Journal of Sound and Vibration (1998) **215**(2), 201–210
Article No. sv961545

VIBRATION AND DAMPING ANALYSIS OF THIN-WALLED BOX BEAMS

R. Suresh and S. K. Malhotra
FRP Research Centre, *Indian Institute of Technology*, *Madras* 600 036 *India*

(*Received* 28 *August* 1996, *and in final form* 26 *November* 1996)

Thin walled box sections are used as load bearing members in various industries, aircraft structures and marine vessels. To avoid undesirable effects of the surrounding vibration environment, the structure can be designed to have natural frequency removed from the range of exciting frequency or by introducing damping. In the present study the vibration and damping behaviour of layered composite box beams is analysed using the finite element method. The finite element formulation is based on first order shear deformation theory, which takes shear deformation of the beam into consideration. The effect of the number of layers, end conditions and fiber angle on frequency and loss factor is studied for two materials, namely, graphite–epoxy and glass–epoxy.

1. INTRODUCTION

Thin walled box sections are used as load bearing members in various industries, as structural part of vehicles (marine, aircraft and underwater vessels etc.), and other applications. When used in a vibratory environment, the structures can receive transmitted vibratory energy which can cause high vibratory stresses and environmental fatigue of the materials. This will cause undesirable effects on the structure. To avoid this, the structure can be designed to have natural frequencies removed from the range of exciting frequency or by introducing damping.

A significant amount of research has been conducted on the dynamic analysis of plates, beams and columns. Classical plate theory is used in some of the earlier works on the laminated composites. Wittrick [1] and Wittrick and Williams [2] have presented an exact analysis of plate structures. A VIPASA computer program by Wittrick and Williams [2] provides an efficient way of calculating exact buckling and natural frequencies of flat plate prismatic structures, but has some limitations concerning the material properties in that the the coupling between extensional and bending stresses cannot be included in the analysis. Shear deformation effects are of critical importance in the analysis of laminated plates [3]. Dawe and Peshkam [4] used shear deformable plate theory with finite strip method to find the natural frequency of the plate structure.

Theoretical analysis of flexural vibration of multi-layered beams, plates and shells employing various arrangements of elastic and viscoelastic layers have been reviewed by Nakra [5, 6]. Damped vibration analysis of fiber reinforced composite plates was carried out by Malhotra *et al*. [7], and Alam and Asnani [8]. Malhotra *et al*. [7] used the Rayleigh–Ritz method to analyze the plate having linear variation in thickness. Alam and Asnani [8] analyzed the simply supported plate using variational methods by including the shear deformation and rotary inertia into account. In the present study, shear deformable

0022–460X/98/320201 + 10 $30.00/0

plate theory is used with the finite element method to find the natural frequencies and loss factor for composite box beam. For damping analysis, the moduli are considered in complex forms. The effect of the number of layers, end conditions and laminate angle on the dynamic behaviour of composite box beams is studied.

2. FINITE ELEMENT MODEL

Figure 1 shows the four noded plate element in the local x-y plane with five degrees of freedom per node namely, u_0, v_0, w_0, mid-plane translations in the x, y, z, directions and θ_x, θ_y, the rotations of the normal, from the undeformed mid-plane, in the yz- and xz-plane respectively. These normals are not necessarily normal to the mid-plane after deformation and consequently, shear deformation is permitted.

The displacements in the laminate can be written as

$$u = u_0 + z\theta_x, \qquad v = v_0 - z\theta_y, \qquad w = w_0. \tag{1}$$

The displacement field, $\{\boldsymbol{\delta}\}_e$, of the element can be expressed as the polynomial shape function, $[\mathbf{N}_i]$, and nodal displacement, $\{\mathbf{a}_i\}_e$, associated with node i is given by

$$\{\boldsymbol{\delta}\}_e = [\mathbf{N}_i]\{\mathbf{a}_i\}_e. \tag{2}$$

The internal strain energy of the element can be determined by integrating the product of the inplane stress resultant, $[\mathbf{N}_m]$, and the extensional strain, $\{\boldsymbol{\varepsilon}_m\}$, moment resultants, $[\mathbf{M}]$, and bending curvature, $\{\boldsymbol{\kappa}\}$, and shear stress resultants, $[\mathbf{N}_s]$, and shear strain, $\{\boldsymbol{\varepsilon}_s\}$, as given by

$$U = \tfrac{1}{2}\left\{\int [\mathbf{N}_m]^{\mathrm{T}}\{\boldsymbol{\varepsilon}_m\}\,\mathrm{d}a + \int [\mathbf{M}]^{\mathrm{T}}\{\mathbf{k}\}\,\mathrm{d}a + \int [\mathbf{N}_s]^{\mathrm{T}}\{\boldsymbol{\varepsilon}_s\}\,\mathrm{d}a\right\}. \tag{3}$$

