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THE THEORETICAL AND APPLICATION STUDY ON A DOUBLE LAYER MICROPERFORATED SOUND ABSORPTION STRUCTURE

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The calculation of resonance and anti-resonance frequencies of a double layer microperforated sound absorption structure is studied, and simplified analytical formulae which will be very useful for the design and analysis of wide bandwidth double layer microperforated sound absorption structures are derived. The results obtained provide an effective method for the design of wide bandwidth microperforated panel silencers.

1. INTRODUCTION

A microperforated Panel Sound Absorption Structure (MPSAS) has been developed to control aerodynamic noise [1–3, 7] on the principle that the acoustic resistance of the structure itself will be sufficient to dissipate sound energy without sound absorption material when the dimensions of the perforation are reduced to sub-millimetre level. The effective sound absorption bandwidth of a single layer MPSAS is often limited, but a double layer one can widen the sound absorption bandwidth significantly.

The calculation of resonance and anti-resonance sound absorption frequencies of a double layer MPSAS had not been given in simplified analytical formulae which would be very useful to have the sound absorption structure tuned for various noise sources with different frequency spectra. In this paper, the impedance characteristic of a double layer MPSAS is analyzed so that simplified analytical formulae for resonance and anti-resonance sound absorption frequencies can be derived. With the application of the results obtained herein, wide bandwidth microperforated panel silencers can easily be developed.

2. SOUND ABSORPTION MECHANISM

The schematic diagrams of a single layer MPSAS and a double layer one and their equivalent circuits are shown in Figures 1 and 2, respectively. Here t, d, p and D are the thickness, the diameter of perforation, the open area ratio and the cavity depth of microperforated panel respectively. ρ is the density of air, c is the sound speed in air and ρc is the characteristic impedance of air. R_a, M_a and C_a are the relative acoustic resistance,

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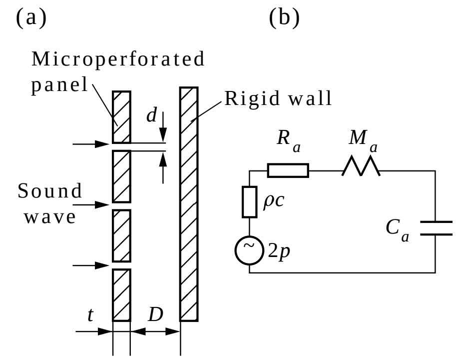

Figure 1. The single layer MPSAS.

relative acoustic mass and relative acoustic compliance of a single layer MPSAS. In terms of acoustic theory [4, 5], R_a, M_a and C_a can approximately be given by

$$R_a = 32\eta\sqrt{1 + x^2/32}l_0/s_0\, d^2 p, \qquad M_a = \rho l/s_0 p, \qquad C_a = s_0 D/\rho c^2, \tag{1–3}$$

where η is the kinematic viscosity of air; $x = 10d\sqrt{f}$ and f is the frequency of sound wave; $l_0 = t + \beta d$ and β is the end correction coefficient for calculating acoustic resistance; s_0 is the cavity area of each hole; $l = t + \beta_0 d + pD/3$ and β_0 is the end correction coefficient for calculating the effective neck length.

Therefore, the relative acoustic impedance of a single layer MPSAS can be expressed as:

$$Z = R_a + \mathrm{j}\omega M_a + 1/\mathrm{j}\omega C_a. \tag{4}$$

The grazing flow has an effect on the impedance of perforates. Rao and Munjal [6] found that the acoustic reactance does not change significantly with the variation in grazing flow velocity, whereas the acoustic resistance increases with grazing flow velocity. Later it will be shown that the resonance and anti-resonance sound absorption frequencies of an MPSAS mainly depend on the acoustic reactive characteristics, and are more or less independent of acoustic resistance. Therefore, the effect of grazing flow on the calculation of resonance and anti-resonance sound absorption frequencies is considered negligible.

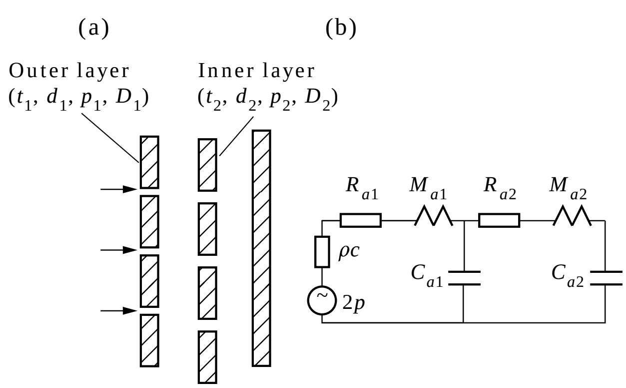

Figure 2. The double layer MPSAS.

