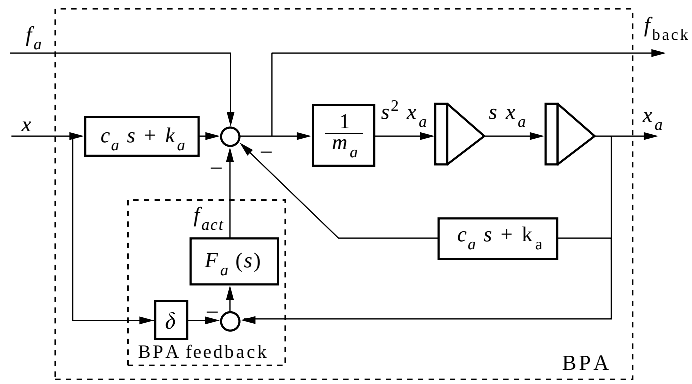

Figure 4. Dynamic model of the vibration absorber.

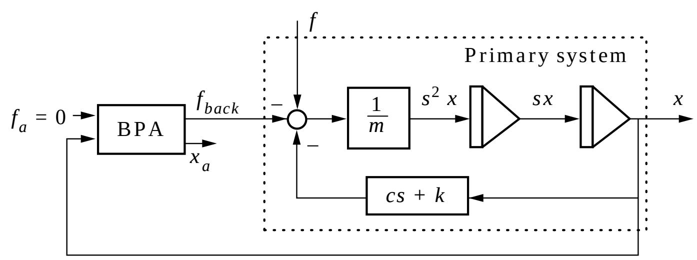

Figure 5. Dynamic model of the global (combined) system.

Consequently, the absorber feedback compensator should be designed so as to achieve the given stable global transfer function $G(s)$. This should be attained with a local absorber feedback as in the case of resonant absorbers.

The system depiction is given in Figure 3. For this purpose the primary will be a single-d.o.f. system. The active force f_{act} should depend only on the absorber position. The transfer function of the feedback filter $F_a(s)$, shown in Figure 4, should be determined by the BPA design. It is assumed that the parameters of the passive part of the absorber (m_a, c_a, k_a) are already known. The model of the global system is given in Figure 5. First, the following transfer functions are derived: $G_a(s)$ of the BPA alone, $G_p(s)$ of the primary alone, and $G(s)$ of the global system. The transfer functions are derived simultaneously for two cases: when the input signal into the feedback filter is the absolute absorber

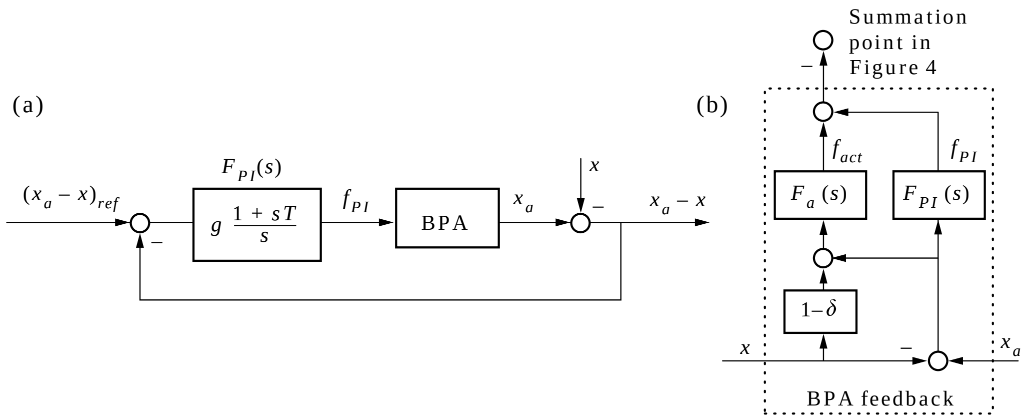

Figure 6. (a) Control structure for the DC compensation; (b) the active part.

position x_a ($\delta = 0$), as well as when the input signal is the relative position $x_a - x$ ($\delta = 1$). This is achieved by using the general feedback signal $x_a - \delta x$ for the following transfer functions.

The primary system ($f_{back} = 0$):

$$G_p(s) = \frac{x(s)}{f(s)} = \frac{1}{M(s)}. \tag{2}$$

The BPA ($x = 0$):

$$G_a(s) = \frac{x_a(s)}{f_a(s)} = \frac{1}{M_a(s) + F_a(s)}. \tag{3}$$

The global system ($f_a = 0$):

$$G = \frac{x(s)}{f(s)} = \frac{1}{M(s) + m_a s^2 (C_a(s) + \delta F_a(s)) G_a(s)}, \tag{4}$$

where

$$M(s) = ms^2 + cs + k, \qquad M_a(s) = m_a s^2 + c_a s + k_a, \qquad C_a(s) = c_a s + k_a.$$

