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VIBRATIONS OF SHALLOW SHELLS DUE TO REMOVAL OF FORMWORK

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This paper is about the forced vibrations of thin-walled shallow shells having rectangular planform and constant thickness and curvature. Assumptions of linear behaviour in both material and kinematic properties of the system are assumed, and an analytical solution is obtained for natural frequencies and mode shapes. The forced vibrations of the shell are computed using the modal superposition approach. The formulation is applied to solve the transient response of a reinforced concrete shallow shell due to removal of the framework. The influence of structural defects is included in an approximate way to estimate forced vibrations in a deteriorated shell. Parametric studies indicate the influence of the main parameters affecting the behaviour or the shell.

1. INTRODUCTION

Very thin concrete shells constructed using a formwork can have important vibration amplitudes following the operation of removal of the formwork. For such slender shells, the vibrations have an instantaneous effect produced by the sudden application of the self weight to the structure; and a delayed effect due to the viscoelastic nature of the structural system. The latter problem was studied by Ballesteros[1]; this paper concentrates on the first problem: the initial stages of the transient response.

Problems of sudden application of self weight to a structure have been studied for prismatic bars by Chen [2] and Laura *et al*. [3]. In practice, the removal of the form will not be performed in an instantaneous fashion at every point of the structural system, as assumed in the present study and which may be considered as a "critical" situation.

There is limited information available in the literature on the forced vibrations of shallow shells. Free vibrations of shallow elastic shells were studied by Reissner [4] based on earlier work by Apeland [5]. Leissa and Kadi [6] considered shallow shells supported on all edges by shear diaphragms. They used linear and non-linear kinematic relations, and showed that the frequencies of spherical and circular shallow shells are much higher than for flat plates, but the differences were small for hyperbolic paraboloidal shells. The influence of boundary conditions has been studied by Leissa and co-workers: Cantilevered shells were studied in reference [7]; shells supported on all four corners in reference [8]; shells with free edges were the subject of reference [9]; and shells with two edges clamped and the other two free were studied in reference [10]. All the above papers employed the Ritz or Galerkin methods and studied natural frequencies of shells with constant curvature and on a

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rectangular planform. Numerical techniques were developed in reference [11], i.e. and isoparametric spline finite strip technique.

In this paper we employ an analytical solution to obtain the natural frequencies of shallow shells, and a modal superposition analysis to obtain the transient response based on the natural frequencies and mode shapes of the shell. The shell equations leading to the free vibration analysis are presented in Sections 2 and 3. The forced vibration problem is described in Section 4. The problem is specialized to an elliptical paraboloidal shell under self-weight as a step load in Section 5, where results are presented. Some concluding remarks are included in Section 6.

2. SHELL EQUATIONS

To obtain the linear elastic response of the shell, use will be made of the approximations due to Marguerre. With reference to Figure 1, the equations of motion may then be written as

$$\frac{\partial N_{11}}{\partial x_1}+\frac{\partial N_{12}}{\partial x_2}=-\rho h\ddot{u}_1, \qquad \frac{\partial N_{12}}{\partial x_1}+\frac{\partial N_{22}}{\partial x_2}=-\rho h\ddot{u}_2,$$

$$\frac{\partial^2 M_{11}}{\partial x_1^2}+2\frac{\partial M_{12}^2}{\partial x_1\,\partial x_2}+\frac{\partial^2 M_{22}}{\partial x_2^2}+(N_{11}k_1+N_{22}k_2+2N_{12}k_{12})=-\rho h\ddot{u}_3, \tag{1}$$

in which ρ is the density of the material, and an overdot denotes differentiation with respect to time. The curvatures of the shell are given by k_1, k_2, and k_{12}. The stress and moment resultants are N_{ij} and M_{ij}.

The stress–strain relations take the form

$$N_{11}=K(\varepsilon_{11}+v\varepsilon_{22}), \qquad N_{22}=K(\varepsilon_{22}+v\varepsilon_{11}), \qquad N_{12}=K(1-v)\varepsilon_{12},$$

$$M_{11}=D(\chi_{11}+v\chi_{22}), \qquad M_{22}=D(\chi_{22}+v\chi_{11}), \qquad M_{12}=D(1-v)\chi_{12}, \tag{2}$$

