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# METHOD OF JUDGING THE SELF-EXCITED VIBRATION OF ROLLING MAIN DRIVE SYSTEM IN ROLLING SLIPPAGE

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For the rolling slipping state of a main drive system, the present paper established the nonlinear mechanical model of the main drive system and used the Krilov–Bogolubov method to produce the solution of the nonlinear mechanical model, and ascertained the condition of generating self-excited vibration. It also presented a method of calculating the upper and the lower limit angular velocities between which self-excited vibration of the main drive system occurs. A relationship of rolling force, friction coefficient between rool and rolled piece, and rolling angular velocity was derived to judge whether self-excited vibration happens or not. The method may provide a valuable way to avoid self-excited vibration and thus prevent the instantaneous breakage of the system due to the self-excited vibration.



## 1. INTRODUCTION

With the development of the steel industry, the accretion of rolling mill capacity, and the increase of electromotor number, it often occurs that the parts of a rolling mill, fasten parts and the rotor elements of an electromotor are damaged due to torsional vibration in the spindle system [1, 2], especially due to the self-excited torsional vibration. Once the self-excited vibration arises, the torque amplification factor (TAF) of the system abruptly increases to a very high value which brings about instantaneous breakage of the whole rolling system. The destruction owing to the self-excited vibration is far more serious than other damage. A practical way of judging whether the system will generate the self-excited vibration in certain conditions and preventing it in the rolling process is needed for designers and operators of the system. The present endeavor is meant to meet this need.

## 2. MATHEMATICAL MODEL OF MAIN DRIVE SYSTEM IN ROLLING SLIPPING STATE

In the case of rolling slipping, the main drive system is separated into two rolling systems from the rolled piece: an upper rolling system and a lower rolling system in a blooming mill. The model of a spring–mass system is shown in Figure 1.

### 2.1. THE NONLINEAR ELASTIC RESTORING FORCE

Due to the wear, the total backlash between two inertias is assumed as $2\Delta$. The elastic restoring force between the $i$th and the $i+$ th inertias can be expressed as:

$$F_{i,i+1}(\theta_i-\theta_{i+1})=\begin{cases}0 & -\Delta\leqslant\theta_i-\theta_{i+1}\leqslant\Delta\\ k_{i,i+1}(\theta_i-\theta_{i+1})-k_{i,i+1}\Delta \quad \text{for} & \Delta\leqslant\theta_i-\theta_{i+1}<\infty\\ k_{i,i+1}(\theta_i-\theta_{i+1})+k_{i,i+1}\Delta & -\infty\leqslant\theta_i-\theta_{i+1}\leqslant-\Delta\end{cases}$$

 

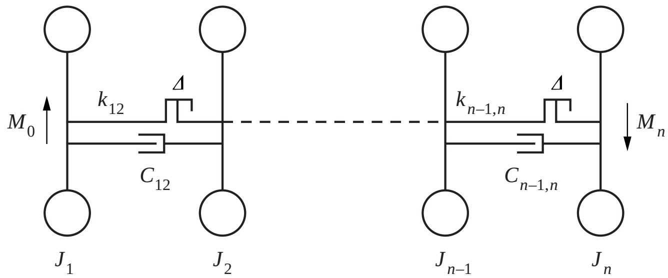


Figure 1. ($N$) Inertia model of typical mill stand.

$$F_{i,i+1}(\theta_i - \theta_{i+1}) = k_{i,i+1}\lambda_{i,i+1} - \varepsilon f(\lambda_{i,i+1})$$

$$\varepsilon f(\lambda_{i,j+1}) = \begin{cases} k_{i,i+1}\lambda_{i,i+1} & -\Delta \leqslant \lambda_{i,i+1}\Delta \\ k_{i,i+1}\Delta \quad\quad \text{for} & \Delta \leqslant \lambda_{i,i+1} \leqslant \infty \\ -k_{i,i+1}\Delta & -\infty \leqslant \lambda_{i,i+1} \leqslant -\Delta \end{cases}$$