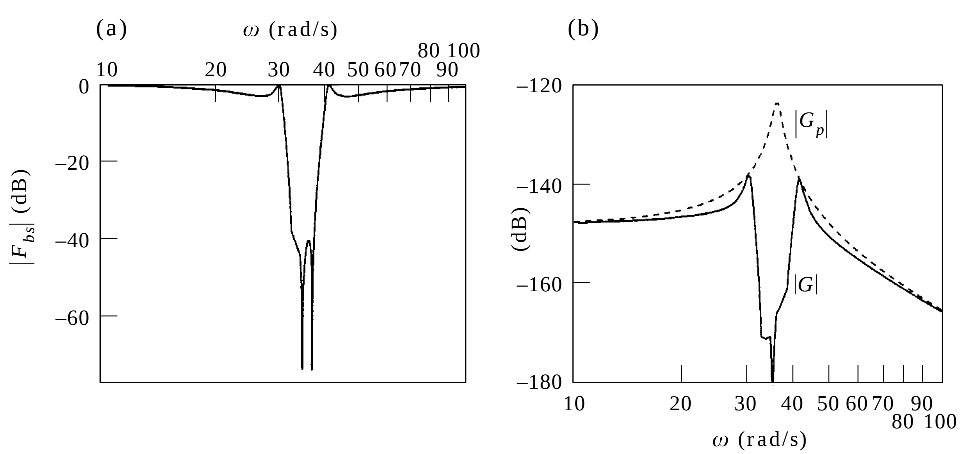

Figure 7. Frequency characteristic of (a) F_{bs}, (b) the primary G_p and the desired combined system G.

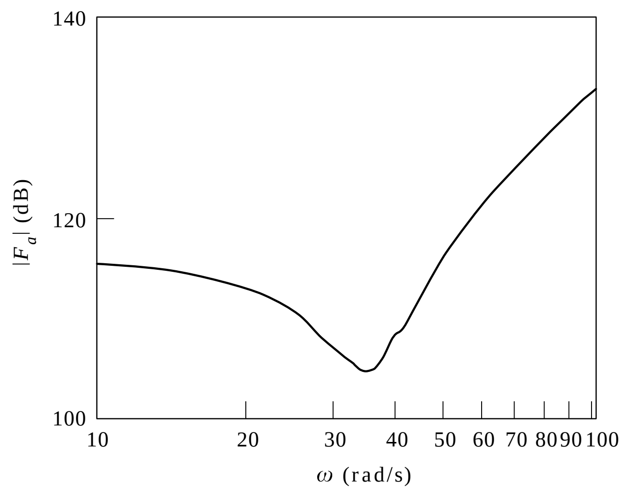

Figure 8. Frequency characteristic of the filter $F_a(s)$ in the speed feedback.

If the transfer functions are denoted by their numerator and denominator polynomials

$$G(s) = \frac{N(s)}{D(s)}, \qquad F_a(s) = \frac{N_a(s)}{D_a(s)},$$

then the global transfer function is (the dependency on s dropped)

$$G = \frac{N}{D} = \frac{M_a D_a + N_a}{M(M_a D_a + N_a) + m_a s^2 (C_a D_a + \delta N_a)}. \tag{5}$$

From equation (5) the degree of the polynomials can be determined:

$$\deg N = \max\,(2 + \deg D_a, \deg N_a), \tag{6}$$

$$\deg D = 2 + \deg N. \tag{7}$$

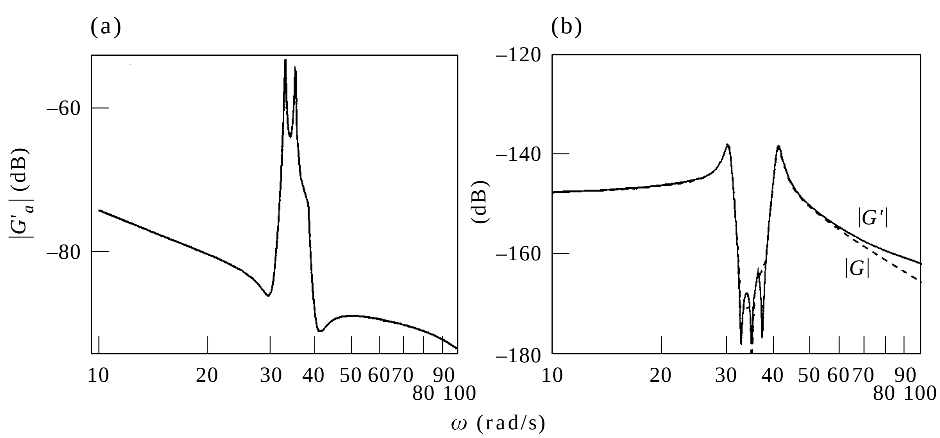

Figure 9. Frequency characteristics of (a) the BPA alone, $G'_a(s)$, (b) the global system, $G(s)$.