The internal strain energy can be written in the form of stiffness and nodal displacement as

$$U = \tfrac{1}{2}\{\mathbf{a}_i\}_e[\mathbf{K}_e]\{\mathbf{a}_i\}_e. \tag{4}$$

To avoid the numerical overstiffness effect or locking caused by shear terms as the thickness of the plate is reduced, the Gaussian quadrature formula adopting selective integral [9] is used in deriving the element stiffness matrix.

For damping analysis, the complex stiffness matrix $\overline{\mathbf{K}}_e = [\mathbf{K}_e + \mathrm{j}\mathbf{H}_e]$, (where $\mathbf{H}_e$ is the imaginary part of the complex stiffness matrix) is obtained by replacing the elasticity moduli E_L, E_T, G_{LT} by complex moduli $\mathbf{E}_{LC}$, $\mathbf{E}_{TC}$, $\mathbf{G}_{LTC}$ when deriving the stiffness matrix.

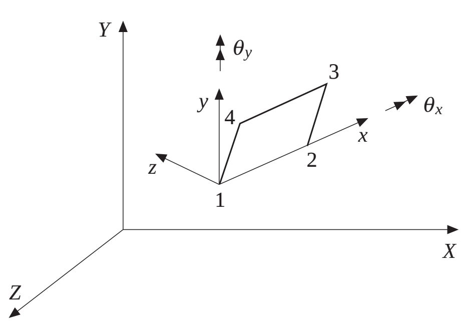

Figure 1. Globally inclined plate element (the element lies in the local xy-plane).

The complex moduli are given in the following form $\mathbf{E}_{LC} = E_L(1 + \mathrm{j}\eta_L)$, $\mathbf{E}_{TC} = E_T(1 + \mathrm{j}\eta_T)$, $\mathbf{G}_{LTC} = G_{LT}(1 + \mathrm{j}\eta_{LT})$, where η_L, η_T, η_{LT} are the loss factors for E_L, E_T, G_{LT}. The value of loss factor, η, is computed from corresponding specific damping capacity, ψ, divided by 2π. The material loss factor for fiber material, in general, is very small in comparison with that of matrix materials. The material loss factor for a matrix material is different for different materials and its value depends upon frequency, temperature and strain. For the present study, the value of material loss factor, η, is taken from Lin *et al*. [10] for glass–epoxy and graphite–epoxy.

The kinetic energy of the vibrating plate is given by

$$T = \tfrac{1}{2}\int \rho[u_{,t}]^2 + [v_{,t}]^2 + [w_{,t}]^2 \,\mathrm{d}v, \tag{5}$$

where ρ is the material density, which is uniform throughout the volume and the subscript $,t$ indicates the derivative with respect to time. Using equation (1) and integrating through the thickness, the kinetic energy becomes:

$$T = \tfrac{1}{2}\rho h \int_A \{(u_{0,t})^2 + (v_{0,t})^2 + (w_{0,t})^2 + h^2/12((\theta_{x,t})^2 + (\theta_{y,t})^2)\}\,\mathrm{d}A. \tag{6}$$

The same shape function as described in the displacement variation is used in the consistent mass matrix for the constant thickness element is expressed by

$$\mathbf{M}^e_{ij} = C_{ij}\langle h, h, h, h^3/12, h^3/12\rangle, \tag{7}$$

where $\langle h, h, h, h^3/12, h^3/12\rangle$ are the diagonal elements of the consistent mass matrix, the remaining elements of the 5×5 element mass matrices are zero, and C_{ij} are the mass matrix coefficients.