where

$$\lambda_{i,i+1} = \theta_i - \theta_{i+1}.$$

### 2.2. THE FRICTION TORQUE ACTING ON THE ROLL

The friction coefficient between the roll and the rolled piece changes with the relative slipping linear velocity, and its function can be written as: [3] $u = u_0 - cv + dv^3$. The parameters, $u_0$, $c$ and $d$, are determined by experiment. Since the friction torque acting on the roll is: $M_n = N \times u \times (D/2)$ in rolling slipping, we have: $M_n = m_0 - m_1\omega_n + m_3\omega_n^3$, where $m_0 = (1/2)NDu_0$, $m_1 = (1/4)ND^2c$, $m_3 = (1/16)ND^4d$, $D =$ diameter of roll and $N =$ rolling force.

### 2.3. THE EQUATION OF MOTION OF A TYPICAL ROLLING SYSTEM

For the typical rolling system shown in Figure 1, its equation of motion is expressed as:

$$[J][\ddot{\theta}] + [\dot{C}]\{\theta\} + [K]\{\theta\} = \{Q\} \tag{1}$$

where

$$[J] = \begin{bmatrix} J_1 & & & & \\ & \cdot & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & J_n \end{bmatrix},$$

$$[C] = \begin{bmatrix} c_{12} & -c_{12} & & & & \\ -c_{12} & c_{12}+c_{23} & -c_{23} & & & \\ & -c_{23} & c_{23}+c_{34} & \cdot & & \\ & & \cdot & \cdot & \cdot & \\ & & & \cdot & \cdot & -c_{n-1,n} \\ & & & & -c_{n-1,n} & c_{n-1,n} \end{bmatrix},$$

$$\{Q\} = \begin{bmatrix} M_0 \\ \cdot \\ \varepsilon f(\theta_{i-1} - \theta_i) \\ -\varepsilon f(\theta_{i-1} - \theta_i) \\ \cdot \\ -M_n - \varepsilon f(\theta_{n-1} - \theta_n) \end{bmatrix},$$

$$[K] = \begin{bmatrix} k_{12} & -k_{12} & & & & \\ -k_{12} & k_{12}+k_{23} & -k_{23} & & & \\ & -k_{23} & k_{23}+k_{34} & \cdot & & \\ & & \cdot & \cdot & \cdot & \\ & & & \cdot & \cdot & -k_{n-1,n} \\ & & & & -k_{n-1,n} & k_{n-1,n} \end{bmatrix}.$$

## 3. EQUATION SOLVING

### 3.1. TRANSFORM THE EQUATION OF MOTION IN GENERALIZED COORDINATES INTO THE EQUATION IN PRINCIPAL COORDINATES

The spring–mass system without exciting forces, its natural frequencies are $0, p_1, p_2, \ldots, p_n$, respectively, and its modal matrix is:

$$[\theta_m] = [1 \quad \boldsymbol{\phi}_2 \quad \boldsymbol{\phi}_3 \quad \cdot \quad \cdot \quad \cdot \quad \boldsymbol{\phi}_n].$$

Decoupling equation (1) into equations set in the principal coordinates:

$$[J_p]\{\ddot{\theta}\} + [C_p]\{\dot{\theta}_p\} + [K_p]\{\theta_p\} = \{Q_p\}. \tag{2}$$

Where $[C_p] = [\theta_m]^T[C][\theta_m]$ is a tridiagonal matrix, we can consider it as an approximately diagonal matrix and replace the off-diagonal elements with zero and

$$\{Q_p\} = [\theta_m]^T\{Q\},\ Q_{pi} = \sum_{j=1}^{n} Q_j\phi_{ji},\ K_{pi} = J_{pi}P_i^2,\ J_{pi} = \sum_{j=1}^{n} J_j\phi_{ji}^2,\ C_{pi} = 2\zeta_i P_i J_{pi},$$

$\zeta_i$ = modal ratio of damping.