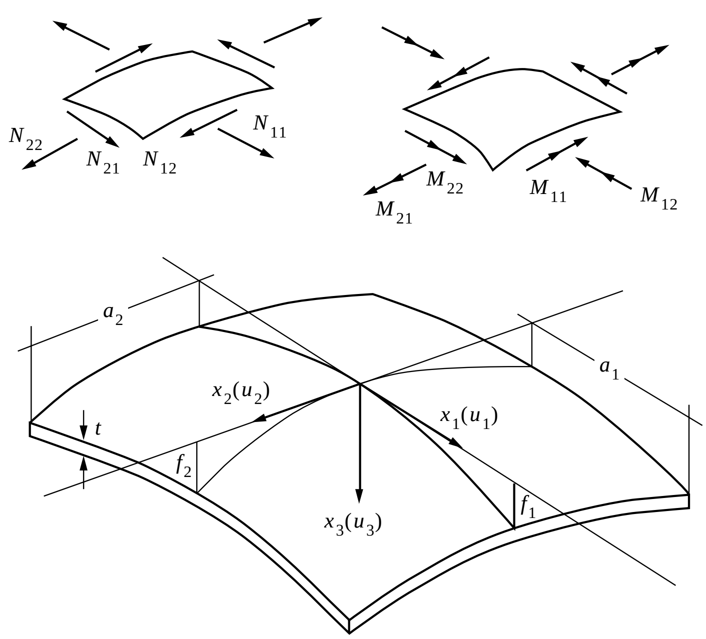

Figure 1. Notation and positive convention of variables used for a shallow shell.

where

$$D = \frac{Eh^3}{12(1-v^2)}, \qquad K = \frac{Eh}{1-v^2}; \tag{3}$$

h is the thickness and E and v are the modulus of elasticity and Poisson's ratio.

Finally, the kinematic relations employed in this work are

$$\varepsilon_{11} = \frac{\partial u_1}{\partial x_1} - k_1 u_3, \qquad \varepsilon_{22} = \frac{\partial u_2}{\partial x_2} - k_2 u_3, \qquad \varepsilon_{12} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right) - k_{12} u_3,$$

$$\chi_{11} = \frac{\partial \beta_1}{\partial x_1}, \qquad \chi_{22} = \frac{\partial \beta_2}{\partial x_2}, \qquad \chi_{12} = \frac{1}{2}\left(\frac{\partial \beta_1}{\partial x_2} + \frac{\partial \beta_2}{\partial x_1}\right), \tag{4}$$

where $\beta_i = -\partial u_3/\partial x_i$.

The displacement components are u_i; ε_{ij} are the strains; and χ_{ij} are the changes in curvature of the shell.

Use of the above relations allows the equilibrium equations to be reformulated in terms of the displacement components in the form

$$K\left[\frac{\partial^2 u_1}{\partial x_1^2} + \frac{1-v}{2}\frac{\partial^2 u_1}{\partial x_2^2} + \frac{1+v}{2}\frac{\partial^2 u_2}{\partial x_1\,\partial x_2} - (k_1 + vk_2)\frac{\partial u_3}{\partial x_1}\right] = -\rho h \ddot{u}_1,$$

$$K\left[\frac{1+v}{2}\frac{\partial^2 u_1}{\partial x_1\,\partial x_2} + \frac{\partial^2 u_2}{\partial x_2^2} + \frac{1-v}{2}\frac{\partial^2 u_2}{\partial x_1^2} - (k_2 + vk_1)\frac{\partial u_3}{\partial x_2}\right] = -\rho h \ddot{u}_2, \tag{5}$$

$$K\left[(k_1 + vk_2)\frac{\partial u_1}{\partial x_1} + (k_2 + vk_1)\frac{\partial u_2}{\partial x_2}\right] - K(k_1^2 + k_2^2 + 2vk_1k_2)u_3 - D\nabla^4(u_3) = -\rho h \ddot{u}_3.$$

Notice that the equations have been restricted to shells with $k_{12} = 0$ in order to obtain simple analytical solutions. The above equations are identical to equations (3)–(5) in reference [6]. They can be applied to solve cylindrical, elliptical paraboloidal and hyperbolic paraboloidal shells.

3. SOLUTION FOR SS BOUNDARY CONDITIONS

For the present purpose of understanding the potential significance of shell vibrations, a simple displacement pattern will be allowed, namely

$$u_1 = u_1^{mn} Z_1^{mn} \cos(\omega^{mn} t), \qquad u_2 = u_2^{mn} Z_2^{mn} \cos(\omega^{mn} t), \qquad u_3 = u_3^{mn} Z_3^{mn} \cos(\omega^{mn} t), \tag{6}$$

where u_i^{mn} are the amplitudes of vibration and the mode shapes are

$$Z_1^{mn} = \sin\left(\frac{\pi}{2} x_1 \eta\right)\cos\left(\frac{\pi}{2} x_2 \mu\right), \qquad Z_2^{mn} = \cos\left(\frac{\pi}{2} x_1 \eta\right)\sin\left(\frac{\pi}{2} x_2 \mu\right),$$

$$Z_3^{mn} = \cos\left(\frac{\pi}{2} x_1 \eta\right)\cos\left(\frac{\pi}{2} x_2 \mu\right), \tag{7}$$

and $\eta = n/a_1$, $\mu = m/a_2$ indicate the number of waves in each coordinate direction. The sides of the shell have dimensions a_1 and a_2, as illustrated in Figure 1; and m and n are odd-integers. The natural frequencies are ω^{mn}.