The assembly of different stiffness matrices for each element must be made in a global co-ordinate system. This can be done by using the transformation matrix [**T**], which relates displacement in local and global systems as follows:

$$\{\bar{\mathbf{a}}\}_e = [\mathbf{T}]\{\mathbf{a}\}_e. \tag{8}$$

Stiffness and mass matrices in the global system are given by

$$[\mathbf{K}] = [\mathbf{T}]\,[\bar{\mathbf{K}}_e]\,[\mathbf{T}]^{\mathrm{T}}, \qquad [\mathbf{M}] = [\mathbf{T}]\,[\mathbf{M}_e]\,[\mathbf{T}]^{\mathrm{T}}. \tag{9}$$

The governing equation for the vibration in the matrix form is

$$[[\mathbf{K}] - \boldsymbol{\lambda}_c\,[\mathbf{M}]]\{\bar{\mathbf{a}}\} = 0. \tag{10}$$

where [**K**] is the global complex stiffness matrix and [**M**] is the global mass matrix, $\boldsymbol{\lambda}_c$ is the complex eigenvalue, and $\{\bar{\mathbf{a}}\}$ is the eigenvector. The above equation is solved by using a simultaneous iteration technique.

3. NUMERICAL EXAMPLES

The finite element formulation described in the earlier section has been used to study the effect of materials and fiber orientation angle on the non-dimensional fundamental frequency and loss factor of composite box beams. In order to validate the above finite element method, the fundamental frequency of isotropic plates, composite plates, and beams are compared.

TABLE 1

Non-dimensional vibration frequencies for an isotropic plate

Aspect ratio	Simply supported		Clamped	
	Leissa [11]	Present	Leissa [11]	Present
1	19·74	19·70	35·99	35·91
2	49·35	49·25	98·59	98·30

TABLE 2

Fundamental vibration frequencies for square antisymmetric angle ply laminate

	Number of layers					
	2		4		6	
Angle	Jones *et al*. [13]	Present	Jones *et al*. [13]	Present	Jones *et al*. [13]	Present
0°	18·81	18·73	18·81	18·73	18·81	18·73
30°	14·20	14·15	22·18	22·10	23·34	23·32
45°	14·64	14·60	23·53	23·43	24·83	24·70

3.1. ISOTROPIC PLATE

The non-dimensional vibration frequencies for an isotropic plate having an aspect ratio of one, two with simply supported and clamped edges are presented in Table 1. The present finite element method results have been compared with those of Leissa [11] and are in good agreement.

TABLE 3

Non-dimensional fundamental frequencies for square antisymmetric cross-ply laminate

		BC			
		ssss		sscc	
Number of layers	b/h	Reddy and Khdeir [14]	Present	Reddy and Khdeir [14]	Present
2	5	8·83	8·78	10·90	10·69
10	5	11·64	11·32	12·92	12·53
10	10	15·78	15·24	20·47	20·26

BC Boundary conditions; s, simply supported; c, clamped.

TABLE 4

Dynamic analysis of the simply supported plate $\lambda_R \times 10^8$

Ply angle (deg.)	0	15	30	45	60	75	90
Alam and Asnani [8]	0·108	0·132	0·180	0·204	0·180	0·132	0·108
Present	0·110	0·133	0·182	0·206	0·182	0·133	0·110

TABLE 5

Natural frequencies of a cantilever box beam

Mode no.	Analytical [15]	FEM. [15]	Present
1	2·912	2·961	3·105
2	17·565	17·868	17·950
3	46·558	47·077	47·252

3.2. ANTI-SYMMETRIC ANGLE PLY LAMINATE

The fundamental vibration frequencies for a square anti-symmetric angle ply laminate are presented in Table 2. The material properties are $E_L/E_T = 40$, $G_{LT}/E_T = 0{\cdot}5$, $v_{LT} = 0{\cdot}25$. The parameters considered are fiber orientation angle and the number of layers. The values obtained for the laminate with S3 (as described by Jones [12]) boundary conditions are compared with those of Jones *et al*. [13] and found to be in good agreement.

3.3. ANTI-SYMMETRIC CROSS-PLY LAMINATE

The non-dimensional fundamental frequencies for a square anti-symmetric cross ply laminate are presented in Table 3. The effect of side to thickness ratio and number of layers are studied. The boundary condition and material properties used for the analysis are the same as those used by Reddy and Khdeir [14]. All the layers are assumed to have the same thickness. The shear correction factors are taken as 5/6. The results are compared with analytical results presented by Reddy and Khdeir. Good agreement between results in the literature and present results is found.