In the vibration of a multi-degree-of-freedom system with the action of exciting forces, the internal damping existing in the system often makes the vibration of high frequency disappear rapidly, and only the vibration of lower frequency will arise. Although there exist vibrations of different frequencies, only one of them dominates in their action on the system. Therefore we suppose that the self-excited vibration in rolling slipping state is the combination of the torsional vibration of the $i$th frequency with the rotation of rigid body in a certain angular velocity $\omega$. Hence, we have:

$$J_{p1}\ddot{\theta}_{p1} = M_0 - (m_0 - m_1\omega_n + m_3\omega_n^3) \tag{3}$$

$$J_{pi}\ddot{\theta}_{pi} + C'_{pi}\theta_{pi} + K_{pi}\theta_{pi} = M_0 - \phi_{ni}(m_0 - m_1\omega_n + m_3\omega_n^3) + (\phi_{2i} - \phi_{3i})\varepsilon f(\theta_2 - \theta_3)$$
$$+ \cdots + (\phi_{n-1,i} - \phi_{n,i})\varepsilon f(\theta_{n-1} - \theta_n) \quad (i = 2, 3, \ldots, n). \tag{4}$$

From $\theta_{n-1} - \theta_n = (\phi_{n-1,i} - \phi_{n,i})\theta_{pi}$ and $\omega_n = \mathrm{d}\theta_n/\mathrm{d}t$, we can get $\omega_n = \dot{\theta}_{pi} + \phi_{ni}\dot{\theta}_{pi}$, substituting them in equations (3) and (4):

$$J_{p1}\ddot{\theta}_{p1} = M_0 - [m_0 - m_1(\dot{\theta}_{p1} + \phi_{ni}\dot{\theta}_{p1}) + m_3(\dot{\theta}_{pi} + \phi_{ni}\dot{\theta}_{pi})^3] \tag{5}$$

$$\begin{aligned} J_{pi}\ddot{\theta}_{pi} + k_{pi}\theta_{pi} = {} & M_0 + M_0\dot{\theta}_{p1} - C_{pi}\dot{\theta}_{pi} - \phi_{ni}[m_0 - m_1(\dot{\theta}_{p1} + \phi_{ni}\dot{\theta}_{pi}) + m_3(\dot{\theta}_{p1} + \phi_{ni}\dot{\theta}_{pi})^3] \\ & + (\phi_{2i} - \phi_{3i})\varepsilon f[(\phi_{2i} - \phi_{3i})\theta_{pi}] + \cdots + (\phi_{n-1,i} - \phi_{n,i})\varepsilon f[(\phi_{n-1,i} - \phi_{n,i})\theta_{pi}]. \end{aligned} \tag{6}$$

### 3.2. THE SOLUTION BY KRILOV–BOGOLUBOV METHOD

For the torsional vibration of the $i$th mode frequency, the first approximation of equation (6) can be calculated: $\theta_{pi} = a\cos\phi$, in which $a$ and $\phi$ can be determined by:

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \varepsilon A_1(a), \qquad \frac{\mathrm{d}\phi}{\mathrm{d}t} = p_i + \varepsilon B_1(a)$$

substituting equation

$$\begin{cases} \theta_{pi} = a\cos\phi \\ \dot{\theta}_{pi} = -ap_i\sin\phi \\ \dot{\theta}_{p1} = \omega \end{cases}$$

in equation (6), and rearranging:

$$\begin{aligned} J_{pi}\ddot{\theta}_{pi} + k_{pi}\theta_{pi} = {} & M_0 - \phi_{ni}m_0 + \phi_{ni}m_1\omega - \phi_{ni}m_3\omega^3 - \tfrac{3}{2}\phi_{ni}^3 m_3 a^2 p_i^2 \omega \\ & + (C_{pi}ap_i - \phi_{ni}m_1 ap_i + 3\phi_{ni}m_3\omega^2 ap_i + \tfrac{3}{4}\phi_{ni}^4 m_3 a^3 p_i^3)\sin\phi + \text{minimal term} \\ & + \{(\phi_{2i} - \phi_{3i})\varepsilon f[(\phi_{2i} - \phi_{3i})a\cos\phi] + \cdots + (\phi_{n-1,i} - \phi_{ni})\varepsilon f \\ & \times [(\phi_{n-1,i} - \phi_{ni})a\cos\phi]\}. \end{aligned}$$

