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LETTERS TO THE EDITOR

DYNAMICS OF TRUSSES: NUMERICAL AND EXPERIMENTAL RESULTS

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1. INTRODUCTION

Some numerical and experimental results on the dynamics of two-dimensional, linear truss structures are presented. It is well-known that such periodic systems act as wave filters, possessing propagation and attenuation zones (PZs and AZs) in the frequency–wavenumber domain [1–4]. Structural waves with frequencies and wavenumbers contained inside PZs, travel unattenuated through the periodic system, whereas waves inside AZs are near field solutions with spatially confined envelopes. What gives rise to these PZs and AZs is a complicated interference pattern of reflected and transmitted waves at each junction between periodic sets, resulting in complex wave interaction phenomena that either enhance or prohibit wave propagation through the periodic structure. Chen and Pierre [5, 6] developed a computational scheme to study mode localization and wave conversion effects in two-dimensional trusses by modelling each structural member by exact linear partial differential equations governing the longitudinal and transverse vibrations, and coupling the members' dynamic responses to form overall transfer matrices. Experimental studies of truss structures have been recorded in references [7–9].

The aim of the present contribution is two-fold. The numerical part is based on the computational procedure of references [5, 6] and analyzes the PZs and AZs of truss structures of infinite spatial extent with clamped and pinned joints. The truss studied in reference [5, 6] is revisited and some missing wave conversion phenomena are discussed. The experimental part aims to verify the theoretical findings by testing a practical flexible truss of finite spatial extent.

2. INFINITE STRUCTURE WITH CLAMPED JOINTS: NUMERICAL RESULTS

The first truss structure considered is depicted in Figure 1 and consists of an infinite number of periodic sets connected by clamped joints. Each periodic set (bay) consists of coupled beams undergoing axial and bending vibrations. The joints connecting the beams of each periodic set transmit longitudinal (axial) and transverse forces, as well as, bending moments. The free dynamics of the truss is analyzed by constructing a linearly exact [5, 6] transfer matrix relating the forces, moments, displacements and rotations at the joints at the left and right boundaries of each periodic set. The detailed construction of the transfer matrix can be found in reference [10]. The values of the parameters for the structural members employed in the numerical simulations were $I = 7{\cdot}98 \times 10^{-11}$ m^4 (moment of inertia), $E = 70 \times 10^9$ Pa (modulus of elasticity), $A = 3{\cdot}167 \times 10^{-5}$ m^2 (cross-section), $\bar{m} = 0{\cdot}0855$ kg/m (mass per unit length), $l_{vert} = 0{\cdot}425$ m (vertical member)

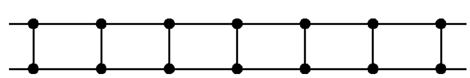

Figure 1. The truss with clamped joints under consideration.

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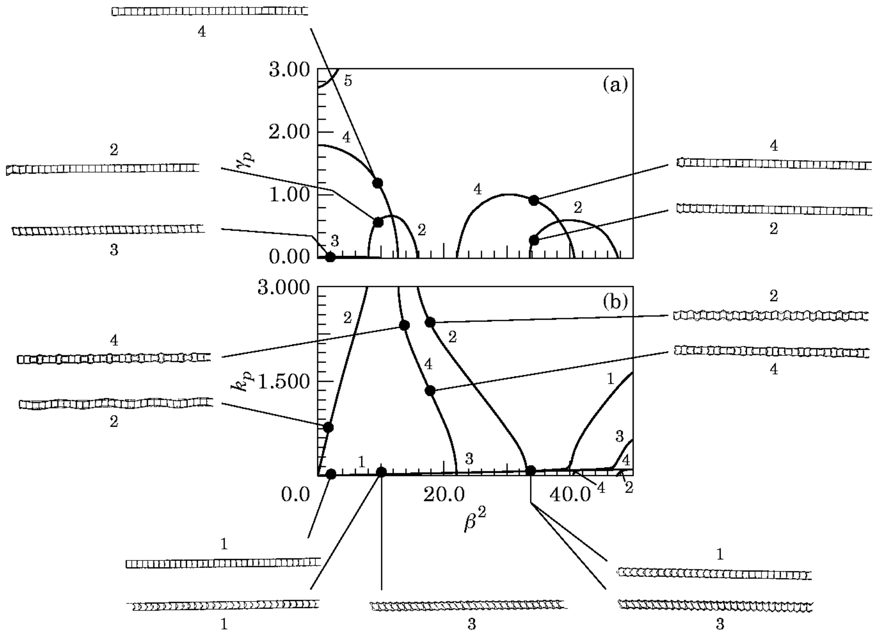


Figure 2. Characteristic waves of the truss with clamped joints: (a) Exponential decay ratees γ_p versus β^2, and (b) phases k_p versus β^2.

and $l_{horiz} = 0{\cdot}903$ m (horizontal member). In addition, the non dimensional frequency $\beta^2 = \omega/(EI/ml^4_{horiz})^{1/2} = 0{\cdot}3176\,\omega$ is introduced. These numerical values match the geometric and material properties of the experimental truss fixture that was tested to verify the numerical results.

