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THE INVESTIGATION OF THE SECOND FIELD OF THE SUMMARIZED FREQUENCY ORIGINATED FROM SCATTERING OF NONLINEARLY INTERACTING SOUND WAVES AT A RIGID SPHERE

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The problems of nonlinear scattering of acoustic waves by a sphere have been considered in references [1, 2], where the processes of nonlinear scattering of sound waves by a pulsating sphere were investigated. A related problem of interest is that of the nonlinear interactions of acoustic waves scattered by a sphere. However, for obtaining full representation of the wave processes happening in scattering by a sphere, one needs a detailed investigation of a second field having four frequency components. The secondary acoustic fields of waves of difference frequency and second harmonics of the initial high-frequency pumping waves have been investigated in references [3, 4]. By a method of successive approximation, the solution of an inhomogeneous wave equation of summarized frequency which includes the processes of nonlinear interaction of the incident and scattered initial pumping waves has been obtained. The solution has an aspect similar to that of the solution for a wave of difference frequency [3]. After integration on angular coordinates the solution for the acoustic pressure of a wave of summarized frequency becomes

$$
\begin{aligned}
P_{+}^{(2)}(r,\theta) &= [P_{+1}^{(2)} + P_{+2}^{(2)} + P_{+3}^{(2)} - P_{+4}^{(2)}] \\
&= C_{+}\int_{a}^{d}\Bigg[\sum_{l=0}^{\infty}(2l+1)^{2}\mathrm{j}_{l}(k_{1}r')j_{l}(k_{2}r')P_{l}^{2}(\cos\theta) \\
&\quad + \sum_{l=0}^{\infty}\sum_{m=l}^{\infty}(2l+1)\mathrm{j}_{l}(k_{1}r')P_{l}(\cos\theta)A_{m}^{2}(-\mathrm{i})D_{m}^{(2)}P_{m}(\cos\theta)\,\mathrm{e}^{-\mathrm{i}\varphi_{m}^{(2)}} \\
&\quad + \sum_{m=0}^{\infty}\sum_{l=m}^{\infty}A_{m}^{(1)}(-\mathrm{i})D_{m}^{(1)}P_{m}(\cos\theta)(2l+1)\mathrm{j}_{l}(k_{1}r')P_{l}(\cos\theta)\,\mathrm{e}^{-\mathrm{i}\varphi_{m}^{(1)}} \\
&\quad - \sum_{m=0}^{\infty}A_{m}^{(1)}D_{m}^{(1)}P_{m}^{2}(\cos\theta)A_{m}^{(2)}D_{m}^{(2)}\,\mathrm{e}^{\mathrm{i}(\varphi_{m}^{(1)}+\varphi_{m}^{(2)})}\Bigg]\sin(k_{5}r')\sin(k_{6}r')\,\mathrm{d}r'. \qquad (1)
\end{aligned}
$$

Here, $C_{+} = 12\,\mathrm{e}^{-\mathrm{i}k_{+}r}\varepsilon\omega_{1}\omega_{2}\rho_{0}\psi_{10}\psi_{20}(\omega_{1}+\omega_{2})^{2}/k_{+}^{2}c_{0}^{4}r\sin 2\theta$, $k_{5} = k_{+}\cos\theta$, $k_{6} = k_{+}\sin\theta$, $k_{+} = (\omega_{1}+\omega_{2})/c_{0}$ (the wave number of a summarized wave), ω_{1}, ω_{2}, are the pumping frequencies, $\mathrm{j}_{l}(k_{n}r)$ is the spherical Bessel function of the lth order, $D_{m}^{(n)}$ and $\varphi_{m}^{(n)}$ are the modulus and phase of a spherical Hankel function of the second kind $h_{m}^{(2)}(k_{n}r)$, $P_{l}(\cos\theta)$, $P_{m}(\cos\theta)$ are Legendre polynomials, $A_{m}^{(n)}$ are coefficients determined from the boundary conditions for a rigid sphere (boundary condition of Neumann), ψ_{n0} is the amplitude of a velocity potential function, ε is a nonlinear parameter, ρ_{0} is the density of the undisturbed

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medium, a is radius of the sphere, and d is the radius of the area of nonlinear interaction of pumping waves around the spherical scatterer.

