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STOCHASTIC NON-LINEAR VIBRATIONS OF HIGHWAY SUSPENSION BRIDGE UNDER INERTIAL SPRUNG MOVING LOAD

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The problem of vertical non-linear vibrations of a single-span suspension bridge due to a random stream of moving vehicles is considered. The inertia forces of car bodies and vehicle suspensions are taken into account because each vehicle is idealized by a set of viscoelastic oscillators. Vehicle arrivals at the bridge as well as the vehicle types and the masses of the truckloads are assumed to be random variables. The bridge model is defined as flat and geometrically non-linear. The numerical simulation method is developed to determine expected values and standard deviations of the bridge response. Solutions for the bridge deflections, the tension of cables and bending moments at the girder are presented and compared with results which have been calculated in simplified cases when a train of concentrated forces moving in the dynamic or static conditions idealizes vehicular traffic loads.

1. INTRODUCTION

In recent literature on probabilistic dynamics of highway bridges, the moving load model is mostly assumed to be an infinite stream of moving forces which arrive at the span at random times and have random values. The inertia forces of car bodies, the vehicle springing and the distances between the vehicle axles are neglected. Thus, analytical methods have been developed to determine the probabilistic characteristics of the bridge response. For instance, this approach has been presented by Tung [1, 2], Iwankiewicz and Śniady [3], Sieniawska and Śniady [4], Zibdeh and Rackwitz [5]. Also Bryja and Śniady [6, 7] have approached the stochastic dynamic problem of a highway suspension bridge from this point of view.

In this paper the simulation method is applied to probabilistic dynamic analysis of a single-span suspension bridge under the action of traffic loads. The bridge model is defined as flat yet the geometrical non-linearity of the construction is considered. The highway traffic load is modelled by a set of viscoelastic oscillators corresponding with vehicles' axles. So, the vertical inertia forces and suspensions of multi-axle road vehicles are taken into account. The inter-arrival times of the moving vehicles are regarded as random variables. It is also assumed that the vehicle types and the masses of the truckloads are random. The expected values and standard deviations of the dynamic bridge response due to a random infinite

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stream of moving vehicles have been determined by the use of the simulation method. Solutions are presented for the tension of cables, bridge deflections and bending moments at the bridge beam. In comparison, calculations are performed for the simplified cases in which vehicles are modelled by forces moving under dynamic or quasi-static conditions.

2. BASIC ASSUMPTIONS

The bridge model is regarded as a single-span prismatic beam which is simply supported and underslung by means of vertical hangers to two whipped cables (see Figure 1). The cables are anchored at their ends and movable at their supporting points on the undeformable pylons. The dead load curve of the cable within the span of the beam forms a parabola $z(x) = 4x(l - x)f/l^2$, the other segments being rectilinear. The mass of the bridge m is assumed to be uniformly distributed and constant along the length of the span. In the paper the vertical vibrations of the bridge are considered with account taken of the cables geometrical non-linearity.

The integro-differential equation of vertical vibrations $w(x, t)$ under the live load $p(x, t)$ has the form

$$EJw^{IV}(x,t) - 2H(t)w''(x,t) + \frac{16kf}{l^2}\int_0^l w(x,t)\,\mathrm{d}x + c\dot{w}(x,t) + m\ddot{w}(x,t) = p(x,t), \quad (1)$$

as was defined by Bryja and Śniady [6, 7]. EJ is the flexural rigidity of the beam, m is the mass of the bridge, c denotes the damping coefficient and $(\cdot)' = \partial/\partial x$, $(\dot{\ }) = \partial/\partial t$. The stiffness factor of a single cable is defined as

$$k = (8f/l^2)(E_c A_c/L_c), \qquad L_c = \int_{(L)} \cos^{-3}\beta\,\mathrm{d}x, \quad (2)$$

where E_c is the cable's modulus of elasticity and A_c is the cable's cross-section area. $H(t)$ denotes the all-horizontal component of the cable tension which is the sum

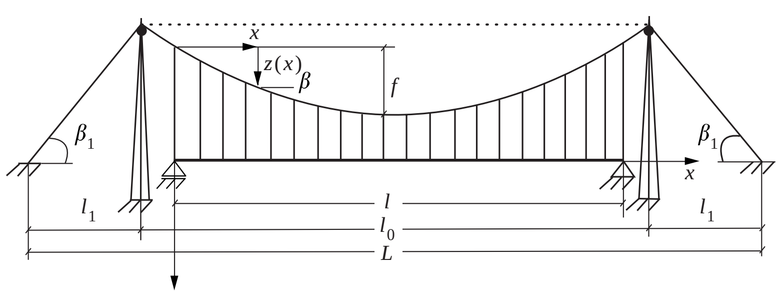

Figure 1. Model of a bridge.

of the initial (dead-load) value $H_0 = mgl^2/16f$ and the vibrational increment $\Delta H(t)$ defined by the integral

$$\Delta H(t) = k \int_0^l w(x, t)\,\mathrm{d}x, \qquad H(t) = H_0 + \Delta H(t). \tag{3}$$

The increment of the cable tension is a function of displacements $w(x, t)$ so equation (1) is non-linear.