TABLE 1

Zeros and poles of the feedback filter, the BPA, and the global system

	Feedback filter $F'_a(s)$	Absorber $G'_a(s)$	Global system $G'(s)$
Zeros	$-4{\cdot}4635 \pm j35{\cdot}3063$	$-1{\cdot}607 \pm j40{\cdot}251$	$-44{\cdot}503 \pm j93{\cdot}631$
	$-1{\cdot}6665 \pm j40{\cdot}5092$	$-1{\cdot}213 \pm j30{\cdot}386$	$-0{\cdot}1692 \pm j34{\cdot}911$
	$-1{\cdot}1547 \pm j30{\cdot}2710$	$-0{\cdot}9925 \pm j35{\cdot}553$	$-0{\cdot}1651 \pm j37{\cdot}333$
	$-1{\cdot}0232 \pm j35{\cdot}5678$	$-10\,000$	$0{\cdot}1540 \pm j32{\cdot}660$
Poles	$-1{\cdot}6069 \pm j40{\cdot}2508$	$-44{\cdot}503 \pm j93{\cdot}631$	$-28{\cdot}759 \pm j102{\cdot}293$
	$-1{\cdot}2131 \pm j30{\cdot}3858$	$-0{\cdot}1692 \pm j34{\cdot}911$	$-14{\cdot}503 \pm j30{\cdot}629$
	$-0{\cdot}9925 \pm j35{\cdot}5530$	$-0{\cdot}1651 \pm j37{\cdot}333$	$-0{\cdot}9925 \pm j35{\cdot}553$
	$-10\,000$	$0{\cdot}1540 \pm j32{\cdot}660$	$-0{\cdot}8811 \pm j40{\cdot}724$
	0	0	$-0{\cdot}6625 \pm j30{\cdot}078$

From equation (7) it can be seen that the system should have a strictly proper transfer function ($\deg N < \deg D$) with the relative degree two. One would like to have the primary system transfer function $G_p(s)$ untouched outside the absorption frequency range. This can be achieved with a multiplicative factor F_{bs} in

$$G(s) = F_{bs}(s)G_p(s) = F_{bs}(s)\,\frac{1}{M(s)}, \tag{8}$$

where $F_{bs}(s) = N_F(s)/D_F(s)$ should have a bandstop frequency characteristic in order to suppress vibrations in a given frequency range.

This is the main goal of the BPA design: given the bandstop frequency characteristic $F_{bs}(s)$, find the feedback filter $F_a(s)$. Because of equation (7) and $\deg M = 2$, it is $\deg N_F = \deg D_F$.

From equation (4) and using equation (8) it is

$$F_a = \frac{C_a K - M_a}{1 - \delta K} \qquad \text{where} \qquad K(s) = \frac{m_a s^2}{(1/F_{bs}(s)) - 1}\,\frac{1}{M} \tag{9}$$

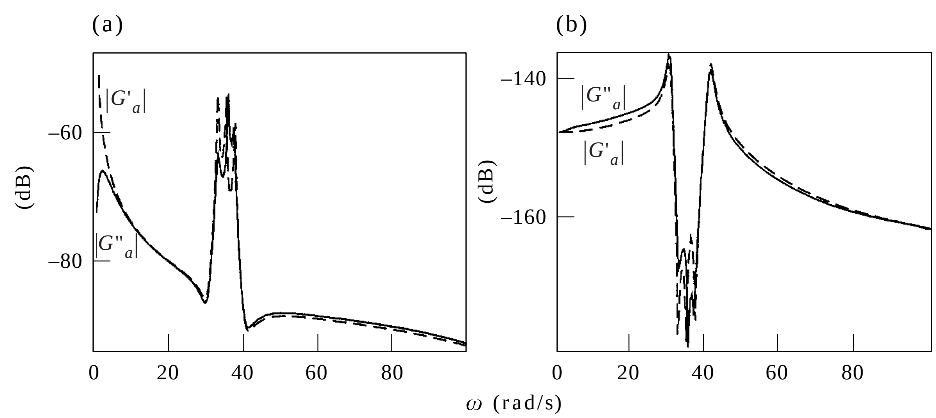

Figure 10. Frequency characteristics of (a) the BPA alone, (b) the global system, with the PI controller.