Equations (4) is accurate enough for engineering application when $\lambda \gg l$ and $\lambda \gg D$. In most cases, both $\lambda \gg l$ and $\lambda \gg D$ are satisfied. Here λ is the sound wavelength.

It can be seen from equation (1) that the smaller the dimensions of the perforation, the larger the acoustic resistance of the MPSAS. When the diameter is small enough, without sound absorption material, the acoustic resistance of the MPSAS is sufficient to provide satisfactory sound absorption. The impedance characteristic of a double layer MPSAS is not the simple combination of two single layer MPSASs due to the coupling reaction between them; however, the acoustic impedance of a double layer MPSAS can be derived from its equivalent circuit. The parameters in the equivalent circuit (see Figure 2(b)) can also be calculated from equations (1), (2) and (3).

3. IMPEDANCE CHARACTERISTIC ANALYSIS OF DOUBLE-LAYER MPSAS

The relative acoustic impedance of the double layer MSPAS shown in Figure 2 can be expressed as

$$Z = R_{a1} + \mathrm{j}\omega M_{a1} + \frac{1}{\mathrm{j}\omega C_{a1}} + \left(\frac{1}{\omega C_{a1}}\right)^2 \frac{1}{R_{a2} + \mathrm{j}\omega M_{a2} + (1/\mathrm{j}\omega C_{a2}) + (1/\mathrm{j}\omega C_{a1})} \tag{5}$$

where ω is the angular frequency of the sound wave.

The imaginary part of Z is the relative acoustic reactance of the double layer MPSAS. With $F(\omega) = \mathrm{Im}\,(Z)$, one has

$$F(\omega) = \left(\omega M_{a1} - \frac{1}{\omega C_{a1}}\right) - \left(\frac{1}{\omega C_{a1}}\right)^2 \frac{\omega M_{a2} - (1/\omega C_{a2}) - (1/\omega C_{a1})}{R_{a2}^2 + \{\omega M_{a2} - (1/\omega C_{a2}) - (1/\omega C_{a1})\}^2}. \tag{6}$$

When the relative acoustic reactance is equal to zero, resonance sound absorption occurs in the sense that the sound absorption becomes maximum. That is

$$\left(\omega M_{a1} - \frac{1}{\omega C_{a1}}\right) - \left(\frac{1}{\omega C_{a1}}\right)^2 \frac{\omega M_{a2} - (1/\omega C_{a2}) - (1/\omega C_{a1})}{R_{a2}^2 + \{\omega M_{a2} - (1/\omega C_{a2}) - (1/\omega C_{a1})\}^2} = 0. \tag{7}$$

When the relative acoustic reactance becomes a maximum, anti-resonance sound absorption happens in the sense that the sound absorption becomes minimum. Therefore, the derivative of the relative acoustic reactance is equal to zero. That is

$$F'(\omega) = 0. \tag{8}$$

Therefore, resonance and anti-resonance sound absorption frequencies of a double layer MPSAS can be obtained from equations (7) and (8), respectively.

4. RESONANCE AND ANTI-RESONANCE SOUND ABSORPTION FREQUENCIES CALCULATION

With the assistance of a computer, although the numerical solutions of equations (7) and (8) can easily be obtained, it is not easy to obtain the analytical solutions. In this paper, simplified analytical formulae are to be given which would be convenient for engineering applications.

The typical relative acoustic reactance characteristic curve of a double layer MPSAS are given in Figure 3. The resonance frequencies can be found from the intersection points