The problem of non-linearity is to be solved here by a numerical method based upon the assumption that equation (1) is considered as linear within each sufficiently short space of time (time step). At each time step the increment of the cable tension, expressed as $\Delta H(t) = \eta H_0$ is initially predicated by the parabolic extrapolation of the parameter η defined by the formula

$$\eta^{(i)} = 3\eta^{i-1} - 3\eta^{i-2} + \eta^{i-3}. \tag{4}$$

After solving equation (1), the increment of tension and corresponding parameter η^i are obtained from the relationship (3). The predicated value $\eta^{(i)}$ is determined by values calculated in previous time steps: η^{i-1}, η^{i-2}, η^{i-3}.

The vertical deflections of the bridge are assumed to be of sine series form,

$$w(x, t) = \sum_n q_{bn}(t) \sin(n\pi\xi) = \mathbf{q}_b^{\mathrm{T}}(t)\mathbf{s}(\xi), \tag{5}$$

in which $\mathbf{q}_b = \mathrm{col}\,(q_{b1}, q_{b2}, \dots, q_{bn_b})$, $\mathbf{s} = \mathrm{col}\,(\sin \pi\xi, \sin 2\pi\xi, \dots, \sin n_b\pi\xi)$, $\xi = x/l \in [0, 1]$ and the symbol $(\cdot)^{\mathrm{T}}$ denotes the transposition operator. Upon taking the variations of solutions in the form of vector $\partial\mathbf{w} = \mathbf{s}(\xi)$, the equations of Galerkin's method can be formulated as

$$\left[EJ \int_0^l \mathbf{s}\mathbf{s}^{IV\mathrm{T}}\,\mathrm{d}x - 2H_0(1+\eta)\int_0^l \mathbf{s}\mathbf{s}''^{\mathrm{T}}\,\mathrm{d}x + \frac{16kf}{l^2}\int_0^l \mathbf{s}\left(\int_0^l \mathbf{s}^{\mathrm{T}}\,\mathrm{d}x\right)\mathrm{d}x\right]\mathbf{q}_b$$

$$+\left(c\int_0^l \mathbf{s}\mathbf{s}^{\mathrm{T}}\,\mathrm{d}x\right)\dot{\mathbf{q}}_b + \left(m\int_0^l \mathbf{s}\mathbf{s}^{\mathrm{T}}\,\mathrm{d}x\right)\ddot{\mathbf{q}}_b = \int_0^l \mathbf{s}p\,\mathrm{d}x. \tag{6}$$

As a result the set of ordinary differential equations is obtained in the matrix form

$$\mathbf{B}_b\ddot{\mathbf{q}}_b + \mathbf{D}_b\dot{\mathbf{q}}_b + \mathbf{K}_b\mathbf{q}_b = \mathbf{F}_b, \tag{7}$$

where

$$\mathbf{K}_b = \frac{EJ}{2l^3}\{\mathbf{d}^4\} + \frac{H_0}{l}(1+\eta)\{\mathbf{d}^2\} + 64kf\mathbf{g}\mathbf{g}^{\mathrm{T}}, \qquad \mathbf{B}_b = \frac{ml}{2}\mathbf{I}, \qquad \mathbf{D}_b = 2\alpha_1\omega_1\mathbf{B}_b,$$

$$\mathbf{F}_b = \int_0^l \mathbf{s}p\,\mathrm{d}x,$$

$$\mathbf{I} = \mathrm{diag}\,(1, 1, \dots), \qquad \{\mathbf{d}\} = \mathrm{diag}\,(\pi, 2\pi, \dots), \qquad \mathbf{g} = \mathrm{col}\left(\frac{1}{\pi}, 0, \frac{1}{3\pi}, 0, \dots\right).$$