The above displacement field satisfies the simple supported conditions along all edges of the shell, for n, $m = 1, 3, 5, 7 \ldots$ Thus, at $x_1 = \pm a_1$ one has

$$u_2 = u_3 = \frac{\partial^2 u_3}{\partial x_1^2} = 0 \tag{8}$$

and similar conditions apply at $x_2 = \pm a_2$.

With the simple displacement field of equation (6) the conditions (5) result in the eigenvalue problem

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{12} & A_{22} & A_{23} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} \begin{Bmatrix} u_1^{mn} \\ u_2^{mn} \\ u_3^{mn} \end{Bmatrix} = \rho h (\omega^{mn})^2 \begin{Bmatrix} u_1^{mn} \\ u_2^{mn} \\ u_3^{mn} \end{Bmatrix}, \tag{9}$$

where the coefficients of the matrix A are

$$A_{11} = -K\left(\frac{\pi}{2}\right)^2\left[\eta^2 + \frac{1-\nu}{2}\mu^2\right], \qquad A_{22} = -K\left(\frac{\pi}{2}\right)^2\left[\mu^2 + \frac{1-\nu}{2}\eta^2\right],$$

$$A_{33} = -D\left(\frac{\pi}{2}\right)^4(\mu^2+\eta^2)^2 - K(k_1^2 + k_2^2 + 2\nu k_1 k_2),$$

$$A_{12} = -K\left(\frac{\pi}{2}\right)^2 \frac{1+\nu}{2}\eta\mu, \qquad A_{13} = K\frac{\pi}{2}(k_1 + \nu k_2)\eta, \qquad A_{23} = K\frac{\pi}{2}(k_2 + \nu k_1)\mu. \tag{10}$$

Solution of the eigenvalue problem (9) leads to values of the natural frequencies ω^{mn} and mode shape amplitudes u_1^{mn}, u_2^{mn}, and u_3^{mn}.

For the class of shells studied the inertia associated with in-plane displacements is much smaller than that associated with out-of-plane displacement. It is then possible to neglect the in-plane inertia terms in equation (9), by assuming $\rho h(\omega^{mn})^2\ u_1^{mn} = \rho h(\omega^{mn})^2\ u_2^{mn} = 0$. Explicit expressions can be derived for natural frequencies and amplitudes of vibration. The displacements u_1^{mn} and u_2^{mn} may now be written in terms of u_3^{mn} as

$$u_1^{mn} = \frac{A_{13}A_{22} - A_{23}A_{12}}{A_{12}A_{12} - A_{11}A_{22}}u_3^{mn} = W_1 u_3^{mn}, \qquad u_2^{mn} = \frac{A_{23}A_{11} - A_{13}A_{12}}{A_{12}A_{12} - A_{11}A_{22}}u_3^{mn} = W_2 u_3^{mn}. \tag{11}$$

Substitution of equation (11) into equation (10) leads to the explicit form for the frequencies,

$$(\omega^{mn})^2 = -\frac{A_{13}W_1 + A_{23}W_2 + A_{33}}{\rho h}. \tag{12}$$

The influence of static stresses on the free vibrations of the shell can be taken into account by introduction of the membrane stress field N_{11}^0, N_{22}^0 in the third of equation (5):

$$-D\nabla^4(u_3) + N_{11}k_1 + N_{22}k_2 + N_{11}^0\frac{\partial^2 u_3}{\partial x_1^2} + N_{22}^0\frac{\partial^2 u_3}{\partial x_2^2} = -\rho h \ddot{u}_3, \tag{13}$$