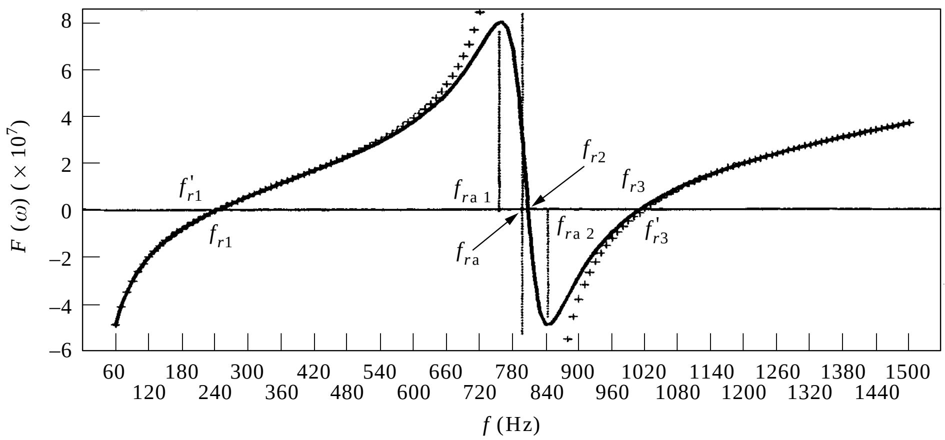

Figure 3. The relative acoustic reactance characteristic curve of a double layer MPSAS. ($t_1 = t_2 = 0{\cdot}75$ mm; $d_1 = d_2 = 0{\cdot}8$ mm, $p_1 = 2{\cdot}5\%$, $p_2 = 1\%$, $D_1 = 80$ mm, $D_2 = 120$ mm). ——, $R_{a2} \neq 0$; $+++$, $R_{a2} = 0$.

on the horizontal axis and the anti-resonance frequencies from the extremum points on the curve.

As shown in Figure 3, three resonance frequencies (f_{r1}, f_{r2}, f_{r3} and $f_{r1} < f_{r2} < f_{r3}$) and two anti-resonance frequencies (f_{ra1}, f_{ra2}) exist for the double layer MPSAS when $R_{a2} \neq 0$. f_{r2} is in between f_{ra1} and f_{ra2}. The absolute value of the relative acoustic reactance increases rapidly when the frequency of sound wave approaches f_{r2}; therefore the half-power sound absorption bandwidth of f_{r2} is very limited. On the other hand, when the frequency of the sound wave approaches f_{r1} or f_{r3}, the absolute value of the relative acoustic reactance changes slowly. As a result, the half-power sound absorption bandwidths of f_{r1} and f_{r3} are much wider than that of f_{r2}. Normally the half-power sound absorption bandwidths of f_{r1} and f_{r3} can be six times a one-third octave band or more. Therefore, it is concluded that f_{r1} and f_{r3} are the two main resonance frequencies of the double layer MPSAS.

Further analysis shows that f_{r2} does not exist when $R_{a2} = 0$. As shown in Figure 3, the relative acoustic reactance curve will be not continuous when $R_{a2} = 0$ and the two resonance frequencies f'_{r1} and f'_{r3} are very close to f_{r1} and f_{r3}, respectively. This can be explained as follows: the difference between the damped resonance frequency and the undamped resonance frequency of the structure is negligible when the damping of any structure is small enough. f_{ra1} is very close to f_{ra2} and f_{ra} is in between f_{ra1} and f_{ra2}, being approximately equal to $(f_{ra1} + f_{ra2})/2$. Therefore, f'_{r1} and f'_{r3} can be used as the approximate solution for f_{r1} and f_{r3}, respectively, and f_{ra} can be used to present the anti-resonance sound absorption characteristics. It is not difficult to find the analytical solutions for f'_{r1}, f'_{r3} and f_{ra} from equations (7) and (8) when $R_{a2} = 0$.

By letting $R_{a2} = 0$, equation (7) can be reduced to

$$\left(\omega M_{a1} - \frac{1}{\omega C_{a1}}\right) - \left(\frac{1}{\omega C_{a1}}\right)^2 \frac{1}{\omega M_{a2} - (1/\omega C_{a2}) - (1/\omega C_{a1})} = 0. \tag{9}$$

By assuming $\omega \neq 0$, $\omega M_{a2} - (1/\omega C_{a2}) - (1/\omega C_{a1}) \neq 0$ and $x = \omega^2$, equation (9) can be reduced to

$$Ex^2 + Fx + G = 0. \tag{10}$$

The two roots of equation (10) represent the two main undamped resonance sound absorption frequencies of the structure and it is easy to obtain the two roots. When $\omega M_{a2} - (1/\omega C_{a2}) - (1/\omega C_{a1}) = 0$, the value of equation (9) will become infinite. Therefore, the anti-resonance sound absorption frequency can be calculated from