Structural waves in the unforced structure of infinite spatial extent are studied by relating the state vectors at the two ends of an arbitrary substructure by the expresssion, $\mathbf{s}_L^{(i+1)} = e^{\mu}\mathbf{s}_L^{(i)}$; where μ is termed the *propagation constant*, and $\mathbf{s}_L^{(i)}$ represents a (12×1) state vector of displacements, rotations, internal forces and internal moments at the left boundary of the ith substructure [10]. Depending on the nature of the propagation constant, one obtains travelling (μ purely imaginary) or attenuating (μ real or complex) characteristic waves. The truss of Figure 1 supports six families of characteristic waves. Within the order of accuracy of the numerical simulations, no complex attenuating waves (with complex μ) were found in the frequency range considered, and only travelling and attenuating waves were detected. In Figure 2 the exponential decay rate (real part of μ) and phase (imaginary part of μ) for five of the six characteristic waves are depicted as functions of the non-dimensional frequency β^2. The waves are labelled from 1 to 5; the sixth characteristic wave (not shown) is strongly attenuating with phase $k_6 = \pi$ and attenuation rate γ_6 of $\mathcal{O}(\pm 12)$ for the entire range of β^2 considered. These plots define the PZs and AZs for each family of characteristic waves. Due to linearity, each wave can propagate (or attenuate) independently from the others, and, as a result, considerable overlap of the PZs and AZs of the various wave families exists in the plots. Wave conversions occur when two families of characteristic waves coalesce to form a branch of complex waves. As mentioned above, no wave conversions were found to exist in the truss with clamped joints examined. The deformation patterns of the truss for characteristic wave oscillations are also depicted in Figure 2 at selected values of β^2.

At sufficiently low frequencies ($\beta^2 < 8$) the truss possesses two travelling and four attenuating characteristic waves. Travelling wave 1 corresponds to longitudinal motions of the horizontal beam members in their first axial clamped–clamped mode, and transverse motions of the vertical beams in their first flexural clamped–clamped mode; when oscillating in this characteristic wave, the truss undergoes predominantly longitudinal motions. Travelling wave 2 corresponds to predominantly bending motions of the truss, with the horizontal beam members performing bending vibrations. Attenuating wave 3 corresponds to predominantly shear motions of the truss, with the upper and lower horizontal beam members oscillating in opposite directions and the coupling joints executing rotations. Attenuating wave 4 involves mainly rotations of the joints, with negligible displacements of the end points of the beam members; the beam members vibrate with nearly simply supported mode shapes. Similar to wave 6, wave 5 is strongly attenuating and involves translations and rotations of the end points of the beam members (cf., Figure 3).

As frequency increases, the characteristic waves can be transformed from travelling to attenuating or *vice versa*, with the exception of waves 1 and 5 which remain travelling and attenuating, respectively, in the entire frequency range considered. This feature creates a complicated pattern of alternating PZs and AZs which, depending on the frequency, can result in as many as four co-existing travelling characteristic waves in the truss. Moreover, as frequency increases the beam members of the truss vibrate in higher longitudinal or flexural modes, as the deformation shapes of Figure 2 indicate. As an example, considering the characteristic wave 2, in its lower PZ the beam members oscillate approximately in rigid body modes, whereas in higher AZs and PZs they oscillate with shapes corresponding to their clamped–clamped bending modes. Similar conclusions can be drawn for the low and high frequency deformation shapes of the characteristic waves 4 (cf., Figure 2).

The complicated pattern of PZs and AZs of the truss provides a justification of the technical difficulties associated with the numerical computation of its response to arbitrary time-varying external excitations. Indeed, different frequency components of the external forces are transmitted by different characteristic travelling waves through the truss, a feature which must be carefully taken into account when performing computations based on numerical integral (and inverse integral) transformations of the force and the response.

3. REVISITING THE TRUSS WITH PINNED JOINTS EXAMINED IN REFERENCE [5]

Before proceeding with experimental results, the infinite truss structure with diagonals examined in reference [5] is revisited, in order to discuss some wave conversion phenomena that were omitted in these previous references. The truss examined has the configuration of Figure 1 with the following structural modifications: (a) the joints connecting the structural members are pinned instead of clamped, allowing free rotation, and (b) there exist diagonal members at each subsystem. The pinned joints introduce additional flexibility to the truss, and give rise to a series of very complicated wave conversion phenomena taking place in very narrow frequency regions (i.e., localized in frequency). These complicated wave mode conversions (not reported in reference [5]) will be discussed in full detail in what follows. To achieve a direct comparison with the results reported in reference [5], the following numerical values for the structural parameters were selected: $EI = 5{\cdot}81482 \times 10^3\ \mathrm{Nm}^2$, $EA = 1{\cdot}93977 \times 10^7\ \mathrm{N}$, $\bar{m} = 0{\cdot}75948\ \mathrm{kg/m}$, and $l_{vert} = l_{horiz} = 1{\cdot}397\ \mathrm{m}$; for this system, the non-dimensional frequency parameter assumes the form, $\beta^2 = 0{\cdot}022304\,\omega$. Note, that in contrast to the truss with clamped joints examined in the previous section, the truss with pinned joints possesses horizontal and vertical beam members of equal lengths. The resulting spatial symmetry introduces degeneracies in the dynamics and complicates the wave conversion process.