3.4. DYNAMIC ANALYSIS OF SIMPLY SUPPORTED PLATE

The variation of the non-dimensional fundamental frequency, λ_R, for the simply supported crisscross-ply laminated plate having four layers is analyzed. The results are compared with those of Alam and Asnani [8]. The results shown in Table 4 are in good agreement with results in the literature.

3.5. ISOTROPIC BOX BEAM

The natural frequencies of a slender box beam, with the following configurations, are shown in Table 5. The span of the beam is 20 m, its width and height is 1 m. Material properties used are the same as used by Petyt [15]. The natural frequencies are compared with the analytical and finite element results of Petyt and good agreement is observed.

3.6. COMPOSITE BOX BEAM

The vibration behaviour of two equal cell orthotropic tubes of square cross-section is analysed. The depth of the beam is B, its span is five times its depth (i.e., 5B) with diaphragm ends, and all the sides of the tube are of the same thickness, h. Two different

TABLE 6

Fundamental frequency parameter of a two cell box beam

	Thick tube $h/B = 0{\cdot}1$		Thin tube $h/B = 0{\cdot}01$	
No.	Dawe and Peshkam [4]	Present	Dawe and Peshkam [4]	Present
1	16·999	16·5	177·0	176·0

thickness values are considered, namely $h/B = 0{\cdot}1$ and $h/B = 0{\cdot}01$. For the present example, a [0/90/0/90/0] lay-up sequence is considered. The thickness of each of the 0° plies is $h/6$ and that of each of the 90° plies is $h/4$. The material properties are taken from Dawe and Peshkam [4]. The frequency parameter $\{(\Omega = fB^2/h(\rho/0{\cdot}517745E_L)^{1/2})$ where f is the frequency, ρ is the mass density, E_L is the longitudinal modulus$\}$ is shown in Table 6. The results show good agreement with those of Dawe and Peshkam.

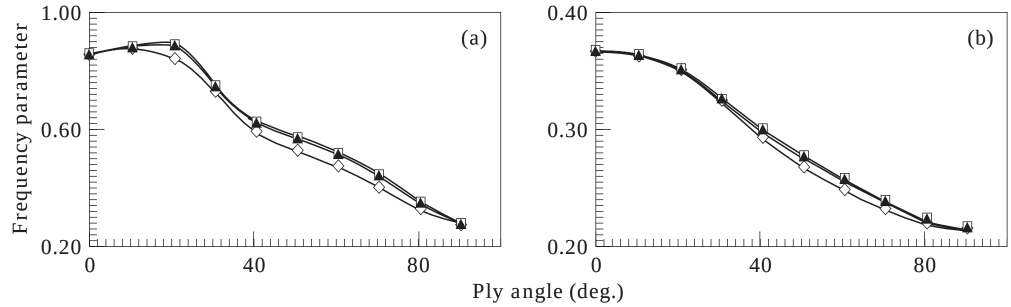

Figure 2. Free vibration analysis of simply supported box beam with symmetrical lay-up: (a) graphite–epoxy; (b) glass–epoxy. Key: —□—, 16 layers; —▲—, 8 layers; —◇—, 4 layers.

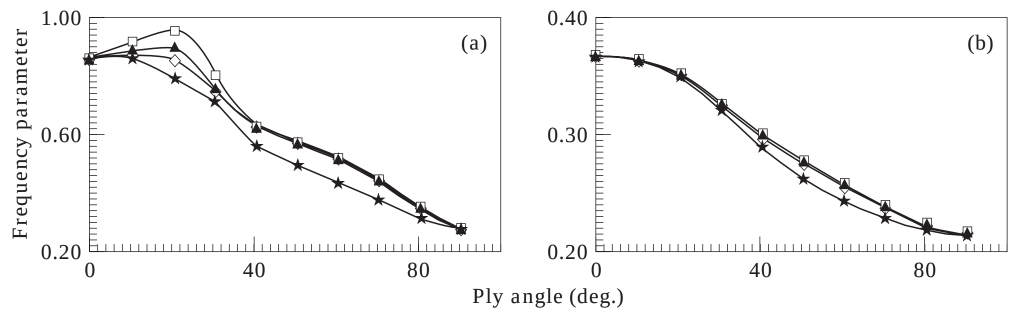

Figure 3. Free vibration of analysis simply supported box beam with antisymmetrical lay-up: (a) graphite–epoxy; (b) glass–epoxy. Key : —□—, 8 layers; —△—; 6 layers; —○—, 4 layers; —★—, 2 layers.