Since the last term at the right-hand side of the above equation is an even function, and the second term is an odd function, we have:

$$\varepsilon A_1(a) = -\frac{1}{2p_i J_{pi}}(C_{pi}ap_i - \phi_{ni}^2 m_1 ap_i + 3\phi_{ni}^2 m_3\omega^2 ap_i + \tfrac{3}{4}\phi_{ni}^4 m_3 a^3 p_i^3) \tag{7}$$

$$\begin{aligned} \varepsilon B_1(a) = {} & -\frac{1}{2\pi a p_i J_{pi}} \int_0^{2\pi} \{(\phi_{2i} - \phi_{3i})\varepsilon f[(\phi_{2i} - \phi_{3i})a\cos\phi] + \cdots \\ & + (\phi_{n-1,i} - \phi_{ni})\varepsilon f[(\phi_{n-1,i} - \phi_{ni})a\cos\phi]\}\cos\phi\,\mathrm{d}\phi. \end{aligned} \tag{8}$$

Let

$$\delta = \frac{m_1\phi_{ni}^2 - C_{pi} - 3\phi_{ni}^2 m_3\omega^2}{J_{pi}}, \qquad k = \frac{m_1\phi_{ni}^2 - C_{pi} - 3\phi_{ni}^2 m_3\omega^2}{3/4 m_3 p_i^2 \phi_{ni}^4}$$

and substitute them in equation (7). Then:

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \varepsilon A_1(a) = \frac{a}{2}\delta\left(1 - \frac{a^2}{k}\right),$$

and its integration is:

$$a = \frac{a_0 \exp[(\delta/2)t]}{\sqrt{1 + \dfrac{1}{k} a_0^2(\exp(\delta t) - 1)}}.$$

Let:

$$\lambda = m_1 \phi_{ni}^2 - C_{pi} - 3\phi_{ni}^2 m_3 \omega^2$$

thus:

$$\delta = \frac{\lambda}{J_{pi}}, \qquad k = \frac{\lambda}{3/4 m_3 p_i^2 \phi_{ni}^4}.$$

From the above outcome,

$$\text{if} \quad \lambda > 0 \quad t \to \infty, \qquad \text{then} \lim_{t \to \infty} a = \sqrt{k}$$

$$\text{if} \quad \lambda < 0 \quad t \to \infty, \qquad \text{then} \lim_{t \to \infty} a = 0.$$

Therefore, that the value of $\lambda$ is positive or negative is the basis of judging whether the self-excited vibration can arise or not, and its steady-state response is:

$$a = \sqrt{k} = \frac{2}{p_i \phi_{ni}} \sqrt{\frac{m_1 - C_{pi}/\phi_{ni}^2}{3m_3} - \omega^2}. \tag{9}$$

Since $\varepsilon f(\theta_{i-1} - \theta_i)$ is a piece-wise linear function, the right-hand side of equation (8) should be a piece-wise linear function too when it is transformed into the function in the principal coordinates:

$$\varepsilon f(\theta_{m-1} - \theta_m) = \begin{cases} K_{m-1,m}(\theta_{m-1} - \theta_m) & -\Delta \leqslant \theta_{m-1} - \theta_m \leqslant \Delta \\ K_{m-1,m}\Delta & \text{for} \quad \Delta \leqslant \theta_{m-1} - \theta_m < \infty \\ -K_{m-1,m}\Delta & -\infty < \theta_{m-1} - \theta_m \leqslant -\Delta \end{cases}.$$

Now, we use $\psi$ to rewrite it and let $\psi_0$ be the minimal root of $\Delta = a \cos \psi$, i.e. minimal value of $\psi_0 = \arccos(\Delta/2)$; we have:

$$\varepsilon f(a \cos \psi) = \begin{cases} K_{m-1,m}(\phi_{m-1,i} - \phi_{mi}) a \cos \psi & \psi_0 \leqslant \psi \leqslant \pi - \psi_0 \\ K_{m-1,m}(\phi_{m-1,i} - \phi_{mi}) a \cos \psi_0 & \text{for} \quad 0 \leqslant \psi \leqslant \psi_0 \\ -K_{m-1,m}(\phi_{m-1,i} - \phi_{mi}) a \cos \psi_0 & \pi - \psi_0 \leqslant \psi \leqslant \pi \end{cases}.$$