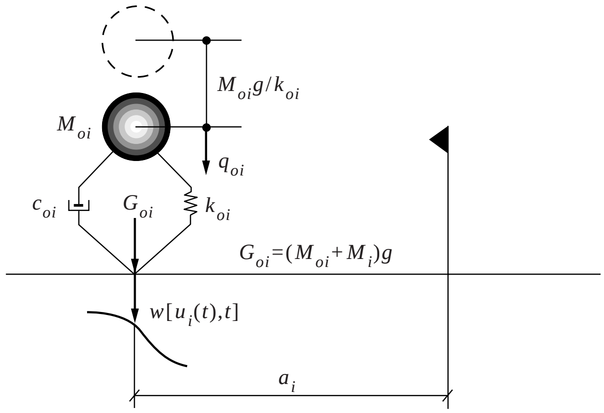

Figure 2. Single oscillator.

Damping properties of the bridge are taken according to the viscous damping model: $\mathbf{D}_b = \mu\mathbf{B}_b$. The damping coefficient is estimated as $\mu = 2\alpha_1\omega_1$, in relation to the fraction of the critical damping α_1 regarded for the first mode of vibrations with natural circular frequency ω_1.

The theoretical model of a road vehicle is assumed to be a set of viscoelastic oscillators. A single ith oscillator is shown in Figure 2, the traffic load model—in Figure 3. The inertia forces of the car body are taken into account whereas the inertia forces of unsprung masses M_i are neglected. $G_{oi} = (M_{oi} + M_i)g$ denotes the part-weight of a vehicle (counted for a single axle). The distances between oscillators are equal to the distances between vehicle axles. The vehicles arrive at the bridge in the static equilibrium state and move at a constant velocity v. The pavement irregularities are neglected.

The dynamic response of a bridge under a semi-infinite random stream of vehicles is considered. Traffic flow is characterized by three basic random parameters: ϑ—time interval between two successive vehicles, K—type of a vehicle, M_t—truckload mass. Usually the arrival process is assumed to constitute the stationary Poissonian process with the intensity λ [8, 9]. It allows unreal cases when $\vartheta \to 0$ or $\vartheta \to \infty$. In this paper the random variable ϑ has the probability

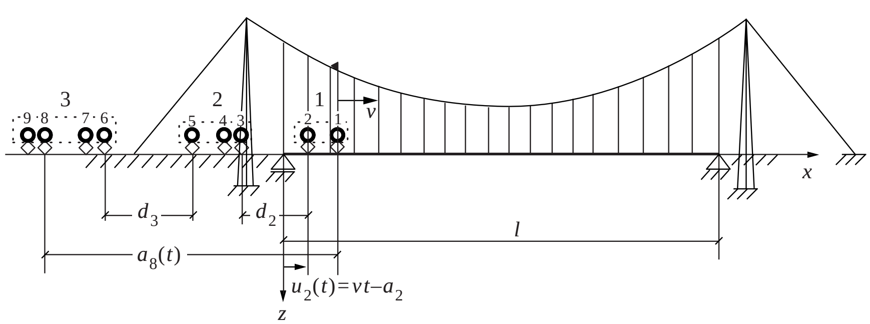

Figure 3. Load pattern of a bridge.

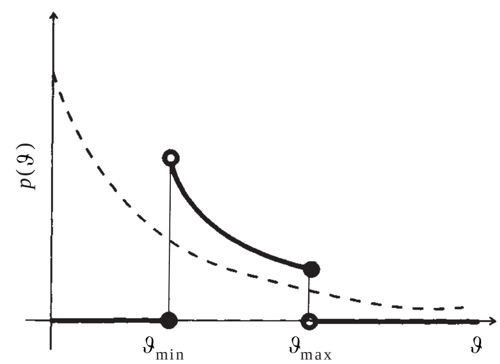

Figure 4. Probability density function of random variable ϑ (time interval between two vehicles); (-----) the Poissonian distribution, (—) the bi-cut exponent distribution.

density function $p(\vartheta)$ in the form of the bi-cut exponent distribution [10] shown in Figure 4 and defined by the formula

$$p(\vartheta) = \left\{ \begin{matrix} \rho\lambda\, e^{-\lambda\vartheta} & \text{for} & \vartheta_{min} < \vartheta \leqslant \vartheta_{max} \\ 0 & \text{for} & \vartheta \leqslant \vartheta_{min} \text{ or } \vartheta > \vartheta_{max} \end{matrix} \right\}, \tag{8}$$

where $\rho = 1/(e^{-\lambda\vartheta_{min}} - e^{-\lambda\vartheta_{max}})$, $\vartheta_{min} = d_{min}/v$, $\vartheta_{max} = d_{max}/v$. The symbols d_{min}, d_{max} denote the minimum and the maximum distance between vehicles at velocity v. The distances between successive vehicles are equal to $d_j = v\vartheta_j$, $j = 1, 2, \ldots$. The sequences of random numbers ϑ_j satisfying formula (8) are generated by making use of the Neumann elimination method [11].