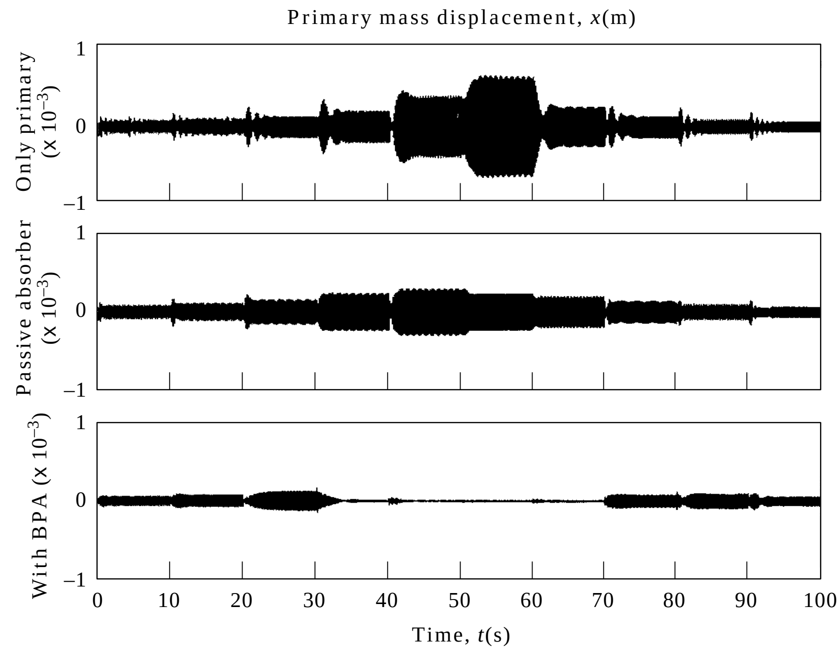

Figure 11. Frequency sweep (26–44) rad/s with the step of 2 rad/s every 10 s.

or

$$F_a = \frac{N_a}{D_a} = \frac{m_a s^2 C_a N_F - M_a M(D_F - N_F)}{M(D_F - N_F) - \delta m_a s^2 N_F} \,. \tag{10}$$

Since $F_{bs}(s)$ has the bandstop characteristic, it is $F_{bs}(0) = F_{bs}(\infty) = 1$; in other words, both $N_F(s)$ and $D_F(s)$ are monic and they both have equal lowest polynomial coefficients. Hence, in the difference $(D_F - N_F)$, the highest and the lowest polynomial coefficients do cancel. Thus, $\deg(D_F - N_F) = \deg N_F - 1$, and also one s can be extracted. Therefore, a new symbol $\Delta(s)$ is introduced, denoting the remaining polynomial after the subtraction:

$$D_F - N_F = s\Delta, \tag{11}$$

where $\deg \Delta = \deg N_F - 2$. The extraction of an s leads in equation (10) to the cancellation of s in the numerator and the denominator:

$$F_a = \frac{N_a}{D_a} = \frac{m_a s C_a N_F - M_a M \Delta}{M\Delta - \delta m_a s N_F} \,. \tag{12}$$

The degrees of the filter polynomials are then

$$\deg N_a = 2 + \deg N_F, \tag{13}$$

$$\deg D_a = \begin{cases} \deg N_F, & \delta = 0 \\ 1 + \deg N_F, & \delta = 1 \end{cases} < \deg N_a \quad ! \tag{14}$$

The feedback filter $F_a(s)$ is then *not proper* ($\deg N_a > \deg D_a$). Since this cannot be realized, one of the following solutions could be implemented. For the absolute position feedback

($\delta = 0$): (a) two new poles should be introduced into $F_a(s)$ which will not influence the feedback in the operating frequency range, i.e., $F_a'(s) = F_a(s)/(1 + sT_c)^2$, where T_c should be much smaller than the smallest system time constant; (b) instead of the position signal, the acceleration signal can be used and then two poles in the origin are included, i.e., $F_a'(s) = F_a(s)/s^2$.

If $\delta = 1$, the feedback is relative, it suffices to use the velocity signal and to include an integrator in the compensator: $F_a'(s) = F_a(s)/s$.

Inserting the solution (12) for $F_a(s)$ into equation (3) gives

$$G_a = \frac{M\Delta - \delta m_a s N_F}{m_a s N_F (C_a - \delta M_a)} . \tag{15}$$

Thus, the BPA transfer function has an integrator which can be unpleasant if the disturbance has a DC component. Therefore, a new additional control of the absorber displacement should remove the low frequency *moving average*.

For that purpose, Figure 6(a) that includes a PI controller in the classic control structure is considered. The parameters g and T of the controller should be designed so to influence only very low frequencies, much lower than the BPA suppression frequencies, $\omega \ll \omega_a = 35$ rad/s.

Since it should be $(x_a - x)_{ref} = 0$, the PI controller operates parallel to the feedback compensator F_a (for $\delta = 1$) and, therefore, they both can be incorporated into one control algorithm (Figure 6(b)). The PI controller is denoted by its transfer function $F_{PI}(s)$. With $F_{PI}(s) = g(1 + sT)/s$ included, the transfer function of the BPA becomes