Employing the technique outlined previously, one constructs a transfer matrix to relate the displacements at the right and left sides of an arbitrary substructure. Since the pinned joints are incapable of transmitting moments, the transfer matrix is now of size (8×8), and the state vectors $\mathbf{s}_L^{(i)}$ of size (8×1) [10]. For an explicit expression of the transfer matrix the reader is referred to reference [5]. Four families of characteristic waves exist, of all three types: attenuating, travelling and complex. Characteristic waves which at low frequencies lead to nearly shearing motions of the truss are labelled by the number 1; these are "shear waves" in the terminology of reference [5]. Waves which attenuate at low frequencies are labelled 2, and are the "evanescent waves" in reference [5]. Characteristic waves which at low frequencies correspond to predominantly transverse vibrations of the truss are labelled

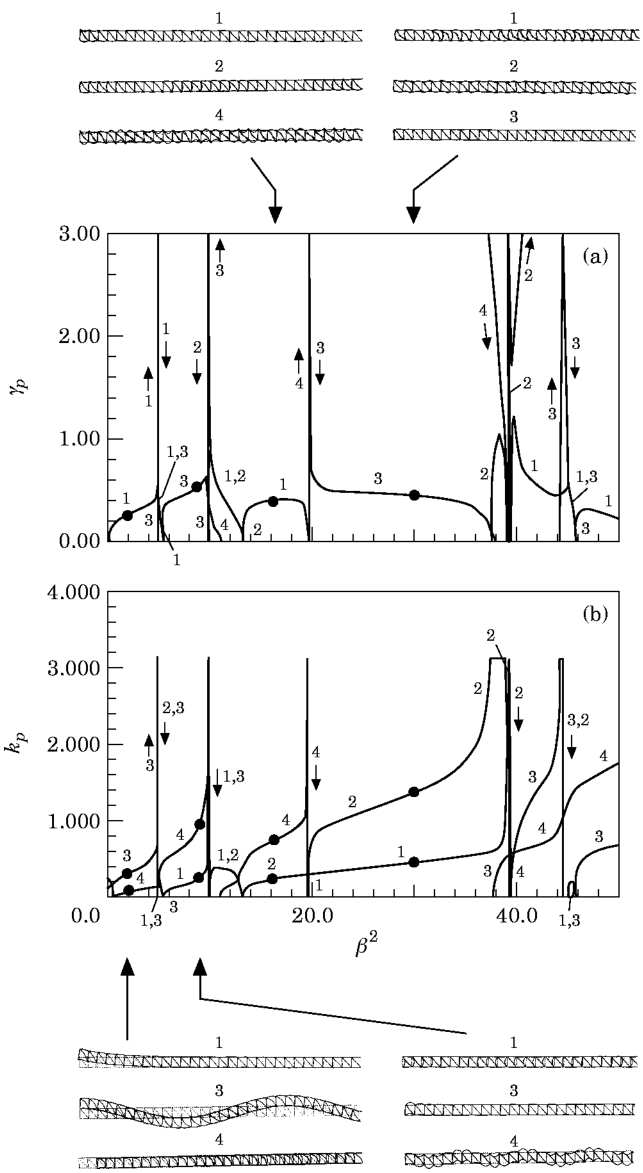


Figure 3. Characteristic waves of the truss with pinned joints and diagonals: (a) Exponential decay rates γ_p versus β^2, (b) phases k_p versus β^2.

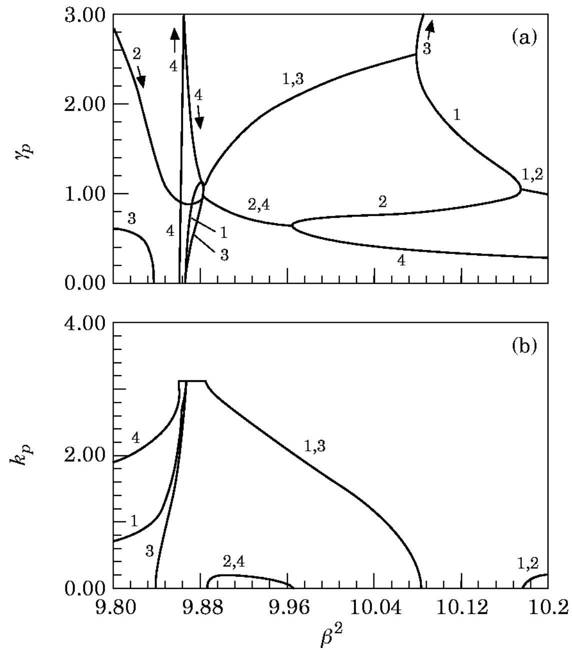


Figure 4. Detail of Figure 3 for $9{\cdot}80 < \beta^2 < 10{\cdot}20$: (a) Exponential decay rates γ_p versus β^2, (b) phases k_p versus β^2.