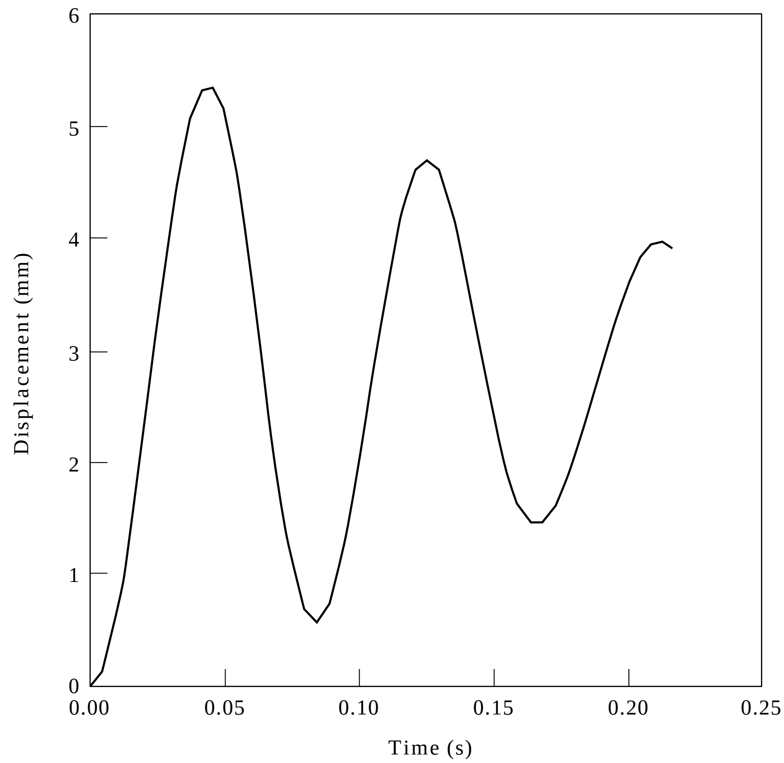

Figure 2. Transient response of a shallow shell. Data: $a_1 = 13{\cdot}7$ m, $a_2 = 13{\cdot}45$ m, $k_1 = 3{\cdot}04 \times 10^{-5}$ 1/mm, $k_2 = 2{\cdot}98 \times 10^{-5}$ 1/mm, $h = 60$ mm, $E = 15\,000$ N/mm^2, $\rho = 2{\cdot}4 \times 10^{-9}$ Ns^2/mm^4, $\nu = 0{\cdot}15$, $\xi = 0{\cdot}02$, $p_3 = 2{\cdot}44 \times 10^{-3}$ N/mm^2.

leading to

$$A'_{33} = A_{33} - \left(\frac{\pi}{2}\right)^2 (N^0_{11}\eta^2 + N^0_{22}\mu^2). \tag{14}$$

As expected, a compressive stress field has the net effect of reducing the natural frequencies of the shell.

4. FORCED VIBRATIONS

The transient response of the shell can be characterized in terms of the out-of-plane displacement

$$m\ddot{u}_3 + c\dot{u}_3 + ku_3 = p_3(t), \tag{15}$$

where m, c, k are the distributed properties of the continuum structure, and p_3 is the time dependent load.

With the mode shapes and natural frequencies computed as in equations (11) and (12), the dynamic response of the structure can be evaluated by mode-superposition techniques (see, for example, references [12] and [13]). Modal superposition analysis is carried out using the approximation

$$u_3 = \sum_n \sum_m Z_3^{mn} q_{mn}, \tag{16}$$

where q_{mn} are generalized coordinates (i.e. the amplitudes of the modal response components), and

$$Z_3^{mn} = \cos\left(\frac{\pi}{2} x_1 \eta\right) \cos\left(\frac{\pi}{2} x_2 \mu\right) \tag{17}$$

are the mode shapes. In general, only a small number of terms is necessary in the summation of equation (16) to obtain acceptable estimates of the displacement amplitudes. Obviously the number of terms must be increased considerably when stress resultants are determined. This is due to the fact that the calculation of the parameters requires obtaining derivatives of the approximate functional relation which constitutes the displacement amplitude.

The load is also written in summation form using the same modes Z_3^{mn}:

$$p_3 = \sum_n \sum_m Z_3^{mn} p_{mn}, \tag{18}$$

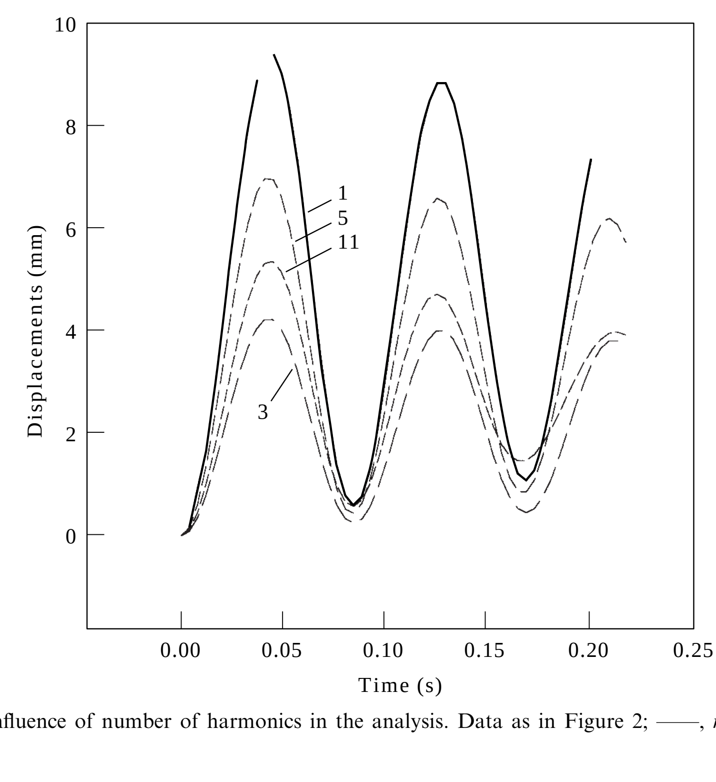

Figure 3. Influence of number of harmonics in the analysis. Data as in Figure 2; ——, $m = n = 1$; ——, 3; ——, 5; ---, 11.

