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TRANSVERSE VIBRATIONS OF CIRCULAR PLATES OF RECTANGULAR ORTHOTROPY CARRYING A CENTRAL, CONCENTRATED MASS

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1. introduction

A circular, solid or annular plate of uniform or non-uniform thickness is probably one of the most common elements that Man has devised in his pursuit of either scientific or technological goals; from optical lens to printed circuit boards passing through pistons of gasoline and diesel engines and compressors, acoustic transducers foundation of machinery, rocket and heart prothesis elements, etc.

In most of these applications the plate performs its function in a dynamic fashion and the design engineer needs an adequate knowledge of its natural vibrational characteristics (frequencies and mode shapes).

On the other hand, several complicated factors may come into play: uniform or non-uniform elastic constraints at the plate edge, presence of elastically or rigidly connected masses, in-plane forces, etc.

When the plate material is isotropic and the structural element possesses uniform thickness, many basic dynamic problems are solved in a classical fashion using Bessel functions [1]. On the other hand if the plate material is characterized as aelotropic, or polarly orthotropic, exact analytical solutions are available for many important structural situations [2]. Certainly the case of a circular plate of polar orthotropy carrying a central, concentrated mass and executing transverse vibrations is amenable to a straightforward approximate solution using simple polynomial coordinate functions [3].

The present study proposes a simple approach for determining the fundamental frequency of transverse vibration of a circular plate of rectangular orthotropy carrying a central, concentrated mass (Figure 1). This problem is also of basic technological importance since steel and aluminum plates do possess, in general, rectangularly orthotropic characteristics due to the metallurgical processes to which they have been exposed.*

Since the boundary of the domain is not natural to the material coordinate axes it seems appropriate to express the plate displacement amplitude in terms of polynomials in the x and y coordinates. Clamped and simply supported edges have been assumed. The frequency determinant has been generated using the classical Rayleigh–Ritz method. The evaluation of the integrals appearing in the functional has been greatly facilitated by the use of MAPLE [4].

* Certainly, the situation is also of interest when dealing with composite materials.

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2. APPROXIMATE ANALYTICAL SOLUTION

Using Lekhnitskii's classical notation [2] one expresses the governing functional in the form

$$J[W]=\frac{1}{2}\int\int\left[D_1\left(\frac{\partial^2 W}{\partial x^2}\right)^2+2D_1\nu_2\frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2}+D_2\left(\frac{\partial^2 W}{\partial y^2}\right)^2\right.$$
$$\left.+4D_k\left(\frac{\partial^2 W}{\partial x\,\partial y}\right)^2\right]\mathrm{d}x\,\mathrm{d}y-\frac{\rho h}{2}\omega^2\int\int W^2\,\mathrm{d}x\,\mathrm{d}y-\frac{M\omega^2}{2}W^2(0,0), \tag{1}$$

where $W(x, y)$ is the amplitude of transverse vibration and M is the concentrated mass.

In the case of a clamped edge the displacement amplitude is approximated using the polynomial expression

$$W\simeq W_a=\left[\left(\frac{x}{a}\right)^2+\left(\frac{y}{a}\right)^2-1\right]^2\left(a_1+a_2\left(\frac{x}{a}\right)^2+a_3\left(\frac{y}{a}\right)^2+a_4\left(\frac{x}{a}\right)^4+a_5\left(\frac{y}{a}\right)^4\right.$$
$$\left.+a_6\left(\frac{x}{a}\right)^6+a_7\left(\frac{y}{a}\right)^6+a_8\left(\frac{x}{a}\right)^8+a_9\left(\frac{y}{a}\right)^8\right] \tag{2}$$

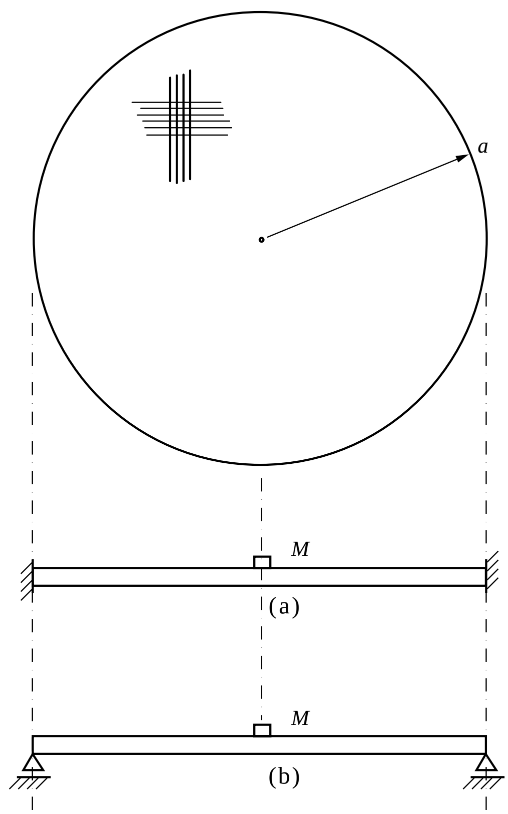

Figure 1. Circular plate of rectangular orthotropy carrying a concentrated mass at its centre. (a) Clamped; (b) simply supported.