$$\omega M_{a2} - (1/\omega C_{a2}) - (1/\omega C_{a1}) = 0. \tag{11}$$

By letting $d_1/d_2 = \gamma$, one can write $s_{02}/s_{01} = \gamma^2 p_1/p_2$. By solving equation (9) for f'_{r1} and f'_{r3}, one obtains

$$f'_{r1} = (1/\sqrt{2})\sqrt{(f_1^2 + f_2^2 + \gamma^2 f_{12}^2) - \sqrt{(f_1^2 + f_2^2 + \gamma^2 f_{12}^2)^2 - 4f_1^2 f_2^2}}, \tag{12a}$$

$$f'_{r3} = (1/\sqrt{2})\sqrt{(f_1^2 + f_2^2 + \gamma^2 f_{12}^2) + \sqrt{(f_1^2 + f_2^2 + \gamma^2 f_{12}^2)^2 - 4f_1^2 f_2^2}}. \tag{12b}$$

By solving equation (11) for f_{ra}, one obtains

$$f_{ra} = \sqrt{f_2^2 + \gamma^2 f_{12}^2}, \tag{13}$$

where

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{1}{M_{a1}C_{a1}}} = \frac{c}{2\pi}\sqrt{\frac{p_1}{D_1 l_1}}, \qquad f_2 = \frac{1}{2\pi}\sqrt{\frac{1}{M_{a2}C_{a2}}} = \frac{c}{2\pi}\sqrt{\frac{p_2}{D_2 l_2}}$$

$$f_{12} = \frac{1}{2\pi}\sqrt{\frac{1}{M_{a2}C_{a1}}} = \frac{c}{2\pi}\sqrt{\frac{p_1}{D_1 l_2}},$$

in which f_1 and f_2 are the resonance sound absorption frequencies of the outer and inner layers of the MPSAS when they are activated, respectively. f_{12} is the result of the coupling reaction.

Some typical results are given in Table 1, where f_{r1}, f_{r2}, f_{r3}, f_{ra1} and f_{ra2} are the numerical solutions of equations (7) and (8), and f'_{r1}, f'_{r3} and f_{ra} are calculated from formulae (12a), (12b) and (13), respectively. It can be seen that the error introduced by the simplified analytical formulae (12a), (12b) and (13) is less than 1% compared to the numerical solution values.

5. APPLICATION STUDY

The validity of the simplified analytical formulae derived in this paper have been checked by sound absorption coefficient measurement. A typical experimental sound absorption coefficient measurement is shown in Figure 4. The experimental results were obtained by

TABLE 1

Comparison of resonance and anti-resonance frequencies

t, d, p, D (mm)	f_{r1} (Hz)	f'_{r1} (Hz)	f_{r3} (Hz)	f'_{r3} (Hz)	f_{ra1} (Hz)	f_{ra2} (Hz)	$(f_{ra1}+f_{ra2})/2$ (Hz)	f_{ra} (Hz)	f_{r2} (Hz)
0·75, 0·8, 2·5%, 80 0·75, 0·8, 1%, 120	242·9	242·2	1011	1017	758	844	801	805	808
0·8, 0·8, 2·5%, 10 0·8, 0·8, 2·0%, 30	761·2	760·9	3141	3143	2327	2426	2376·5	2377	2378
0·8, 0·8, 2·0%, 10 0·8, 0·8, 1·0%, 30	554·7	554·4	2829	2831	2052	2157	2104·5	2106	2107

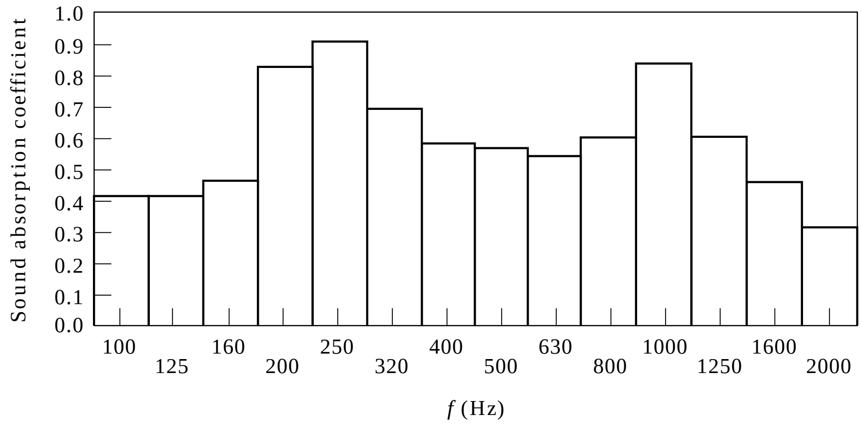

Figure 4. Typical sound absorption coefficient of the MPSAS. ($t_1 = t_2 = 0{\cdot}8$ mm; $d_1 = d_2 = 0{\cdot}8$ mm, $p_1 = 2\%$, $p_2 = 1\%$, $D_1 = 80$ mm, $D_2 = 120$ mm).