$$G_a''(s) = \frac{1}{M_a(s) + F_a(s) + F_{PI}(s)} , \tag{16}$$

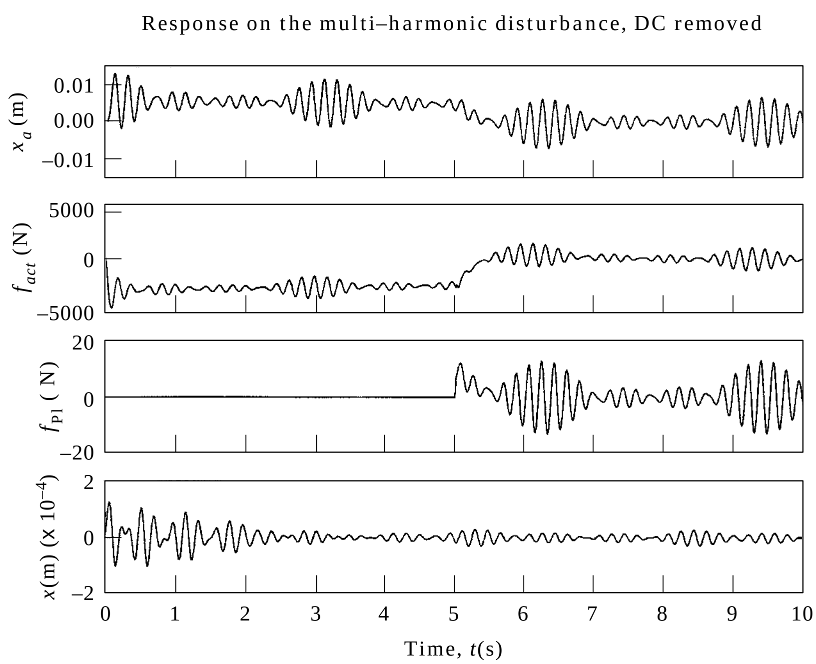

Figure 12. Multi-harmonic disturbance absorption in the system with BPA; at $t = 5$ s the PI controller is turned on.

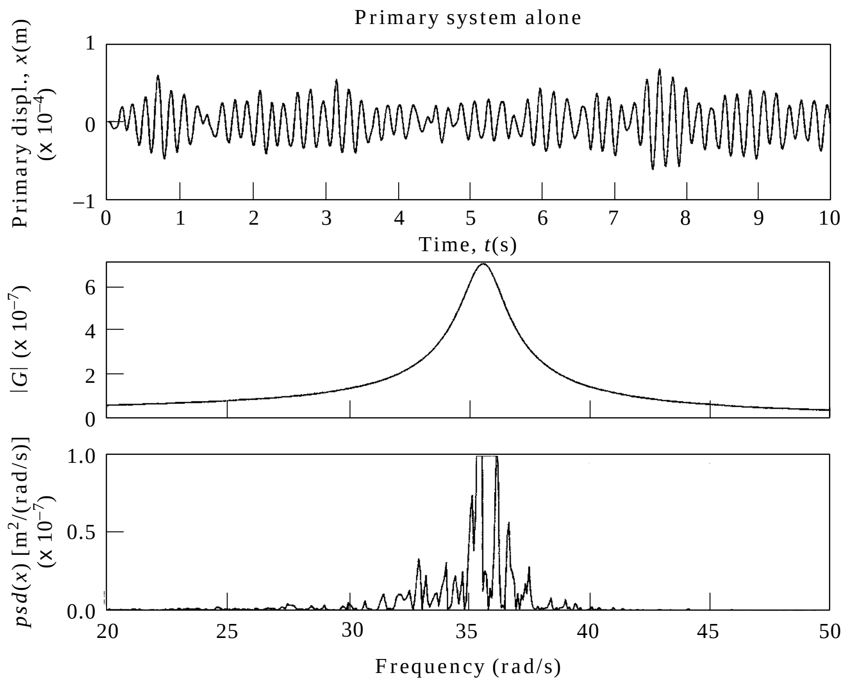

Figure 13. White noise disturbance response of the system without absorber.

which transforms equation (15)—by inserting expression (12) into (16)—to

$$G''_a(s) = \frac{s(M\Delta - \delta s m_a N_F)}{[sM_a + g(1 + sT)](M\Delta - \delta m_a s N_F) + s(m_a s C_a N_F - M_a M\Delta)}. \tag{17}$$

Thus, the integrative property of the absorber is removed.

3. AN EXAMPLE: VIBRATIONS OF A PAPER MILL WINDER

The whole paper mill model is a multi-mass system with some prominent resonant frequencies. The speed of the paper is limited by the vibrations of the winder. The suppression of these vibrations could improve the efficiency of the paper mill since higher paper velocities would be allowed.

The concept of the BPA is applied to a single-d.o.f. model of the paper winder. The primary system parameters are: $m = 20\,000$ kg, $c = 39\,700$ Ns/m, $k = 25\,300\,000$ N/m, with the natural frequency $\omega_n = \sqrt{k/m} = 35{\cdot}7$ rad/s and the damping ratio $\zeta = c/\sqrt{2mk} = 0{\cdot}0279$. The absorber parameters are chosen to be: $m_a = 500$ kg, $c_a = 4900$ Ns/m, $k_a = 632\,500$ N/m, with the natural frequency $\omega_a = \omega_n$ and the damping ratio $\zeta_a = 0{\cdot}1378$.