Despite the analogy of the expressions to those of the scattering of a high-frequency, those for the summarized wave have a geometric character, as against a wave of difference frequency, which encompasses Rayleigh's and resonance areas. Thus it is necessary to note that the first term of expression (1), $P^{(2)}_{+1}$, corresponds to the part of the common acoustic pressure of a secondary field of a wave of summarized frequency, which is generated in a spherical stratum of the region of nonlinear interaction between the incident plane pumping waves of frequencies ω_1 and ω_2; the second term, $P^{(2)}_{+2}$, describes the interaction

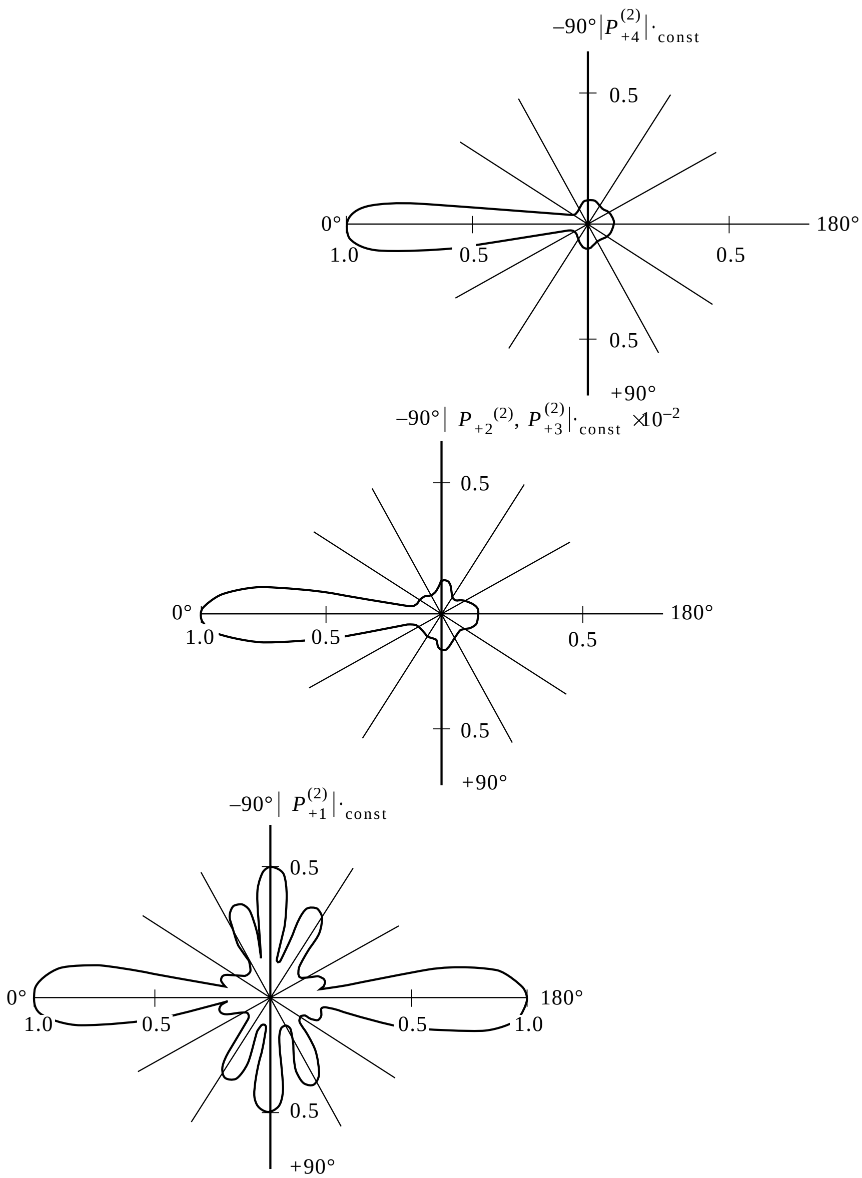

Figure 1. The diagrams of scattering of terms of common acoustic pressure of a wave of summarized frequency $P^{(2)}_{+1}$, $P^{(2)}_{+2}$, $P^{(2)}_{+3}$, and $P^{(2)}_{+4}$ by a sphere with a diameter of 11 mm on a distance $d = 10$ mm with $f_1 = 986{\cdot}6$ kHz, $f_2 = 1030$ kHz, $F_+ = 2016{\cdot}6$ kHz ($k_+a = 46{\cdot}5$).