using an impedance tube. As shown in Figure 4, the resonance and anti-resonance sound absorption occurred as predicted ($f'_{r1} = 242{\cdot}5$ Hz, $f'_{r3} = 911{\cdot}5$ Hz, $f_{ra} = 731{\cdot}5$ Hz, with the dimensions given in Figure 4). The sound absorption coefficient measurement show that the formulae are valid and useful in MPSAS design and analysis.

With the application of the formulae derived in this paper, some wide band sound absorption double layer MPSAS mufflers have successfully been developed by the authors to reduce the noise level in fans. For example, one of these was made up of two double layer MPSASs ($t_1 = t_2 = 0{\cdot}75$ mm, $d_1 = d_2 = 0{\cdot}8$ mm, $p_1 = 2{\cdot}5\%$, $p_2 = 1\%$, $D_1 = 80$ mm, $D_2 = 120$ mm, $t'_1 = t'_2 = 0{\cdot}75$ mm, $d'_1 = d'_2 = 0{\cdot}8$ mm, $p'_1 = 2{\cdot}5\%$, $p'_2 = 2\%$, $D'_1 = 10$ mm, $D'_2 = 300$ mm). It was used to reduce the noise level of a Model T35-11-8 # forced draft fan in a kitchen where a super clean environment is needed and sound absorption material is not preferred or allowed. The combination of two double layer MPSASs provided a silencer having good noise reduction over a wide frequency range. The insertion loss of the silencer is shown in Figure 5. The overall noise level of the fan was decreased by more than 18 dB(A).

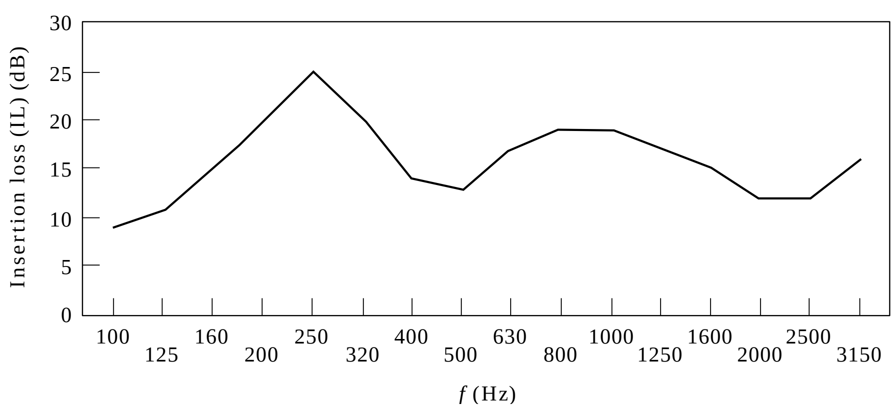

Figure 5. The insertion loss of a typical MPSAS silencer.

REFERENCES

1. D. MAA 1990 *Chinese Journal of Noise and Vibration Control* **3**, 3–9. Composite microperforated panel sound absorption constructions.
2. Y. FENG 1975 *Chinese Journal of Environmental Science* **5**, 68–73. Study of sound absorption by normal and random incidence.
3. D. MAA 1988 *Chinese Journal of Acoustics* **13**, 174–180. Design of microperforated panel sound absorption structure.
4. S. L. ZHAO 1985 *Noise Reduction and Isolation*. Shanghai, P. R. China: Tongji University Press.
5. G. DU, Z. ZHU and X. GONG 1986 *An Introduction to Acoustics*. P. R. China: Shanghai Science and Technology Press.
6. K. N. RAO and M. L. MUNJAL 1986 *Journal of Sound and Vibration* **108**, 283–295. Experimental evaluation of impedance of perforates with grazing flow.
7. D. MAA 1983 *Inter-noise* '83, 415–420. Wide-band sound absorber based on micro-perforated panels.