The bandstop filter function $F_{bs}(s)$ is designed using *Matlab* Signal Processing Toolbox. It is chosen to design an elliptic filter of the third order ($n = 3$) with $rp = 3$ dB of ripple in the passband and a stopband $rs = 40$ dB down from the peak value in the passband (suppression ratio). The bandwidth is $bw = 10$ rad/s and the centre frequency

$wo = 35$ rad/s. The Matlab commands are:

```
[z, p, k] = ellipap(n, rp, rs)                    % prototype low-pass IIR filter
[A, B, C, D] = zp2ss(z, p, k)                     % zero-pole to state space conversion
[At, Bt, Ct, Dt] = 1p2bs(A, B, C, D, wo, bw)      % low-pass to bandstop transformation
[NF, DF] = ss2tf(At, Bt, Ct, Dt)                  % state space to transfer function conv.
[zf, pf, kf] = ss2zp(At, Bt, Ct, Dt)              % state space to zero-pole conversion
```

Then one has

$$F_{bs} = \frac{N_F}{D_F} = \frac{(s^2 + 1391)(s^2 + 1225)(s^2 + 1079)}{(s^2 + 1{\cdot}785s + 1659)(s^2 + 30{\cdot}95s + 1225)(s^2 + 1{\cdot}318s + 904{\cdot}6)}. \quad (18)$$

The frequency characteristic of $F_{bs}(s)$ is shown in Figure 7(a). The characteristics of the primary single-d.o.f. system $G_p(s)$ and the desired combined system $G(s) = F_{bs}(s)G_p(s)$ are depicted in Figure 7(b). Consequently, the peak of the primary system should be removed. Using *MapleV*, the ideal non-proper feedback compensator $F_a(s)$ is calculated and its frequency characteristic is shown in Figure 8. The compensator transfer function is

$$F_a(s) = -496{\cdot}4\,\frac{(s^2 + 2{\cdot}310s + 917{\cdot}66)(s^2 + 8{\cdot}927s + 1266{\cdot}5)}{(s^2 + 2{\cdot}427s + 924{\cdot}77)}$$

$$\cdot\,\frac{(s^2 + 2{\cdot}046s + 1266{\cdot}1)(s^2 + 3{\cdot}334s + 1643{\cdot}9)}{(s^2 + 1{\cdot}985s + 1265)(s^2 + 3{\cdot}215s + 1622{\cdot}8)}.$$

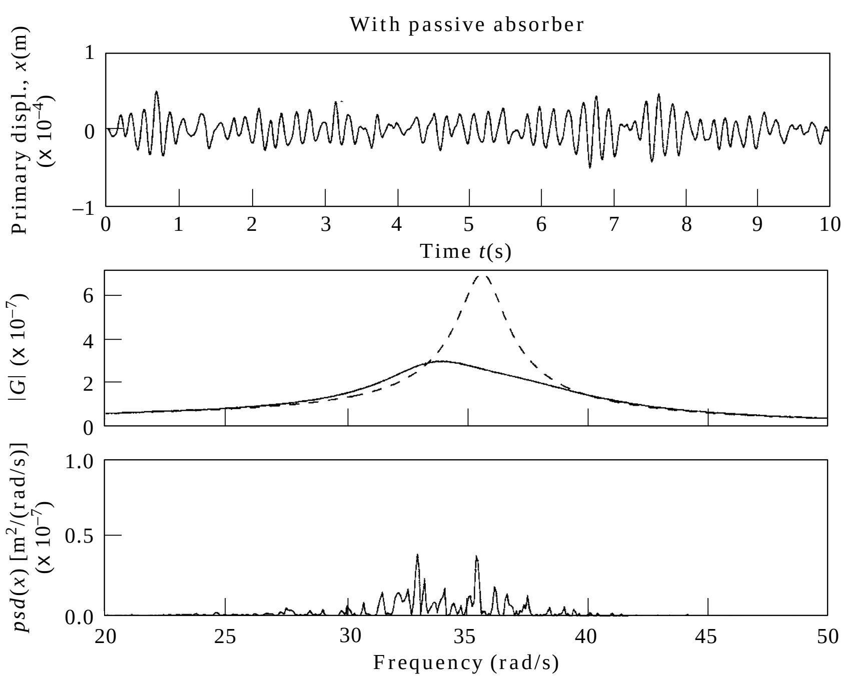

Figure 14. White noise disturbance absorption of the system with the passive absorber.