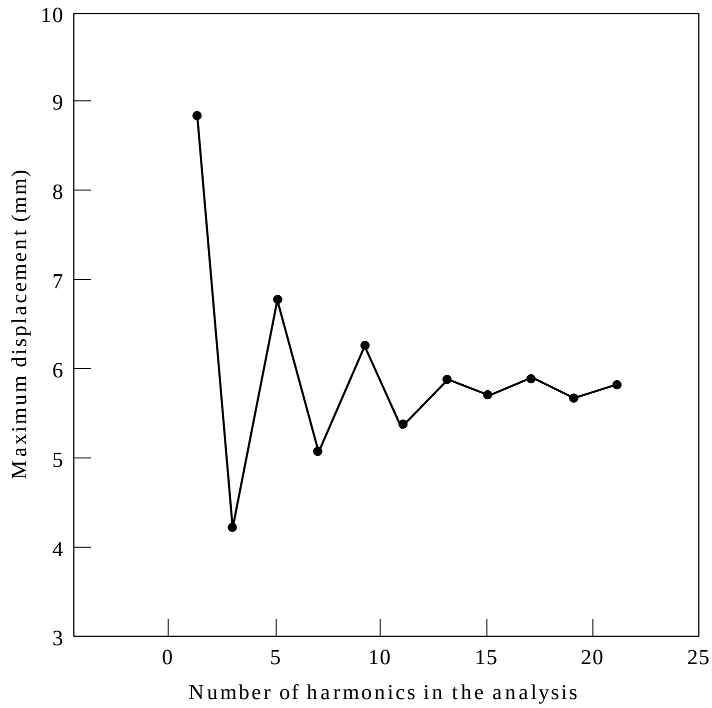

Figure 4. Convergence of the solution with number of harmonics. Data as in Figure 2.

where

$$p_{mn} = \frac{\int\int p_3 Z_3^{mn}\, \mathrm{d}x_1\, \mathrm{d}x_2}{\int\int (Z_3^{mn})^2\, \mathrm{d}x_1\, \mathrm{d}x_2}. \tag{19}$$

Then the transient response of the shell is computed using the approximation

$$\ddot{q}_{mn} + 2\xi\omega^{mn}\dot{q}_{mn} + (\omega^{mn})^2 q_{mn} = \frac{1}{\rho h} p_{mn}, \tag{20}$$

where ξ is the damping ratio.

For load variation with a step function in time, the load participation factor becomes

$$p_{mn} = \left(\frac{4}{\pi}\right)^2 \frac{p_3}{mn} \sin\left(\frac{n\pi}{2}\right) \sin\left(\frac{m\pi}{2}\right). \tag{21}$$

Integration of the above equation is done using Duhammel's integral, as shown in reference [12].

5. RESULTS OF AN ELLIPTICAL PARABOLOIDAL SHELL

5.1. CASE STUDIED

The present formulation has been applied to study the vibration of a shallow reinforced concrete shell which occurs as a consequence of the removal of the formwork. The specific case considered is an elliptical paraboloidal shell previously studied by Ballesteros [1], with the following dimensions: $a_1 = 13{\cdot}7$ m, $a_2 = 13{\cdot}45$ m, $k_1 = 3{\cdot}04 \times 10^{-5}$1/m, $k_2 = 2{\cdot}98 \times 10^{-5}$ 1/m, and $h = 60$ mm. The material properties are taken from reference [1] as $E = 15\,000$ N/mm^2, $\rho = 2{\cdot}4 \times 10^{-9}$ Ns^2/mm^4, $v = 0{\cdot}15$, $\xi = 0{\cdot}02$, and the load is given by $p_3 = 2{\cdot}44 \times 10^{-3}$ N/mm^2. For the present problem, p_3 constitutes the self weight per unit area of the structural element.

The transient response of the shell is illustrated in Figure 2, and shows a maximum amplitude during the initial oscillation, which decreases with time. For this specific case, the maximum amplitude is approximately 5·25 mm.

The accuracy of the approximate response depends on the number of terms included in the analysis. This is shown in Figure 3, where the number of terms in equation (16) is increased from $m = n = 1$ to values of $m = n = 3$, 5, and 11. The convergence process is also shown in Figure 4 for the maximum amplitude in each case, and it is clear that the solution converges as the number of terms is increased. As expected, it is not monotonic convergence, but it exhibits an oscillatory behaviour.

Damping has an influence on the solution, although not very important because the maximum response occurs at the initial stages after the removal of the formwork. This is shown in Figure 5, for values of $\xi = 0{\cdot}5\%$ and 2%. As expected, the differences are small during the first cycle, and the curves become apart as time increases, but with smaller amplitudes.