3, or 'bending waves' [5]. Finally, waves which at low frequencies give rise to predominantly longitudinal truss vibrations are labelled by 4, and are "compression waves" of reference [5]. It is emphasized at this point that the classification of waves as "bending" or "shear", can only have meaning at low frequencies; indeed, at intermediate or high frequencies complicated wave conversions occur (complex branches), and the actual motion of the truss does not possess simple recognizable patterns that would justify the use of such characterizations.

In Figures 3–5 the exponential decays and phases of the four characteristic waves as functions of the frequency are depicted. A first observation regarding these plots is that the truss with pinned joints possesses a more complicated structure of PZs and AZs than the truss with clamped joints studied in the previous section. This is partly due to the increased flexibility of the truss with pinned joints, and to the degeneracies in the dynamics introduced by the spatial symmetries of its periodic sets; in addition, the truss with pinned joints possesses much longer longitudinal beam members, a feature which also increases the flexibility of the system. The overall (large-scale) characteristics of the plots are the same with the ones reported in reference [5], but the wave classification over certain frequency ranges used herein is slightly different. The performed numerical computations reveal very complicated wave conversion phenomena in the two narrow frequency regions $9{\cdot}80 < \beta^2 < 10{\cdot}20$ and $36{\cdot}50 < \beta^2 < 46{\cdot}00$. These wave conversions were omitted in reference [5], and are fully exploited in Figures 5 and 6. In the neighborhood $\beta^2 \approx 5$, where the first bending mode of the pinned–pinned diagonal beam member resonates, the first branch of complex waves occurs, resulting from the merging of waves 1 and 3. The neighborhood $\beta^2 \approx \pi^2$ is especially degenerate, since the first bending modes of the pinned–pinned horizontal and vertical beam members resonant simultaneously there. As seen in Figure 4, there are three branches of complex waves in that neighborhood, with all four families of characteristic waves participating in the wave conversions. In the

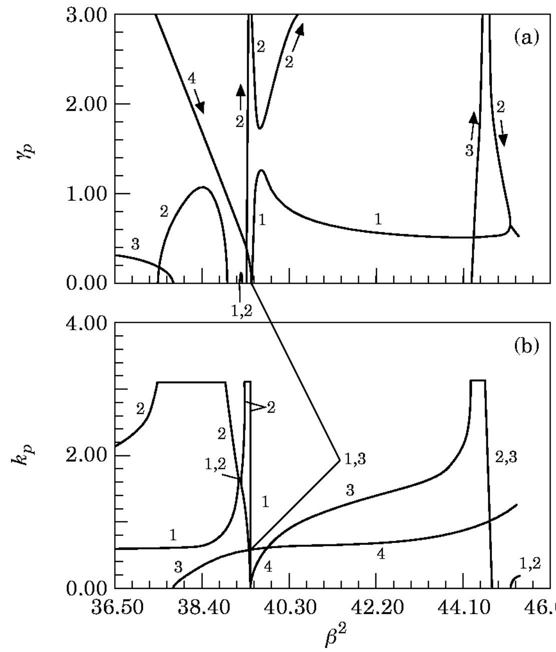


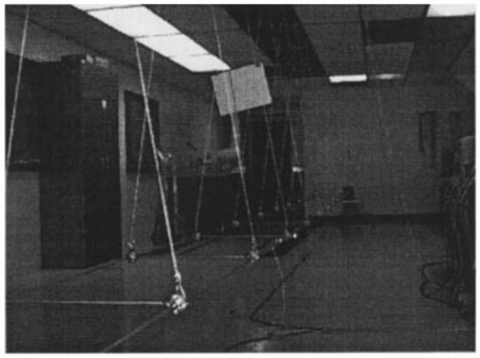
Figure 5. Detail of Figure 3 for $36{\cdot}50 < \beta^2 < 46{\cdot}00$: (a) Exponential decay rates γ_p versus β^2, (b) phases k_p versus β^2.

neighborhood $\beta^2 \approx 19{\cdot}74$ the second resonance of the pinned–pinned diagonal beams occurs and, consequently, resonances in the plots are again observed. In Figure 5 the neighborhood of the second resonance of the pinned–pinned horizontal and vertical beams at $\beta^2 = 4\pi^2$ is studied, and two additional branches of complex waves are detected involving the wave families 1, 2 and 3.