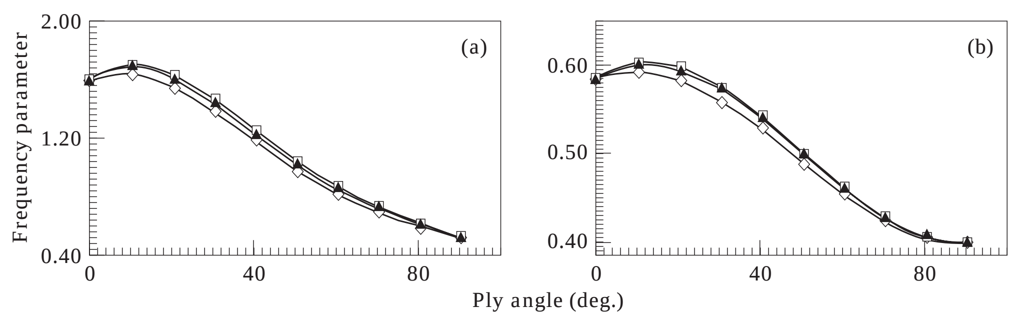

Figure 4. Free vibration analysis of fixed box beam with symmetrical lay-up: (a) graphite–epoxy; (b) glass–epoxy. Key as for Figure 2.

4. RESULTS AND DISCUSSIONS

4.1. FREE VIBRATION WITHOUT DAMPING

The results of the natural frequency of single cell laminated composite thin walled box beams made of glass–epoxy and graphite–epoxy are presented. The material properties are: graphite–epoxy $E_L/E_T = 40{\cdot}0$, $G_L/E_T = 0{\cdot}6$, $G_T/E_T = 0{\cdot}5$, $v_{LT} = 0{\cdot}25$; glass–eopxy $E_L/E_T = 4{\cdot}70$, $G_L/E_T = G_T/E_T = 0{\cdot}5$, $v_{LT} = 0{\cdot}26$. Configuration of the box beam is, $a/b_f = 12{\cdot}0$, $b_w/b_f = 1{\cdot}0$, $t_w/t_f = 1{\cdot}0$ (where a is the span of the beam, b_w, b_f are the web and flange width respectively, t_w, t_f are the thickness of the web and flange respectively). Symmetric and antisymmetric angle ply laminates with simply supported and fixed end conditions are analysed in the present study. In this analysis, the imaginary part in the complex stiffness matrix is zero. Figures 2–5 show the variation of frequency parameter with ply angle for a single cell box beam. The non-dimensional frequency parameter for free vibration without damping is given by $(fb^2(\rho/t^2E_T)^{1/2})$, where f is the fundamental frequency, ρ is the mass density, E_T is the transverse modulus, and b and t are the width and thickness of the flange.

4.1.1. *Simply supported box beam*

Figures 2 and 3 show the variation of frequency parameter with ply angle, for symmetric and anti-symmetric angle ply composite laminated box beams. It is observed from the figures that, as the number of layers increases, the natural frequency increases. This is due to the fact that inplane–shear coupling (A_{16} and A_{26}) and bending–twisting coupling (D_{16} and D_{26}) decreases as the number of layers increases for symmetric lay-up, and for anti-symmetric lay-up the inplane–twisting coupling (B_{16} and B_{26}) effect decreases as the number of layers is increased, keeping the total thickness constant.

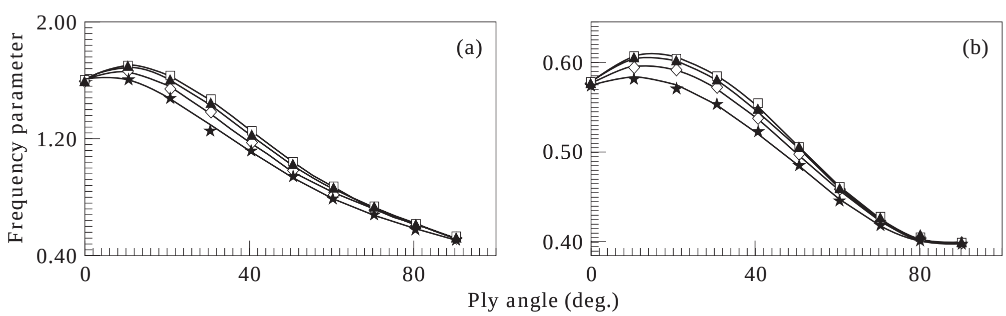

Figure 5. Free vibration analysis of fixed box beam with antisymmetrical lay-up: (a) graphite–epoxy; (b) glass–epoxy. Key as for Figure 3.