Substituting the above equation in equation (8) and rearranging:

$$\varepsilon B_1(a) = \frac{-1}{2\pi a p_i J_i} \int_0^{2\pi} \varepsilon f(a \cos \psi) \cos \psi \, d\psi \varepsilon f(a \cos \psi) = \begin{cases} Ka \cos \psi & \psi_0 \leqslant \psi \leqslant \pi - \psi_0 \\ Ka \cos \psi_0 & 0 \leqslant \psi \leqslant \psi_0 \\ -Ka \cos \psi_0 & \pi - \psi_0 \leqslant \psi \leqslant \pi \end{cases}$$

where

$$K = \sum_{j=2}^{n} K_{j,j+1}(\phi_{ji} - \phi_{j+1,i})^2.$$

As $F(\psi) = \varepsilon f(a\cos\psi)\cos\psi$ is an even function

$$\varepsilon B_1(a) = -\frac{4}{2\pi a p_i J_i}\left(\int_0^{\psi_0} Ka\cos\psi_0\cos\psi\,\mathrm{d}\psi + \int_{\psi_0}^{\pi/2} Ka\cos^2\psi\,\mathrm{d}\psi\right)$$

$$= -\frac{K}{\pi p_i J_i}\left[\arcsin\frac{\Delta}{2} + \frac{\Delta}{a}\sqrt{1-\left(\frac{\Delta}{a}\right)^2}\right].$$

From $(\mathrm{d}\phi/\mathrm{d}t) = p_i + \varepsilon B_1(a)$, we know that the backlash in the main drive system makes the frequency of self-excited vibration somewhat smaller than its natural frequency. Its frequency is:

$$p_i' = p_i - \frac{K}{\pi p_i J_i}\left[\arcsin\frac{\Delta}{a} + \frac{\Delta}{a}\sqrt{1-\left(\frac{\Delta}{2}\right)^2}\right].$$

## 4. EXAMPLE AND ANALYSIS

Using a 1150 blooming mill as an example and applying the above results, we analyse its dynamic characteristics in rolling slipping state. Due to the rolling slipping, the whole main drive system is separated into an upper and a lower rolling system. Now we analyse the upper rolling system shown in Figure 2.

### 4.1. GIVE THE PARAMETERS [4] OF THE UPPER ROLLING SYSTEM AND PRODUCE ITS NATURAL FREQUENCIES AND MODES

$$J_1 = 0{\cdot}60389\ \text{t.m.s}^2,\qquad J_2 = 0{\cdot}75662\ \text{t.m.s}^2,\qquad J_3 = 0{\cdot}88628\ \text{t.m.s}^2,$$

$$J_4 = 0{\cdot}29208\ \text{t.m.s}^2,\qquad J_5 = 0{\cdot}573\ \text{t.m.s}^2,\qquad k_{12} = 46267{\cdot}0918\ \text{t.m/rad},$$

$$k_{23} = 25352{\cdot}34735\ \text{t.m/rad},\qquad k_{34} = 7356{\cdot}073762\ \text{t.m/rad},$$

$$k_{45} = 97549{\cdot}1243\ \text{t.m/rad},\qquad N = 694\ \text{t},\qquad D = 1100\ \text{mm},\qquad \Delta = 0{\cdot}008\ \text{rad}.$$

When the linear velocity of the roll surface is in the interval $v = \ = 0{\cdot}3$–$3$ m/s, by experiment, [3] we have: $u_0 = 0{\cdot}2$–$0{\cdot}49$, $c = 0{\cdot}03$–$0{\cdot}09$, $d = 0{\cdot}0015$–$0{\cdot}0033$. Then $m_0 = 76{\cdot}34$–$187{\cdot}03$, $m_1 = 6{\cdot}30$–$18{\cdot}89$, $m_3 = 0{\cdot}095$–$0{\cdot}2096$. According to the given