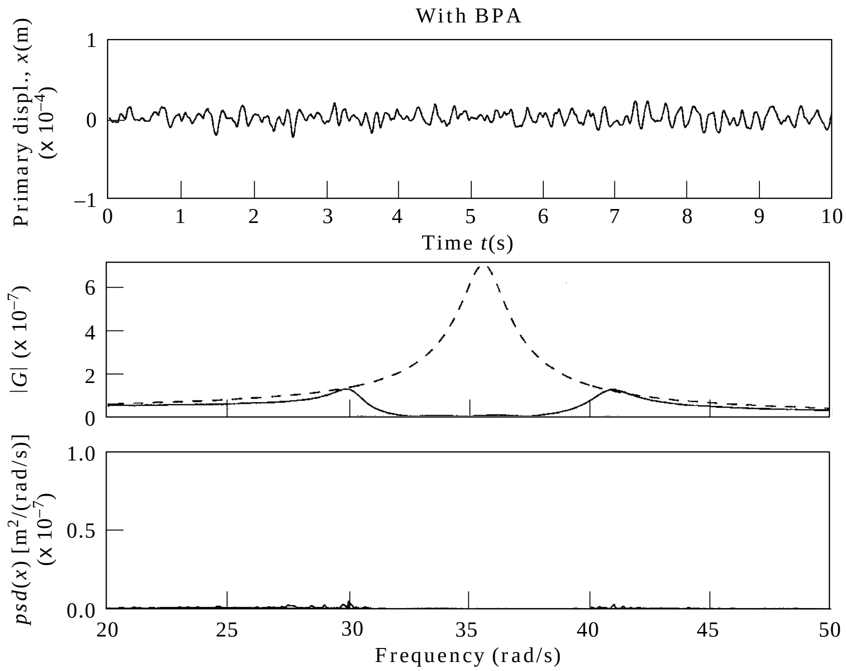

Figure 15. White noise disturbance absorption in the system with BPA.

In order to find the proper $F_a'(s)$ with the same frequency characteristic in the operating frequency range like $F_a(s)$, the suggested solutions in the previous section give the following results: (Ad a) the additional double pole $-1/T_c$ should be larger than $-10^9\,s^{-1}$ in order not to change the $|F_a(s)|$ significantly, which caused unsolvable numerical problems in Matlab; (Ad b) using the acceleration signal for the feedback produces one algebraic loop in the Simulink model and any further analysis in Matlab is therefore not possible.

The problem has been solved by using a "middle" solution: the velocity signal is used for the feedback and one new pole is added with $T_c = 10^{-4}\,s$; thus

$$F_a'(s) = F_a(s)\,\frac{1}{s(1 + sT_c)}\,. \tag{19}$$

Accordingly, in equations (3) and (4) the $F_a(s)$ should be exchanged with $sF_a'(s)$.

The resulting frequency characteristics of the BPA alone, $G_a'(s)$, and the global system, $G'(s)$, are given in Figure 9.

In Table 1 the poles and zeros of the feedback filter F_a', of the absorber G_a' and of the global system G'without the PI controller are given. Note that, though the global system and the feedback filter are stable, the BPA alone is *unstable* (the pole with the positive real part). Thus, with this design method the stability of the combined system is guaranteed, but it is achieved with an unstable absorber.

Obviously, the poles of the feedback filter are the zeros of the absorber, and the poles of the absorber are the zeros of the global system.

With the PI controller the very low part of the BPA frequency characteristic $G_a''(s)$ in Figure 10(a) is changed and does not contain an integrator. The global frequency characteristic $G''(s)$ in Figure 10(b) is not significantly changed.

4. SIMULATION RESULTS

The simulations in this section use the model from Figure 3 with the same parameters as before.

4.1. DISTURBANCE FREQUENCY SWEEP

The BPA suppression characteristics are shown at different frequencies by frequency sweep in Figure 11. The excitation has the amplitude of 1000 N and the frequency changes from 26 to 44 rad/s. The upper graph shows the response of the primary system alone. The largest amplitudes are at 36 rad/s. Attaching the passive absorber, the amplitudes around the peak frequency are lowered (except at 30 and 32 rad/s); see the middle graph in Figure 11. However, the absorber with the feedback designed according to the BPA concept suppresses vibrations at the bandpass frequencies much more efficiently, see the lower graph in Figure 11. This is exactly what is wanted by the BPA design.

4.2. MULTI-HARMONIC DISTURBANCE

The response of the combined system to the four-harmonic disturbance force

$$f = A \sum_{i=1}^{4} \sin \omega_i t,$$

where $A = 1000$ N and $\omega_{1,2,3,4} = \{32, 34, 36, 38\}$ rad/s is studied next. Without the absorber the maximal amplitude in the steady state is 1·45 mm.

Applying the BPA, all four frequencies inside the given absorption frequency range are suppressed (Figure 12). Obviously, the PI controller removes the DC component of the absorber displacement and, therefore, the actuator is also disburdened. This is achieved with the PI controller parameters $g = 2000$ N/m and $T = 1$ s. Absorption properties are