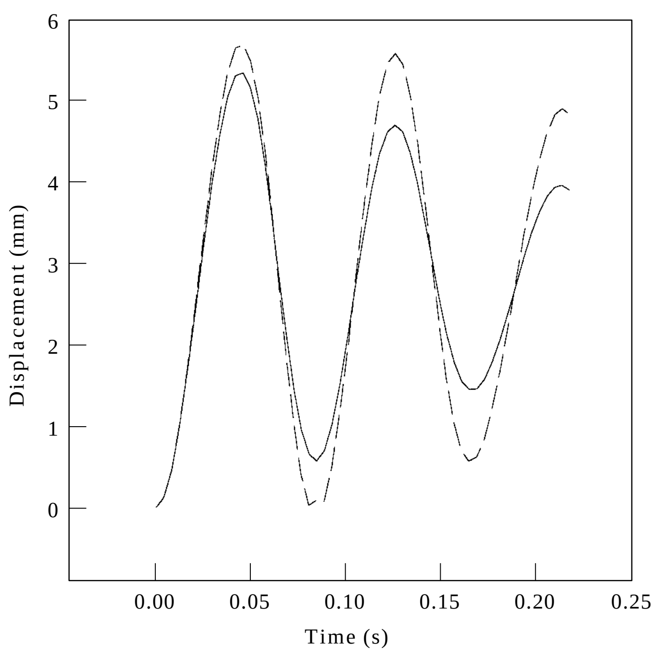

Figure 5. Influence of damping ration on transient response. Data as in Figure 2; ——, 2%; – – –, 0·5%.

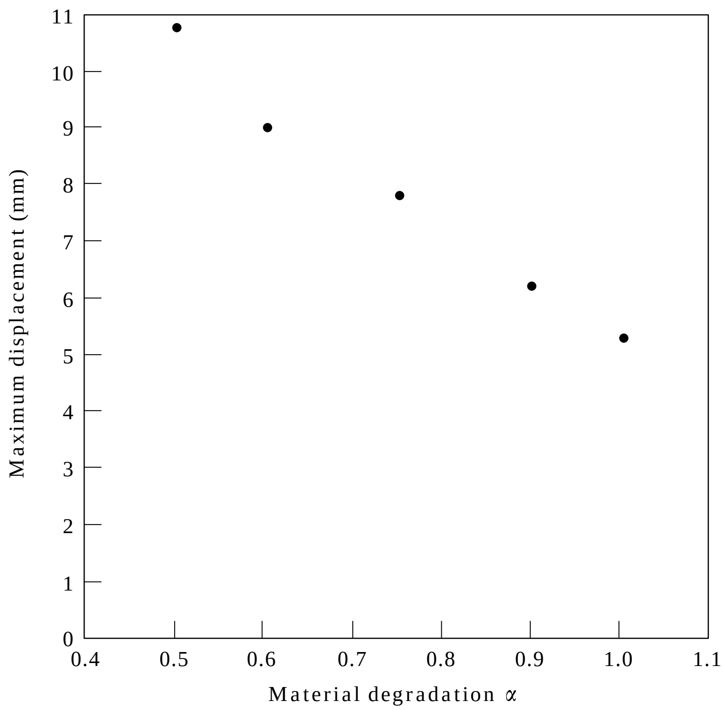

Figure 6. Influence of material degradation on maximum displacement amplitude. Data as in Figure 2.

Concrete shells such as those studied in reference [1] often have degradation of the material properties. To investigate this effect on the dynamic response we employ a degradation factor α related to the thickness, so that the membrane and bending stiffness in equation (3) are modified by $K' = \alpha K$ and $D' = \alpha^3 D$. Parametric studies were carried out using $0{\cdot}5 \leqslant \alpha \leqslant 1$, and the results of sensitivity analysis are plotted in Figure 6. The maximum displacement is increased to about twice the value in the shell with the initial properties ($\alpha = 1$).

The last chapter of the present study is concerned with the shape of the shell. Reports from reference [1] indicate severe geometric imperfections in concrete shells due to errors in the construction. This type of defect is extensively reported in the literature for concrete shells, and for static behaviour the main results have been collected in a recent book [14]. In the present case, deviations from the original curvature have been studied using a factor β, affecting the curvatures as follows: $k_1' = (1 + \beta)k_1$, $k_2' = (1 + \beta)k_2$. Results of the maximum displacement for $-0{\cdot}15 \leqslant \beta \leqslant 0{\cdot}1$ are plotted in Figure 7. The superposition of both material degradation and geometric defects is also shown in Figure 7, for $\alpha = 0{\cdot}6$ and different values of β. Both curves ($\alpha = 1$ and $\alpha = 0{\cdot}6$) show the same trends, with increasing amplitudes of the response as the curvature is decreased. In shells with defects in material and geometry, as reported by Ballesteros [1], the amplitude of transient displacements may increase by a factor of three.

5.2. INFLUENCE OF SHELL PARAMETERS

The case study presented above refers to one specific shell geometry, and it would be interesting to understand how the different parameters of the shell influence the forced vibration response under self weight. In all cases, the material properties are as in the previous section, i.e. $E = 15\,000$ N/mm^2, $\rho = 2{\cdot}4 \times 10^{-9}$ Ns^2/mm^4, and $\nu = 0{\cdot}15$.