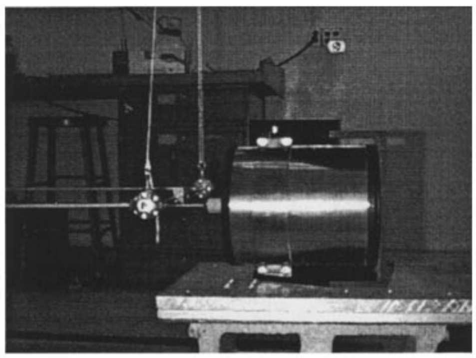
The previous numerical results indicate that complicate wave conversion phenomena can occur in flexible trusses that possess spatial symmetry degeneracies. The effects of structural modifications (such as, removing the diagonals or replacing the pinned by clamped joints) on the characteristic waves are studied in detail in reference [10], where the interested reader is referred.

4. EXPERIMENTAL TESTS

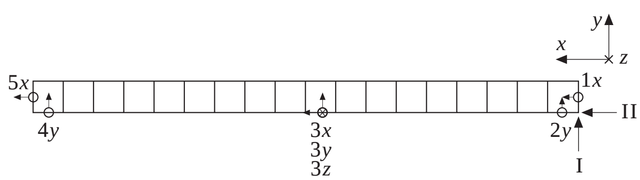
A picture of the experimental truss is given in Figure 6(a); in Figure 6(b) a schematic diagram indicating the forcing and measurement positions and directions is shown. The setup consists of 18 periodic sets coupled by means of clamped joints, with a total length of 17·34 m. To simulate free boundary conditions, the truss was hung from the ceiling of the laboratory, with strings of length 1·8415 m. The ‘pendulum modes’ introduced by the hanging strings were confined at low frequencies and did not interfere with the measurements of the dynamics of the truss. All beam members and joints were constructed out of aluminum, and the configuration, material properties and dimensions of each periodic set were chosen identical to ones of the truss with clamped joints considered in section 2. Each joint consisted of a hollow sphere with multiple threaded entries. Each beam member was securely threaded into the corresponding joint entry in an effort to establish clamped boundary conditions. As a result, each joint introduced added mass (that of the hallow sphere) to the truss, a feature which was not accounted for in the theoretical



(a)



(b)



(c)

Figure 6. The experimental truss fixture: (a) general configuration, (b) detail of the connection of the shaker to the joint, (c) forcing types I and II, and measurement positions–directions (⊘).

model of section 2. The results of the previous two sections were for trusses of infinite spatial extent, and assumed no reflections of characteristic waves from any boundary. Clearly, such a condition is unrealizable in an experimental setting, and the effects of wave reflections from the boundaries can only be minimized by including a sufficiently large number of bays.

The question of the relation between the dynamics of the *infinite* (theoretical) and *finite* (experimental) trusses is now addressed. This relation stems from the fact that resonances (standing waves) of the finite truss are generated by superpositions of characteristic waves propagating in opposite directions. Since wave conversions occur when characteristic waves encounter a boundary, resonances in finite trusses may result as superpositions of more than one families of characteristic waves; this holds when the truss possesses multiple PZs at the resonance frequency. For trusses with a sufficiently large number of periodic sets, only travelling waves can produce resonances (standing waves), since only such waves can propagate through the entire truss and reach both boundaries (attenuating or complex waves are near-field solutions). Using these ideas, Mead [1] argued that nearly all of the resonances of the finite truss occur inside PZs of the corresponding infinite one. Moreover, when the truss possesses a single (non-overlapping) PZ in a given frequency band and is composed of a sufficiently large number of periodic sets, say N, there are up to N natural frequencies inside this PZ [1].

Considering the PZs of the theoretical truss of section 2, one notes that the predominantly tranverse (“bending”) characteristic waves 2 possess three low frequency PZs separated by two AZs lying approximately in the intervals 12·6–25·2 Hz and 52·7–74·9 Hz (the frequency conversion relation $f = 1{\cdot}57768\ \beta^2$ is used, with f in Hz). The predominantly longitudinal (“pressure”) characteristic waves 1 possess an uninterrupted PZ at low frequencies. However, from the plot of Figure 2(b) one notes that the phases of these waves become noticeable only at frequencies above (approximately) 63·9 Hz; at lower frequencies the phases are close to zero and wave transmission is nearly non-existent. Similarly for “shear” waves 3, propagation becomes evident only above (approximately) 74·1 Hz. Finally, the characteristic waves 4 possess a single PZ in the range 19·9–35·0 Hz (partially overlapping with the second PZ for “transverse” waves), which is expected to contribute to the resonance phenomena. In conclusion, the theoretical truss possesses dominant PZs at the frequency bands 0–12·6 Hz (wave 2), 19·9–52·7 Hz (waves 2 and 4), and above 63·9 Hz (wave 1). As discussed previously, most of the resonances of the experimental truss should “cluster” inside these dominant PZs.