TABLE 7

Material properties of unidirectional composite for 0·5 *volume fraction* [10]

Material	E_L (Gpa)	E_T (Gpa)	G_{LT} (Gpa)	ψ_L	ψ_T	ψ_{LT}	γ_{LT}	Specific gravity
Glass–DX-210 (Glass–epoxy)	37·78	10·9	4·91	0·0087	0·0505	0·0691	0·3	1·87
HMS–DX-210 (Graphite–epoxy)	172·7	7·2	3·76	0·0045	0·0422	0·0705	0·3	1·55

For graphite–epoxy the natural frequency increases from 0° to 20° ply angle and then starts decreasing. Unlike graphite–epoxy, the natural frequency steadily decreases from 0° to 90° for glass–epoxy (0° angle orientation coincides with axial direction of the beam).

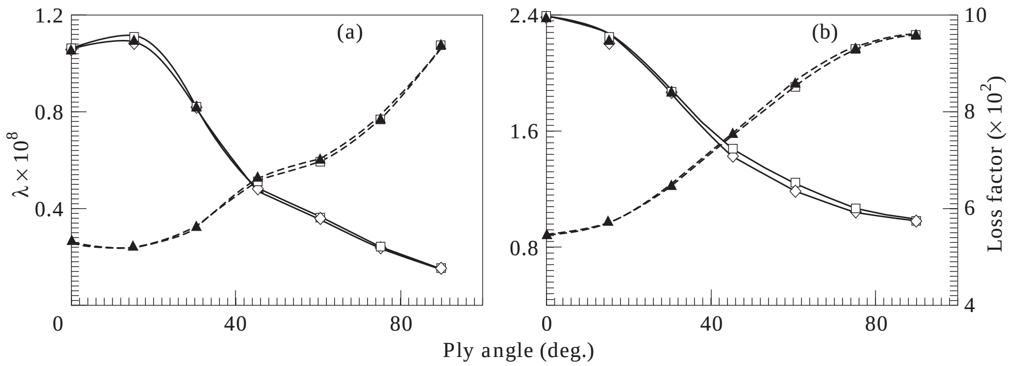

Figure 6. Free vibration analysis (with damping) of simply supported box beam with symmetrical lay-up: (a) graphite–epoxy; (b) glass–epoxy. Key as for Figure 2.

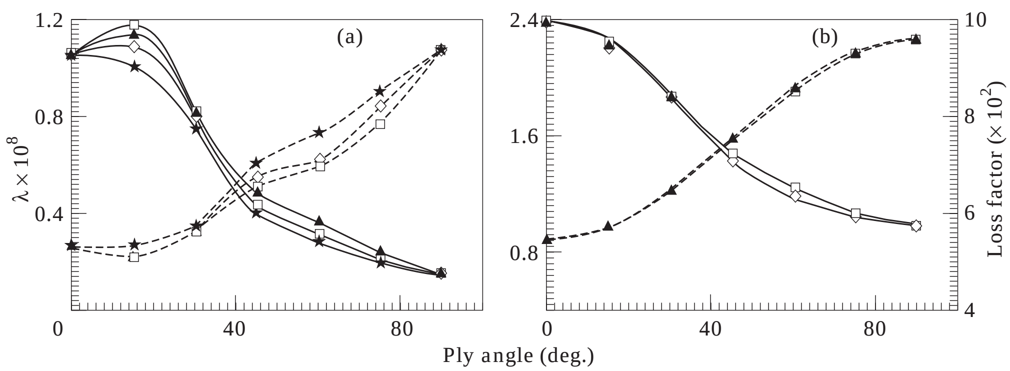

Figure 7. Free vibration analysis (with damping) of simply supported box beam with antisymmetrical lay-up: (a) graphite–epoxy; (b) glass–epoxy. Key as for Figure 3.