A decrease in the thickness of the shell has the effect of increasing the flexibility of the shell, but is also decreases the self weight. For a shell with square planform and the same curvature in both directions, results for $150 \geqslant a_1/h \leqslant 250$ are presented in Table 1. First, we notice that only small changes in w_{max} occur as a consequence of thickness modifications. Second, for the small variation computed, there is a minimum at $a_1/h = 170$, and the displacements increase with decreasing thickness. The time T_{max} necessary to obtain the maximum displacement is the same for all cases in Table 1. Observe that although the maximum displacement in absolute value is the same, the relation between the displacement and the thickness is not constant.

The curvature of the shell has been shown to modify the natural frequencies of a simply supported shallow shell [6], and it would be expected to have an influence on the transient response. Results have been computed in Table 2 for a square and symmetric shell with increasing k_1 for fixed values of a_1 and h. As the product $k_1 a_1$ decreases, the amplitude w_{max} also increases. For example, for $k_1 a_1 = 0{\cdot}4$ one gets $w_{max} = 5{\cdot}442$, while for a reduction in k_1 of 25% the displacement is increased by 77%. The time to reach the maximum displacement also increases with the radius of curvature of the shell.

In Tables 1 and 2 we assume symmetry in the shell, in the sense that the curvature was the same in both directions ($k_2/k_1 = 1$). Let us now modify the ratio k_2/k_1 for a shell with fixed dimensions in planform. The thickness remains constant with $a_1/h = 200$. With increasing curvature k_2 the values of w_{max} decrease as a consequence of the larger stiffness due to the geometry, while a decrease in the curvature k_2 leads to larger displacements. The time at which the maximum displacement is reached is almost constant (Table 3).

In our final study let us consider how the actual dimensions of the shell in planform affect the amplitude of the forced vibrations under self weight. The shells studied are square

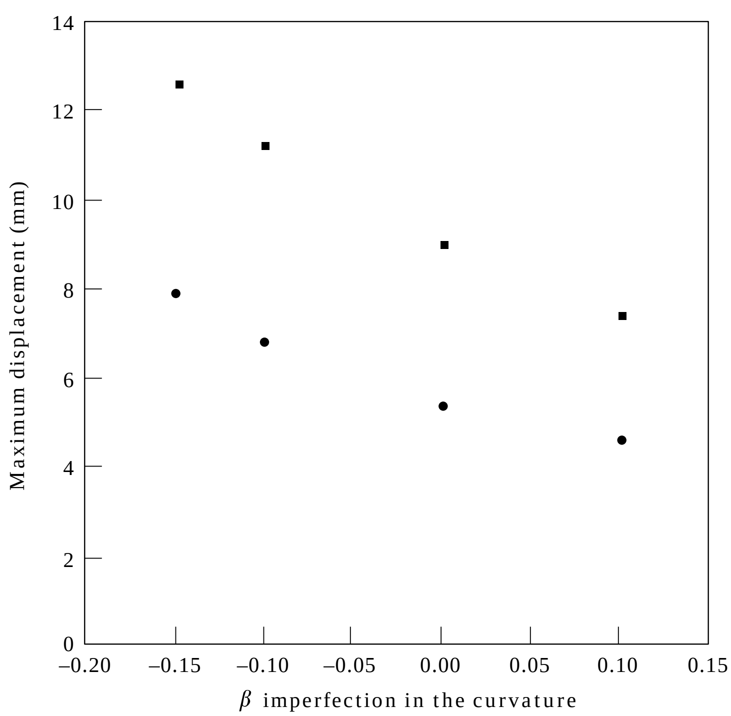

Figure 7. Influence of imperfections in the curvature on maximum displacement amplitude. Data as in Figure 2; ●, $\alpha = 1{\cdot}0$; ■, $\alpha = 0{\cdot}6$.

TABLE 1

Influence of the thickness of the shell on the maximum displacement for $k_2/k_1 = 1$; $a_2/a_1 = 1$; $a_1k_1 = 0{\cdot}39$; $a_1 = 13\,000$ mm

a_1/h	w_{max} (mm)	T_{max} (s)
125	5·755	0·04
150	5·709	0·044
175	5·682	0·04
200	5·704	0·04
225	5·725	0·04
250	5·735	0·04
275	5·734	0·04

TABLE 2

Influence of the curvature of the shell on the maximum displacement for $k_2/k_1 = 1$; $a_2/a_1 = 1$; $a_1 = 13\,000$ mm; $a_1/h = 200$

k_1a_1	w_{max} (mm)	T_{max} (s)
0·50	3·503	0·032
0·45	4·318	0·036
0·425	4·802	0·04
0·40	5·442	0·044
0·375	6·174	0·044
0·35	7·064	0·048
0·30	9·661	0·056
0·25	14·038	0·064
0·20	21·803	0·080