The experimental truss was excited by means of a PCB impact hammer or a 50 lb MB-Dynamics electromagnetic shaker at the excitation positions and directions indicated in Figure 6. The modal hammer was used to produce an impact force which led to simultaneous excitation of the truss at a wide range of frequencies; the modal shaker was utilized to harmonically excite the truss at a single frequency and study its steady state response at that particular frequency. The excitation was applied at a single point of the truss, at the two directions labelled I and II in Figure 6(c). The response of the truss was recorded by means of uni-axial (at measurement positions 1, 2, 4 and 5, cf., Figure 6(b)), and tri-axial (at measurement position 3) accelerometers. The forcing signal (recorded by a force transducer at the tip of the impact hammer, or at the point of contact of the truss with the sting of the modal shaker), and the seven accelerometer signals at the measurements positions were fed to conditioning amplifiers, and, subsequently to an 8-channel Zonic WCA Spectral Analyzer for post processing. The Analyzer possessed Fast Fourier Transform (FFT), Modal Analysis, Time History Animation and Signal Generating capabilities (the signal generation for the modal shaker was provided by the Analyzer).

In Figure 7 the experimental acceleration spectra at measurement points 1–5, for type I (transverse) impact excitation is presented; the frequency spectrum of the force is also presented in that figure. The impact force excites predominantly “bending” characteristic waves, but, due to wave conversions at the boundaries, other families of characteristic waves are expected to be present in the response; one expects, however, that characteristic waves 2 will dominate the response. The acceleration spectrum at position 4 (Figure 7(g) provides the best measure of the transverse response of the truss without near-field effects, since it is recorded at the periodic set farthest from the point of excitation. At this measurement position the effects of attenuating (near-field) characteristic waves generated at the forcing point are minimal, and one expects to identify clusters of resonances lying in the theoretically predicted PZs. Indeed, one observes three clusters of resonances in the frequency bands 0–8 Hz, 16–32 Hz and 59–90 Hz. The isolated resonance close to 48 Hz corresponds to a local mode which is discussed later. The experimental results confirm the existence of three passing bands in the frequency range of interest, and, hence, are in qualitative agreement with the theoretically predicted PZs for characteristic waves 2 and 1. However, the locations of the three PZs in the frequency domain are in qualitative discrepancy with the theoretically predicted passing band ranges of 0–12·6 Hz, 25·2–52·7 Hz and above 63·9 Hz. These quantitative differences can be attributed to resonances produced by other types of travelling characteristic waves (both waves 3 and