The road vehicle type K is a discrete random variable with the probability density function in the form of jump distribution illustrated in Figure 5, where P_j denotes the probability of the event when a vehicle of the jth type arrives at the bridge, n_K is the number of vehicle types. Successive vehicles are of the types K_j, $j = 1, 2, \ldots$. The sequences of random numbers K_j are generated by use of the method of inversion of distribution function [11].

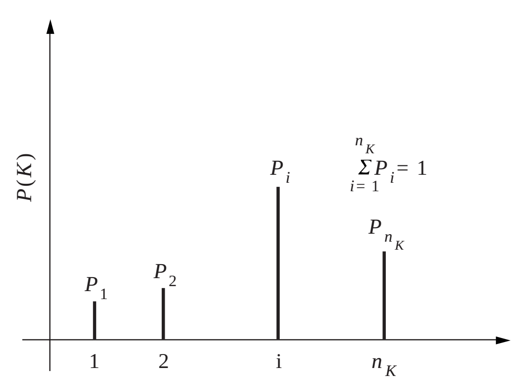

Figure 5. Jump distribution of random variable K (vehicle type).

The truckload mass M_t for successive vehicles varies randomly in the continuous interval [0, $M_{t,max}$]. $M_{t,max}$ is the maximum mass determined for every vehicle type. It is assumed that $M_t = zM_{t,max}$, where the truckload rate z is a random continuous variable with the uniform probability distribution in the interval [0, 1]. Successive vehicles have the truckload mass $M_{t,j} = z_j M_{t,max,j}$, $j = 1, 2, \ldots$. To generate sequences of random numbers z_j the standard numerical procedure may be adopted.

3. ESTIMATION OF THE BRIDGE RESPONSE CUMULANTS

By applying the method presented by Klasztorny and Langer [12], the matrix equation of motion of the bridge and moving load system can be formulated in the form

$$\mathbf{B}\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{F}, \tag{9}$$

where

$$\mathbf{B}(t) = \left[\begin{array}{c|c} \mathbf{B}_b & \mathbf{S}\{\mathbf{M}_\mathrm{o}\} \\ \mathbf{0} & \{\mathbf{M}_\mathrm{o}\} \end{array}\right], \qquad \mathbf{D}(\mathbf{t}) = \left[\begin{array}{c|c} \mathbf{D}_b & \mathbf{0} \\ -\{\mathbf{c}_\mathrm{o}\}\mathbf{S}^\mathrm{T} & \{\mathbf{c}_\mathrm{o}\} \end{array}\right], \qquad \mathbf{F}(t) = \begin{bmatrix} \mathbf{S}\mathbf{G}_\mathrm{o} \\ \mathbf{0} \end{bmatrix},$$

$$\mathbf{K}(t) = \left[\begin{array}{c|c} \mathbf{K}_b & \mathbf{0} \\ -\{\mathbf{k}_\mathrm{o}\}\mathbf{S}^\mathrm{T} - (v/l)\{\mathbf{c}_\mathrm{o}\}\mathbf{C}^\mathrm{T}\{\mathbf{d}\} & \{\mathbf{k}_\mathrm{o}\} \end{array}\right]. \tag{10}$$