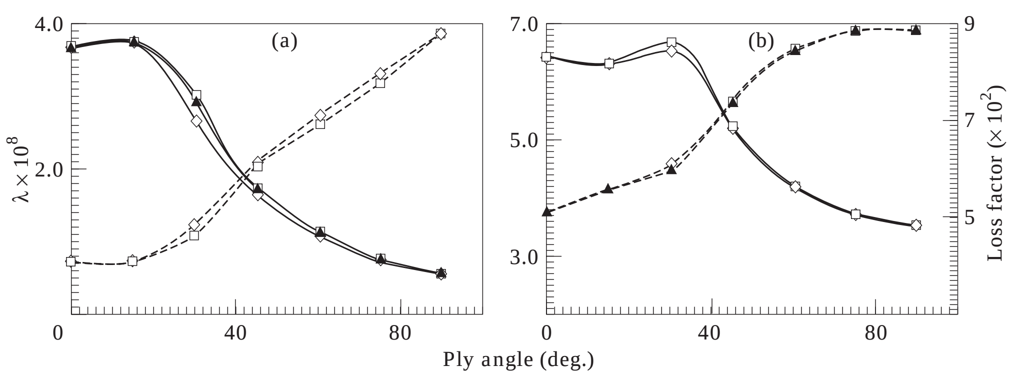

Figure 8. Free vibration analysis (with damping) of fixed box beam with symmetrical lay-up: (a) graphite–epoxy; (b) glass–epoxy. Key as for Figure 2.

4.1.2. *Fixed box beam*

Variation of the frequency parameter with respect to fiber angle is shown in Figures 4 and 5. Similar to the simply supported cases, the inplane–shear coupling and bending–twisting coupling decreases for the symmetric lay-up sequence. For antisymmetric laminate along with inplane–shear and bending–twisting coupling, the inplane twisting coupling effect decreases as the number of layers increases, while keeping the total thickness of the laminate as constant. Due to this reduction in coupling effect, the natural frequency increases. The maximum natural frequency occurs between 0° and 20° for all the cases.

4.2. FREE VIBRATION WITH DAMPING

The variation of the non-dimensional fundamental frequency, λ, and loss factor with fiber angle orientation for graphite–epoxy and glass–epoxy materials are shown in Figures 6–9. The same box beam configuration as before is taken for this analysis. Symmetric and anti-symmetric laminates with simply supported and fixed end conditions are presented in this study. The material properties are taken from Lin *et al*. [10] and it is shown in Table 7. Non-dimensional fundamental frequency for free vibration with damping is given by $\lambda = f\rho t/E_L$, where f is the frequency corresponding to the real part of the complex eigenvalue, ρ is the mass density, E_L is the longitudinal modulus and t is the thickness. The loss factor is given by the ratio of imaginary part to real part of the complex eigenvalue.

4.2.1. *Simply supported box beam*

The non-dimensional fundamental frequency (λ) increases slightly with fiber angle up to 15° and then starts decreasing (as shown in Figures 6 and 7) for symmetric and antisymmetric lay-up sequences for the graphite–epoxy material, whilst glass–epoxy laminate shows a steady decrease in non-dimensional fundamental frequency (λ) from 0° to 90°. As the number of layers increases, by keeping the total thickness of the laminate as constant, the natural frequency increases. This is due to the coupling effect as explained earlier but this effect is smaller compared with the free vibration without damping. The loss factor follows an inverse pattern of variation with fiber angle. As the number of layers increases, by keeping the total thickness constant, the loss factor decreases.