TABLE 3

Influence of the curvature ratio of k_2/k_1 the shell on the maximum displacement for $a_1k_1 = 0{\cdot}4$; $a_2/a_1 = 1$; $a_1/h = 200$; $a_1 = 13\,000$ mm

k_2/k_1	w_{max} (mm)	T_{max} (s)
0·6	9·243	0·052
0·7	7·987	0·048
0·8	6·952	0·044
0·9	6·131	0·044
1·0	5·442	0·044
1·2	4·210	0·04
1·4	3·554	0·032

TABLE 4

Influence of the size of the shell for $k_2/k_1 = 1$; $a_2/a_1 = 1$; $a_1/h = 200$; $a_1k_1 = 0{\cdot}4$; and $a_1^* = 13\,000$ mm

a_1/a_1^*	w_{max} (mm)	T_{max} (s)
0·6	1·960	0·024
0·8	3·483	0·032
1·0	5·442	0·04
1·2	7·836	0·048
1·4	10·664	0·056
1·6	13·929	0·064

and have the same curvature in both directions but differ in the dimensions a_1 and a_2. The thickness is not constant, but the ratio a_1/h was kept constant. As a reference value we consider a dimension $a_1^* = 13\,000$ mm. Table 4 shows that the displacements increase with the dimensions of the shell; for example, a 40% increase in the dimensions leads to a 96% increase in the displacements.

6. CONCLUSIONS

A linear dynamic analysis of shallow shells with rectangular planform has been presented to obtain the displacement response. This consideration of the transient response under suddenly applied loads leads to several conclusions:

- Knowledge about the amplitude of the vibrations of a shallow concrete shell after removal of the formwork is important in itself, and may be an indication that the gradual rather than sudden removal of the form is necessary in some cases.
- The maximum amplitude of displacements during the vibration cycle is small, so that according to reference [6] it is expected that, for the present example, kinematic non-linearity should not be important for the natural frequencies of the shell. Because we presented a modal superposition analysis for the transient response, which is heavily dependent on the natural frequencies of the shell, it is also expected that the transient response itself should not be affected by geometric non-linearity.
- Parametric studies showed that changes in the thickness of the shell do not affect the maximum displacement response; but the curvature and other dimensions of the shell have a significant influence on the transient response.
- For the response of buckling loads of the shell, the amplitudes of out-of-plane deformations which occur could be critical. For the shell considered in this paper, it is shown that the out-of-plane displacements may be above 25% of the thickness of the shell. This could act as an "effective" imperfection so that this form of failure could be greatly increased. Thus, the induced transient response could have a major deleterious effect upon the resistance of the shell to incipient buckling collapse.

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REFERENCES

1. P. Ballesteros 1978 *IASS Bulletin*, 66. Non-linear dynamic and creep buckling of elliptical paraboloidal shells.
2. Y. Chen 1966 *Vibrations: Theoretical Methods*. Reading, MA: Addison–Wesley.
3. P. A. A. Laura, R. Gelos, R. E. Rossi and D. V. Bambill 1994 *Anal. Acad. Nac. Cs. Ex. Fis. Nat., Buenos Aires* **46**, 139–144. Some educational points when teaching undergraduate Applied Mechanics courses.
4. E. Reissner 1975 *Quarterly Applied Mathematics* **13**(2), 169–176. On transverse vibrations of thin shallow elastic shells.
5. K. Apeland 1963 *Acta Polytechnica Scandinavica*. Analysis of bending stresses in translational shells including anisotropic and inhomogeneous properties.
6. A. W. Leissa and A. S. Kadi 1971 *Journal of Sound and Vibration* **16**(2), 173–187. Curvature effects on shallow shell vibrations.

7. A. W. LEISSA, J. K. LEE and A. J. WANG 1981 *Journal of Sound and Vibration* **78**, 311–328. Vibrations of cantilevered shallow cylindrical shells having rectangular planform.
8. Y. NARITA and A. W. LEISSA 1984 *Earthquake Engineering and Structural Dynamics* **12**, 651–662. Vibrations of corner-supported shallow shells of rectangular planform.
9. A. W. LEISSA and Y. NARITA 1984 *Journal of Sound and Vibration* **96**, 207–218. Vibrations of completely free shallow shells of rectangular planform.
10. M. S. QATU and A. W. LEISSA 1993 *Mechanics of Structures and Machines* **21**(3), 285–301. Vibration of shallow shells with two adjacent edges clamped and the others free.
11. F. T. K. AU and Y. K. CHEUNG 1996 *Thin-Walled Structures* **24**, 53–82. Free vibrations and stability analysis by the isoparametric spline finite strip method.
12. R. W. CLOUGH and J. PENZIEN 1993 *Dynamics of Structures*. New York: McGraw–Hill.
13. L. A. GODOY, V. C. M. SOUZA and J. G. A. CROLL 1985 *International Journal of Thin Walled Structures* **3**, 67–79. Transient dynamics of cylinders under wave slamming.
14. L. A. GODOY 1996 *Thin-Walled Structures with Structural Imperfections*: *Analysis and Behavior*. Oxford: Pergamon Press.