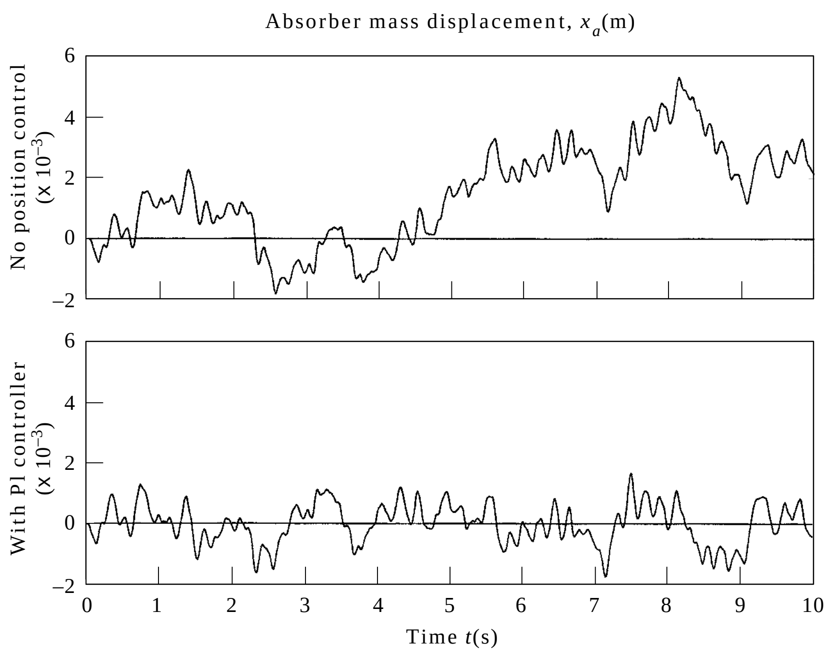

Figure 16. Absorber displacements during the absorption with BPA, with and without DC control.

not influenced by the additional moving average control. The amplitude of the primary displacement is now much smaller and amounts to 0·026 mm. Comparing maximal amplitudes in steady state, the absorption with the BPA is 34·9 dB better than without an absorber (and 31·6 dB better than with the passive absorber).

The figure shows that the actuator should be able to produce forces around 1000 N and amplitudes around 8 mm.

4.3. RANDOM VIBRATIONS

Thus far, the absorption of forced disturbances with discrete spectra has been examined. Here, the efficiency of the BPA attached to the primary system that is subjected to *random* vibrations with a (pseudo)white noise *continuous spectrum* is inspected.

The disturbance force f and its power spectral density *psd* are generated by means of the Matlab–Simulink block *Band-Limited White Noise*. The noise power is 1000 W and the sample time 0·01 s.

The response of the primary system alone is given in Figure 13. The frequency response has a peak at $\omega_p = 35{\cdot}54$ rad/s and the spectrum of mass movements is modulated correspondingly. The peak of the frequency response is $G_M = |G(j\omega_p)| = 7{\cdot}08 \times 10^{-7}$ m/N. The maximal magnitude of the primary mass displacements is $x_M = x(t)_{max} = 6{\cdot}89 \times 10^{-5}$ m. The r.m.s. primary mass displacement is $\sigma_x^o = 2{\cdot}32 \times 10^{-5}$ m. These three parameters, G_M, x_M and σ_x^o, are used as a reference for further results.

A standard solution for vibration suppression is the application of the passive absorber. The frequency response of the system with the passive absorber, Figure 14, shows that the peak is lowered and moved to a lower frequency. This passive absorber is not optimized by any criterion (this is not the issue here). The peak is at the frequency $\omega = 33{\cdot}9$ rad/s and amounts to $3{\cdot}02 \times 10^{-7}$ m which is 42·6% of G_M. The maximal amplitude is 71·1% of x_M, and the standard deviation (r.m.s. value) is reduced to 67·3% of σ_x^o. The power spectral density of the primary mass displacement, *psd*(x), is modulated by the frequency transfer function $G(j\omega)$. Hence, the frequencies around $\omega = 32$ rad/s are even amplified compared to Figure 13.

If the feedback is designed so that the absorber becomes a bandpass absorber, the absorption is much better. From Figure 15 it is obvious that the frequency response is *always* lower than or equal to the frequency response of the primary alone. The use of the BPA gives significant improvements: the peak frequency amplitude is 18%, the maximal displacement amplitude 35·7% and the r.m.s. displacement 33·5% of the initial primary results, respectively.

The band-limited white noise generated in Matlab is not DC free. Therefore, the absorber mass displacements without PI control, and thus the active actuator force, include the DC component (Figure 16). However, the PI controller keeps absorber displacements inside the 2-mm limits.

5. CONCLUSION

The concept of the bandpass absorber (BPA) has been introduced. The BPA comprises the standard passive absorber and a single local feedback with a compensator designed to obtain a desired bandstop system characteristic. With such an absorber the vibrations of the primary mass can be suppressed in a given range of frequencies. The design procedure given in the paper guarantees the stability of the system. The suppression degree is a design parameter.

The integrative operation of the BPA is eliminated by the additional PI control of the relative absorber displacement. This control operates with a slower dynamics than the main compensator, and so does not deteriorate the absorption quality.

The application of the BPA can be justified in systems acted upon by disturbances with variable frequency, or more frequencies, in some fixed frequency range, as well as for suppression of "coloured" vibrations. If the frequency range itself is time variable, the feedback of the BPA could be made adaptive with the self-tuning of the compensator parameters.

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