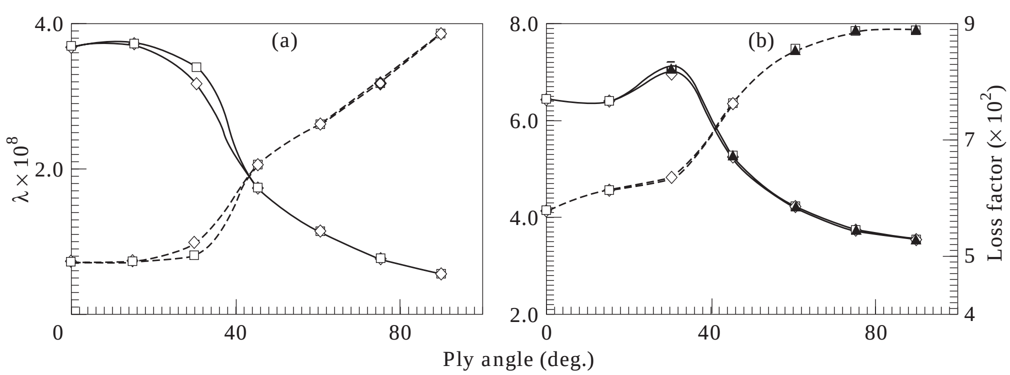

Figure 9. Free–vibration analysis (with damping) of fixed box beam with antisymmetrical lay-up: (a) graphite–epoxy; (b) glass–epoxy. Key as for Figure 3.

4.2.2. *Fixed box beam*

Figures 8 and 9 show the variation of non-dimensional fundamental frequency (λ) with fiber angle, for symmetric and anti-symmetric angle ply composite laminated box beams. The non-dimensional fundamental frequency increases up to 30° fiber angle and then starts decreasing for glass–epoxy, whereas graphite–epoxy shows almost constant frequency parameter for fiber angle up to 15° and then starts decreasing. The coupling effects are small compared to the vibration without damping. The loss factor follows an inverse pattern of variation with the fiber angle i.e., the loss factor decreases as the number of layers increases.

5. CONCLUSIONS

The vibration and damping behaviour of layered composite box beams have been presented. The effect of end conditions, number of layers, and fiber angle on frequencies and loss factor have been studied for the glass–epoxy and graphite–epoxy systems.

REFERENCES

1. W. H. Wittrick 1968 *International Journal of Mechanical Science* **10**, 946–966. General sinusoidal stiffness matrix for buckling and vibration analysis of thin-walled structure.
2. W. H. Wittrick and F. W. Williams 1974 *International Journal of Mechanical Science* **16**, 209–239. Buckling and vibration of anisotropic or orthotropic plate assemblies under combined loading.
3. R. K. Kapania and Stefano Raciti 1989 *Journal of the American Institute of Aeronautics and Astronautics* **27**, 923–934. Recent advances in analysis of laminated beam and plates, part-1: shear effects and buckling.
4. D. J. Dawe and V. Peshkam 1990 *International Journal of Mechanical Science* **32**, 743–766. Buckling and vibration of long plate structure by complex finite strip methods.
5. B. C. Nakra 1981 *Shock and Vibration Digest* **13**, 17–20. Vibration control with viscoelastic materials—II.
6. B. C. Nakra 1984 *Shock and Vibration Digest* **16**, 17–22. Vibration control with viscoelastic materials—III.
7. S. K. Malhotra, N. Ganesan and M. A. Velusamy 1988 *Journal of Sound and Vibration* **120**, 617–628. Damped vibration of orthotropic square plates having variable thickness, (linear variation).
8. N. Alam and N. T. Asnani 1986 *Journal of Composite Materials* **20**, 2–18. Vibration and damping analysis of fiber reinforced composite material plates.
9. T. J. R. Hughes, M. Cohen and M. Haroun 1978 *Nuclear Engineering Design* **46**, 203–222. Reduced and selective integration technique in the finite element analysis of plates.
10. D. X. Lin, R. G. Ni and R. D. Adams 1984 *Journal of Composite Materials* **18**, 132–152. Prediction and measurement of the vibrational damping parameters of carbon and glass fiber reinforced plastics plates.
11. A. W. Leissa *NASA*, *ST*160 *Washington*. Vibration of plates.
12. R. M. Jones 1985 *Mechanics of composite materials*. Washington, D.C.: Scripta.
13. R. M. Jones, S. Morgan and J. M. Whitney 1973 *American Society of Mechanical Engineers*, *Journal of Applied Mechanics* **40**, 1143–1144. Buckling and vibration of anti-symmetrically laminated angle ply rectangular plates.
14. J. N. Reddy and A. A. Khdeir 1989 *Journal of the American Institute of Aeronautics and Astronautics* **27**, 1808–1817. Buckling and vibration of laminated plates using various plate theories.
15. M. Petyt 1990 *Introduction to finite element vibration analysis*. New York: Cambridge University Press.