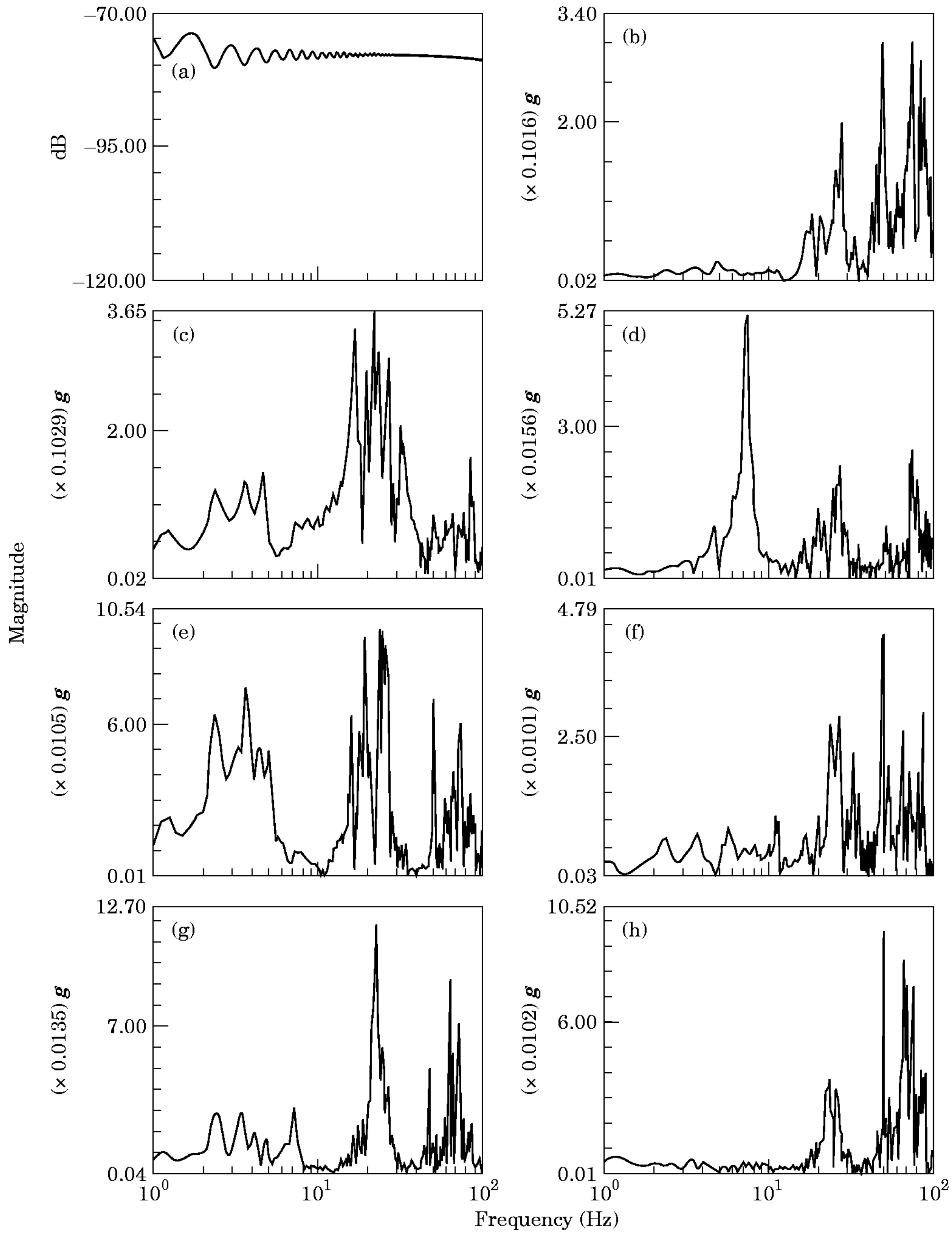


Figure 7. Experimental measurements for a type I impact excitation: (a) Frequency spectrum of the force, and frequency acceleration spectra at measurement positions, (b) $1x$, (c) $2y$, (d) $3x$, (e) $3y$, (f) $3z$, (g) $4y$, (h) $5x$.