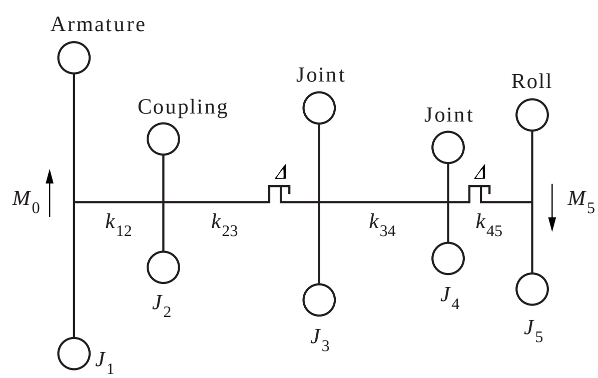


Figure 2. Upper rolling system of 1150 blooming mill.

parameters and using the QR method, we can obtain its natural frequencies and modes of the upper rolling system. Since the ratio of damping can be shown as:

$$\zeta_i = \frac{1}{2\pi j} \ln \frac{x_i}{x_{i+j}},$$

we can calculate each ratio of damping. According to

$$J_{pi} = \sum_{j=1}^{5} \phi_{ij}^2 J_j \quad \text{and} \quad C_{pi} = 2\zeta_i p_i J_{pi},$$

we have $J_{p1} = 3{\cdot}11196$, $J_{p2} = 4{\cdot}2544$, $J_{p3} = 2{\cdot}2215$, $J_{p4} = 1{\cdot}54$, $J_{p5} = 7412521{\cdot}029$ and $C_{p1} = 0$, $C_{p2} = 16{\cdot}938$, $C_{p3} = 19{\cdot}2589$, $C_{p4} = 24{\cdot}579$, $C_{p5} = 214113045{\cdot}6$.

### 4.2. JUDGE AND ANALYSE THE SELF-EXCITED VIBRATION OF THE ROLLING SYSTEM

With $m_1 = 6{\cdot}30$–$18{\cdot}89$, $m_3 = 0{\cdot}095$–$0{\cdot}2096$, and according to $\lambda > 0$, i.e.

$$F(\omega, m_1, m_3) = m_1 - C_{pi} \frac{1}{\phi_{5i}^2} - 3\phi_{5i}^2 m_3 \omega^2 > 0,$$

we can get the relationship between $m_1$, $m_3$ and a certain mode self-excited vibration shown in Figure 3. From Figure 3, we can see that the dashed area is the self-excited vibration range of $m_1$ and $m_3$. To a certain mode self-excited vibration whether it can arise or not is also determined by angular velocity of the rolling system, such as the two judging lines $L_1$ and $L_2$ (shown in Figure 3) that correspond to the lower and the upper limit angular velocities between which the self-excited vibration may arise. In other words, if the angular velocity of the rolling system is not between the lower limit angular velocity and the upper limit angular velocity, this mode self-excited vibration will never arise. To the system mentioned here, its lower limit angular velocity and upper limit angular velocity of the second mode self-excited vibration are 30·1 rpm and 77·5 rpm respectively.

Substituting $\dot{\theta}_{pi} = -ap_i \sin\phi$ and $\dot{\theta}_{p1} = \omega$ in equation (5) and rearranging:

$$J_{p1}\ddot{\theta}_{p1} = M_0 - m_0 - m_1\omega - m_3\omega^3 - \tfrac{3}{2} m_3 \phi_{ni}^2 a^2 p_i^2 \omega + \text{minimal term}$$
$$+(-\phi_{ni} m_1 a p_i + 3\phi_{ni} m_3 \omega^2 a p_i + \tfrac{3}{4}\phi_{ni}^3 m_3 a^3 p_i^3) \sin\phi.$$

Since the constant term on the right-hand side of the above equation is the secular term, we have

$$M_0 - m_0 - m_1\omega - m_3\omega^3 - \tfrac{3}{2} m_3 \phi_{ni}^2 a^2 p_i^2 \omega = 0.$$

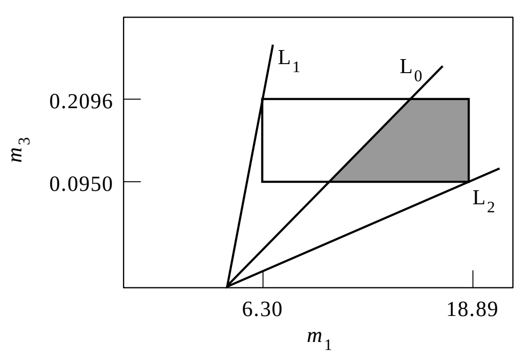


Figure 3. Criterion drawing of a certain mode self-excited vibration.