The vector of generalized co-ordinates $\mathbf{q} = \mathrm{col}\,(\mathbf{q}_b, \mathbf{q}_\mathrm{o})$ consists of the "permanent" subset $\mathbf{q}_b$ relating to the bridge and the "ephemeral" subset $\mathbf{q}_\mathrm{o} = \mathrm{col}\,(q_{\mathrm{o}n_r}, \ldots, q_{\mathrm{o}i}, \ldots, q_{\mathrm{o}n_l})$ which describes the vibrations of the oscillators directly acting on the bridge at the moment t. The diagonal matrices $\{\mathbf{M}_\mathrm{o}\} = \mathrm{diag}\,(M_{\mathrm{o}n_r}, \ldots, M_{\mathrm{o}i}, \ldots, M_{\mathrm{o}n_l})$, $\{\mathbf{k}_\mathrm{o}\} = \mathrm{diag}\,(k_{\mathrm{o}n_r}, \ldots, k_{\mathrm{o}i}, \ldots, k_{\mathrm{o}n_l})$, $\{\mathbf{c}_\mathrm{o}\} = \mathrm{diag}\,(c_{\mathrm{o}n_r}, \ldots, c_{\mathrm{o}i}, \ldots, c_{\mathrm{o}n_l})$ and column matrix $\mathbf{G}_\mathrm{o} = \mathrm{col}\,(G_{\mathrm{o}n_r}, \ldots, G_{\mathrm{o}i}, \ldots, G_{\mathrm{o}n_l})$ contain the oscillators' parameters. Columns of matrices $\mathbf{S}(t) = [\mathbf{S}_{n_r}, \ldots, \mathbf{S}_i, \ldots, \mathbf{S}_{n_l}]$ and $\mathbf{C}(t) = [\mathbf{C}_{n_r}, \ldots, \mathbf{C}_i, \ldots, \mathbf{C}_{n_l}]$ are function vectors of the form

$$\mathbf{S}_i(t) = \begin{Bmatrix} \mathbf{s}(u_i(t)/l) & \text{for} & u_i \in [0, l] \\ \mathbf{0} & \text{for} & u_i \notin [0, l] \end{Bmatrix}, \qquad \mathbf{C}_i(t) = \begin{Bmatrix} \mathbf{c}(u_i(t)/l) & \text{for} & u_i \in [0, l] \\ \mathbf{0} & \text{for} & u_i \notin [0, l] \end{Bmatrix}, \tag{11}$$

where $u_i(t) = vt - a_i$, $\mathbf{c} = \mathrm{col}\,(\cos \pi\xi, \cos 2\pi\xi, \ldots, \cos n_b \pi\xi)$.

By making use of the Newmark method in the unconditionally stable variant, the equation of motion (9) has been solved numerically for the random samples of traffic load. Let $U(t)$ denote the bridge response in question—either deflection $w(x, t)$ or bending moment $M(x, t) = -EJw''(x, t)$ at the beam cross-section, or the vibrational increment of the cable tension $\Delta H(t)$. So, one has

$$U(t) = \begin{Bmatrix} \mathbf{s}^\mathrm{T}(\xi)\mathbf{q}_b(t) & \text{for} & w(x, t) \\ (EJ/l^2)\mathbf{s}^\mathrm{T}(\xi)\{\mathbf{d}^2\}\mathbf{q}_b(t) & \text{for} & M(x, t) \\ 2kl\mathbf{g}^\mathrm{T}\mathbf{q}_b(t) & \text{for} & \Delta H(t) \end{Bmatrix}. \tag{12}$$

In order to calculate the probabilistic parameters of the bridge random response, the following hypothesis has been formulated—the bridge response $U(t)$ is a stationary ergodic random process for $t \gg 0$. This hypothesis is going to be confirmed by numerical analysis. Based on the ergodic properties, it can be stated that probabilistic parameters of the response process $U(t)$ can be determined from their empirical counterparts obtained by time-averaging of one simulated process sample [13].

For such a process $U(t)$, the first and second order moments are defined by well-known formulae [13, 14],

$$m_1(U, t) = \frac{1}{t}\int_0^t U(t)\,\mathrm{d}t \xrightarrow{t\to\infty} E[U(t)] = \text{const}, \tag{13}$$

$$m_2(U, t) = \frac{1}{t}\int_0^t U^2(t)\,\mathrm{d}t \xrightarrow{t\to\infty} E[U^2(t)] = \text{const}, \tag{14}$$

in which $E[\,\cdot\,]$ denotes the expected value. In the case considered the response $U(t)$ is calculated from equation (12) by using the numerical solution of equation (9) obtained for the random sample of traffic load. The long enough observation time t is equally spaced with the step $h = \Delta t \ll t$. Thus, the process $U(t)$ is randomly simulated at equidistant time points $t_j = jh$, $j = 1, 2, \ldots$. The steady state values of the mean and variance are equal to

$$E[U] = m_1(U, t_n) \cong \frac{1}{n}\sum_{j=1}^{n} U(t_j), \tag{15}$$

$$\mathrm{Var}\,[U] = m_2(U, t_n) - m_1^2(U, t_n), \qquad m_2(U, t_n) \cong \frac{1}{n}\sum_{j=1}^{n} U^2(t_j), \tag{16}$$

where $t_n = nh \gg 0$.