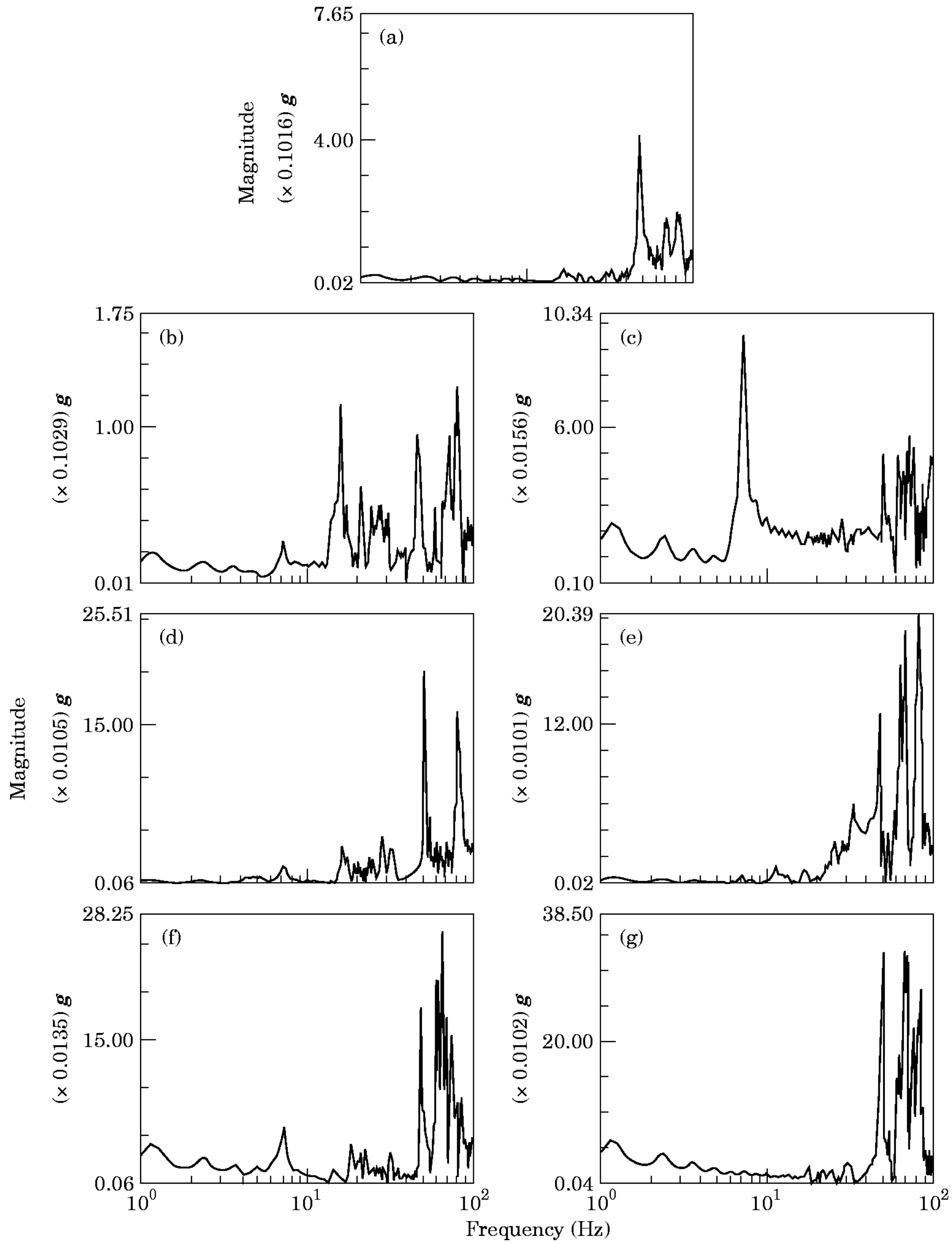


Figure 8. Experimental measurements for a type II impact excitation: frequency acceleration spectra at measurement positions; (a) $1x$, (b) $2y$, (c) $3x$, (d) $3y$, (e) $3z$, (f) $4y$, (g) $5x$.

4 possess PZs in the frequency range considered); to non-perfect realization of clamped conditions at the experimental joints; to dynamical effects of the added-mass of the joints (which is not taken into account in the theoretical model); or to out-of-plane, three-dimensional dynamical effects. Similar clusters of resonances are observed in the other acceleration spectra of Figure 7, confirming the clustering of natural frequencies of the truss.

In Figure 8, the acceleration spectra at measurement points 1–5 for a type II (longitudinal) impact excitation are presented. The impact force excites predominantly longitudinal characteristic waves 1, but, again, wave conversions at the boundaries are expected to introduce additional families of characteristic waves in the response. The acceleration spectrum at position 5 (cf., Figure 8(g)) captures the longitudinal response of the truss at the periodic set farthest from the point of excitation (and, thus, nearly free of near-field effects). The resonances appearing in this spectrum are mainly produced by traveling waves 1 and are clustered at high frequencies, above 60 Hz. A local mode close to 48 Hz appears in the spectrum. The high frequency clustering is in qualitative agreement with the theoretical prediction that travelling waves 1 become more evident at high frequencies above 63·9 Hz, but, again a slight quantitative discrepancy between theory and experiment exists. Similar high frequency clustering of resonances is observed for all spectra depicting longitudinal accelerations (cf., Figures 8(a, c, g)), where characteristic waves 1 dominate the response. Considering the spectra of the transverse accelerations (cf., Figures 8(b, d, f)), one observes cluster of resonances at three frequency bands; this is physically meaningful, since these spectra are influenced by travelling waves 2, which possess a different structure of PZs than waves 1.

A second series of experimental tests was performed by forcing the truss with a single harmonic excitation of varying frequency by means of an electromagnetic shaker. First, shaker tests were performed to confirm the existence of AZs for "longitudinal" or "transverse" characteristic waves. This was achieved by forcing the truss either longitudinally or transversely at a frequency inside a predicted AZ, letting it reach steady state, and observing the ensuing pattern of oscillation. When in an AZ, the main body of the truss remained virtually motionless at steady state, and only a small number of bays close to the excitation point showed any significant motion. This steady state localization of vibrational energy close to the forcing point indicated the inability of either the "longitudinal" or "transverse" characteristic waves to propagate through the truss, and established that the frequency of excitation was located in an AZ of these waves. In additional series of shaker tests, the dynamics of the truss close to certain resonances of the acceleration spectra of Figures 7 and 8 was examined. The strong resonance at approximately 48 Hz observed in all spectra was found to correspond to an isolated local mode of the truss, coinciding with the first bending mode of the out-of-plane, *clamped–pinned*, horizontal beam members. On this local mode, the joints of the truss were, alternatively, stationary or rotating harmonically, so that each horizontal beam member vibrated with the shape of its first clamped-pinned mode. When the frequency of excitation was slightly increased or decreased this local mode vanished and the truss vibrated as in an AZ. Hence, the local mode at 48 Hz appears to be isolated in and AZ, and not to be contained in any cluster of resonances. Similar shaker tests close to the resonances of the acceleration spectra confirmed the existence of "bending", "shear" and "longitudinal" resonances of the truss. Depending on the frequency, the longitudinal and vertical beam members of the truss vibrated in their bending or longitudinal clamped–clamped modes.

5. CONCLUSIONS

Complicated wave conversion phenomena in a flexible truss with pinned joints and diagonals were detected in very narrow frequency bands close to the natural frequencies for bending vibrations of individual beam members. Replacement of pinned with clamped joints and removal of the diagonals simplified greatly the topological structure of the exponential decay and phase curves in the frequency domain, in spite of the increase in the number of families of characteristic waves from four to six. Experimental frequency spectra at different measurement points of a truss with clamped joints showed qualitative agreement with theoretical results, and confirmed the existence of the numerically computed PZs and AZs for the "transverse" and "longitudinal" families of characteristic waves. Quantitative differences between theory and experiment concerned mainly the locations of the PZs, and were attributed to non-perfect clamped conditions at the experimental joints, to unaccounted dynamical effects due to added mass at the joints, or to out-of-plane effects.

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