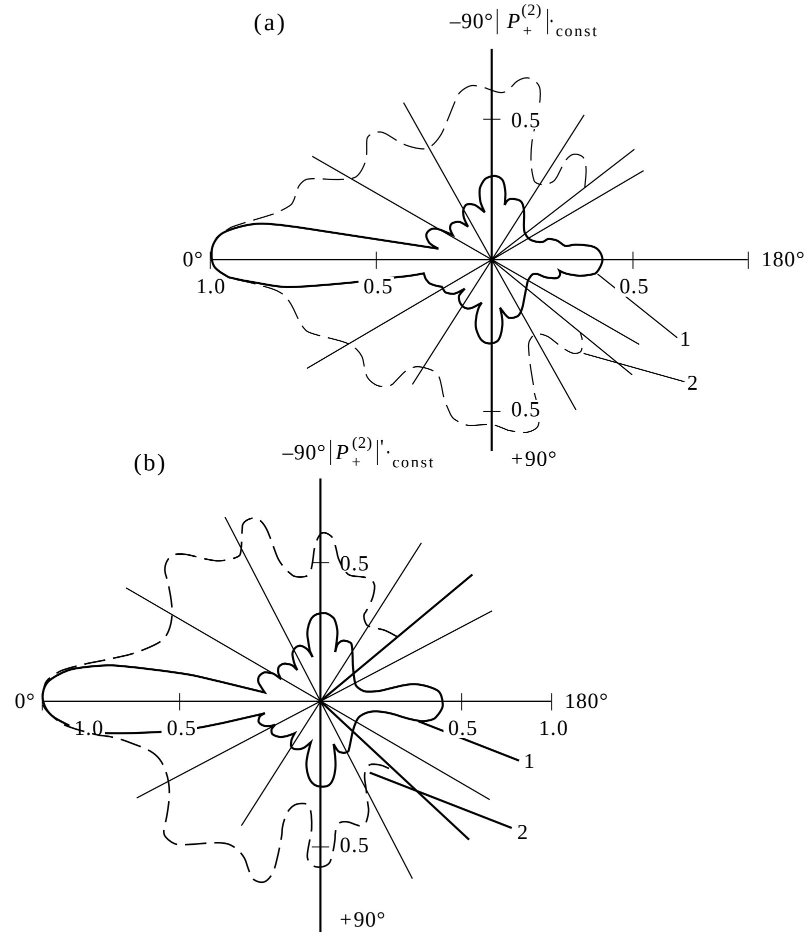

Figure 2. Calculated (1) and experimental (2) diagrams of scattering of common acoustic pressure of a wave of summarized frequency $P_+^{(2)}$ by a sphere with a diameter of 11 mm at a distance $d = 20$ cm with (a) $f_1 = 986{\cdot}6$ kHz, $f_2 = 1030$ kHz, $F_+ = 2016{\cdot}6$ kHz ($k_+a = 46{\cdot}5$); (b) $f_1 = 943{\cdot}2$ kHz, $f_2 = 1030$ kHz, $F_+ = 1973{\cdot}2$ kHz ($k_+a = 45{\cdot}5$).

of an incident plane wave of frequency ω_1 and a scattered spherical wave of frequency ω_2; the third term, $P_{+3}^{(2)}$, corresponds to the interaction of an incident plane wave of frequency ω_2 with a scattered spherical wave of frequency ω_1; the fourth term, $P_{+4}^{(2)}$, to the interaction of scattered spherical waves with frequencies ω_1 and ω_2 in a spherical stratum of the medium around the scatterer.

Thus the common acoustic pressure of a secondary field of a wave of summarized frequency represents a population of acoustic pressures of all spatial components of a second field with different amplitude and phase relations.

After a final integration on the coordinate r of expression (1), one obtains high frequency asymptotic expressions for all four spatial components of the common acoustic pressure of the second field of a wave of summarized frequency. These expressions for the scattering of components $P_{+1}^{(2)}$, $P_{+2}^{(2)}$, $P_{+3}^{(2)}$, $P_{+4}^{(2)}$, have been evaluated and the results are shown in Figure 1. The calculations were for a sphere with a diameter of 11 mm, $k_+a = 46{\cdot}5$, at a distance $d = 10$ mm. Figure 1 shows that the first term $P_{+1}^{(2)}$ has dominant levels of scattering in both inverse and direct directions, $\theta = 0°$ and 180°; the scattering of the second and third terms $P_{+2}^{(2)}$ and $P_{+3}^{(2)}$ has a lower level of scattering in the direct direction $\theta = 0^\circ$ and the scattering of the fourth term $P_{+4}^{(2)}$ has a unique maximum in the opposite direction. Note also that the second and third terms, $P_{+2}^{(2)}$ and $P_{+3}^{(2)}$ have lower levels of acoustic pressure because of destructive interaction of the initial high frequency waves.

In Figure 2 the compared, calculated and experimental results for the scattering of the common acoustic pressure of a wave of summarized frequency $P_{+}^{(2)}$ are shown (the experiments were carried out in water with a steel sphere of diamter 11 mm). The figure shows that although the agreement between calculated and experimental results is satisfactory, the calculated intermediate maxima are somewhat smaller than the experimental ones. This is because of the transient character of the experimental excitation, casuing the duration of the interaction of counter pumping waves to be limited in time. One can note also that with increase in the Helmholtz number of the sphere the levels of the intermediate maxima undergo insignificant changes. This is because, although the volume of the region of nonlinear interaction increases, the amplitudes of the interacting waves become correspondingly weaker at increased radial distances.

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