TABLE 1

Isotropic circular plate with a central, concentrated mass ($\nu_2 = \nu = 0{\cdot}30$): comparison of values of the fundamental frequency coefficient $\Omega_1 = \sqrt{\rho h/D}\omega_1 a^2$ as a function of M/M_p

	M/M_p	Number of terms 3	7	9	[5]	Exact
Clamped	0	10·217	10·216	10·216	10·22	10·215
	0·05	9·046	9·028	9·026	9·01	
	0·10	8·188	8·151	8·148	8·11	
	0·20	7·006	6·944	6·937	6·87	
	0·5	5·204	5·121	5·112	5·02	
Simply supported	0	4·940	4·937		4·93	4·935
	0·05	4·562	4·553			
	0·10	4·259	4·243		4·23	
	0·20	3·797	3·772		3·75	
	0·5	2·986	2·948		2·92	

M = concentrated mass; M_p = plate mass.

while, when dealing with the simply supported plate, W is expressed in the form

$$W \simeq W_a = \left[\left(\frac{x}{a}\right)^2 + \left(\frac{y}{a}\right)^2 - 1\right]\left(a_1 + a_2\left(\frac{x}{a}\right)^2 + a_3\left(\frac{y}{a}\right)^2 + a_4\left(\frac{x}{a}\right)^4 + a_5\left(\frac{y}{a}\right)^4 + a_6\left(\frac{x}{a}\right)^6 + a_7\left(\frac{y}{a}\right)^6\right]. \tag{3}$$

Clearly, expression (2) satisfies both essential governing boundary conditions at $r = a$:

$$W = \frac{\partial W}{\partial r} = 0 \tag{4}$$

while equation (3) satisfies the null displacement requirement at the plate edge but not the natural boundary condition which requires that the bending moment normal to the boundary be equal to zero.

Substituting W_a, as given by equations (2) or (3), in the energy functional (1) and requiring that

$$\frac{\partial J[W_a]}{\partial a_i} = 0, \tag{5}$$

one obtains a linear, homogeneous system of equations in the a_i's. A determinantal equation is finally obtained from the non triviality condition, its lowest root being the fundamental frequency coefficient $\Omega_1 = \sqrt{\rho h/D_1}\omega_1 a^2$.

3. NUMERICAL RESULTS

In the case of a clamped edge the eigenvalues were determined using 3, 7 and 9 terms, respectively, while, when dealing with a hinged boundary, the fundamental frequency coefficients were determined using approximations of 3 and 7 terms.† In all cases the numerical values were truncated after the third decimal figure.

† Previous studies have shown that the present polynomial approach converges faster in the case of a simply supported edge [5].

TABLE 2

Orthotropic circular plate with a central, concentrated mass ($D_2/D_1 = D_k/D_1 = 0{\cdot}5$; $\nu_2 = 0{\cdot}30$): *analysis of the convergence of the values of* $\Omega_1 = \sqrt{\rho h/D_1}\,\omega_1 a^2$

	M/M_p	Number of terms 3	7	9
Clamped	0	9·619	9·619	9·618
	0·05	8·513	8·492	8·490
	0·10	7·703	7·661	7·656
	0·20	6·588	6·517	6·509
	0·5	4·890	4·797	4·786
Simply supported	0	4·483	4·482	
	0·05	4·142	4·138	
	0·10	3·868	3·859	
	0·20	3·451	3·433	
	0·5	2·716	2·686	

Table 1 depicts a comparison of eigenvalues for clamped and simply supported isotropic plates ($\nu = 0{\cdot}30$). There is good engineering agreement with values previously determined using the optimized Galerkin approach where all the governing boundary conditions were satisfied [5].

Tables 2 and 3 deal with orthotropic circular plates: $D_2/D_1 = D_k/D_1 = 0{\cdot}5$ and $D_2/D_1 = 1$; $D_k/D_1 = 0{\cdot}5$, respectively, while ν_2 was taken equal to 0·3 for both configurations. It can be seen that the effect of increasing the number of terms is greater as M/M_p increases.

The present approach is quite simple and straightforward specially when using an oriented mathematical algorithm [4]. The case of elliptical plates constitutes, obviously, an extension of the present treatment.

TABLE 3

Orthotropic circular plate with a central, concentrated mass ($D_2/D_1 = 1$; $D_k/D_1 = 0{\cdot}5$; $\nu_2 = 0{\cdot}30$): *analysis of the convergence of the values of* $\Omega_1 = \sqrt{\rho h/D_1}\,\omega_1 a^2$

	M/M_p	Number of terms 3	7	9
Clamped	0	10·593	10·592	10·592
	0·05	9·379	9·359	9·358
	0·10	8·490	8·450	8·446
	0·20	7·264	7·196	7·189
	0·5	5·396	5·306	5·296
Simply supported	0	4·978	4·977	
	0·05	4·600	4·593	
	0·10	4·296	4·283	
	0·20	3·833	3·811	
	0·5	3·017	2·982	

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