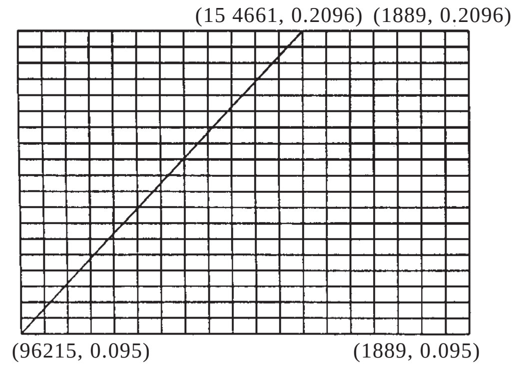


Figure 4. Mesh of $m_1$, $m_3$.

Substituting equation (9)

$$f(\omega, m_0, m_1, m_3) = (M_0 - m_0) - \left(m_1 - \frac{2C_{pi}}{\phi_{ni}}\right)\omega + 5m_3\omega^3 = 0.$$

Now, we divide the dashed area (shown in Figure 3) into mesh (shown in Figure 4) so that the range of $m_1$ and the range of $m_3$ are divided into 20 steps. To each node $(m_1, m_3)$ of the mesh, we use Newton iteration method to calculate $\omega$ in the range of $m_0$, divided into 200 steps, and then under the condition of $F(\omega, m_1, m_3) > 0$, sift out the values $(m_0, m_1, m_3)$ in which the rolling system generates self-excited vibration. The sifted results are shown in Figure 5.

According to Figure 5, we can easily find that the greater the value of $m_0$, the more possible it is for the rolling system to generate the self-excited vibration. From the equation $m_0 = (1/2)NDu_0$, we know that if the rolling system is in rolling slipping state with large enough rolling force, it is quite possible to generate self-excited vibration. Since the TAF due to the self-excited vibration is very large, it often causes the instantaneous breakage of the whole main drive system.

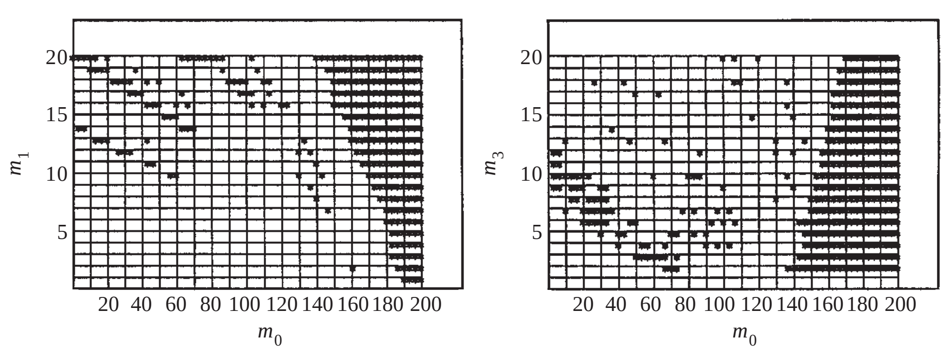


Figure 5. $m_0$, $m_1$, $m_3$ producing self-excited vibration.

## 5. CONCLUSION

The present author established a universal nonlinear mechanical model for the main drive system of a rolling mill. This paper presents a method of calculating the amplitude and frequency of self-excited vibration in rolling slipping state and the limit angular velocities between which the system may generate self-excited vibration. The paper also analyses the relationship between the system parameters and the self-excited vibration. This work may offer a method to analyse the dynamic characteristics of the nonlinear rolling systems and serve as a basis for the prevention of self-excited vibration in rolling production technology. Also engineers may use the method developed here as a reference in an attempt to reduce or eliminate self-excited vibration in their design process.

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