4. NUMERICAL EXAMPLE AND CONCLUSIONS

The algorithm presented above has been applied to the single-span suspension bridge with the following design parameters that are denoted in Figure 1: $l = 300$ m, $l_0 = 315$ m, $l_1 = 98$ m, $f = 30$ m, $\beta_1 = 0{\cdot}61$ rad, $m = 12\,000$ kg/m, $EJ = 1{\cdot}98 \times 10^{11}$ Nm^2, $E_c A_c = 2{\cdot}2 \times 10^{10}$ N. The first natural frequency of the bridge is $\omega_1 = 2{\cdot}188$ rad/s and the damping coefficient is taken as $\alpha_1 = 0{\cdot}01$. The bridge is subjected to the random stream of two-axle road vehicles modelled by two oscillators. Irrespective of the speed, vehicles arrive with expected rate λ which is equal to $0{\cdot}5\ \mathrm{s}^{-1}$ and the extreme distances between vehicles are: $d_{min} = 5$ m, $d_{max} = 300$ m. Five classes of vehicles with the following arrival probabilities are distinguished: $K = 1$ (automobiles), $P_1 = 0{\cdot}05$; $K = 2$ (delivery trucks), $P_2 = 0{\cdot}10$; $K = 3$ (buses), $P_3 = 0{\cdot}10$; $K = 4$ (trucks), $P_4 = 0{\cdot}45$; $K = 5$ (heavy trucks), $P_5 = 0{\cdot}30$. The average parameters of vehicles belonging to these classes are

TABLE 1

Average parameters of vehicles of the five classes

K	M_{vf} (kg) / M_f (kg)	M_{vb} (kg) / M_b (kg)	$M_{t,max}$ (kg)	$\varepsilon = e_f/e$ / e (m)	k_f (kN/m) / c_f (kNs/m)	k_b (kN/m) / c_b (kNs/m)
1	700 / 80	600 / 90	400	0·80 / 2·75	36 / 2·0	50 / 2·2
2	1000 / 300	900 / 580	2500	0·90 / 3·70	156 / 5·0	636 / 9·6
3	3100 / 850	2500 / 1300	6000	0·75 / 5·15	250 / 11·1	300 / 11·0
4	2700 / 700	1700 / 1500	8200	0·82 / 4·00	406 / 13·2	1000 / 16·5
5	3600 / 800	4400 / 2600	12 000	0·95 / 5·75	510 / 17·1	1900 / 36·6

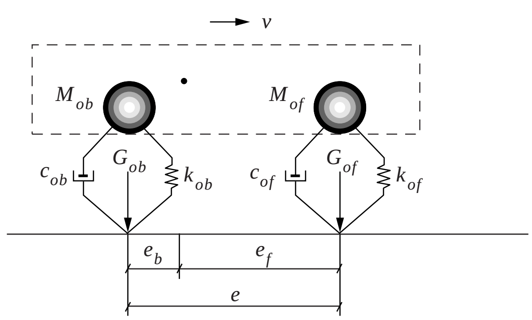

Figure 6. Model of a two-axle vehicle.

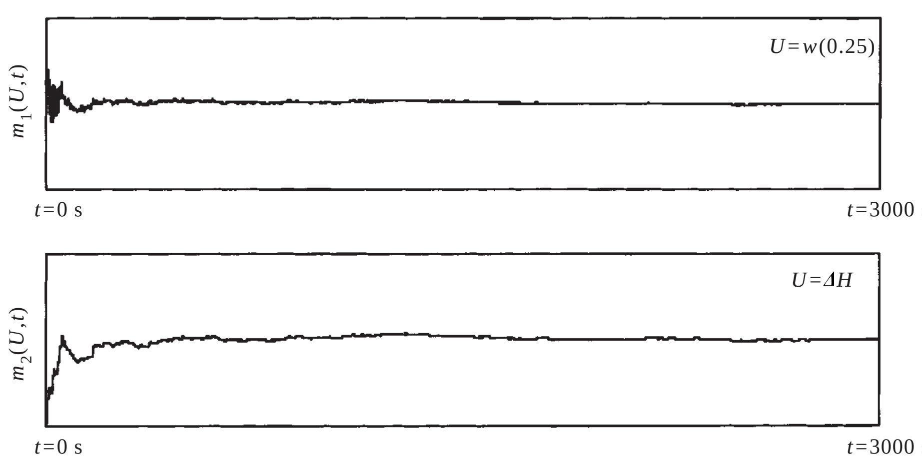

Figure 7. Exemplary stabilization histories of bridge response moments (first and second order).

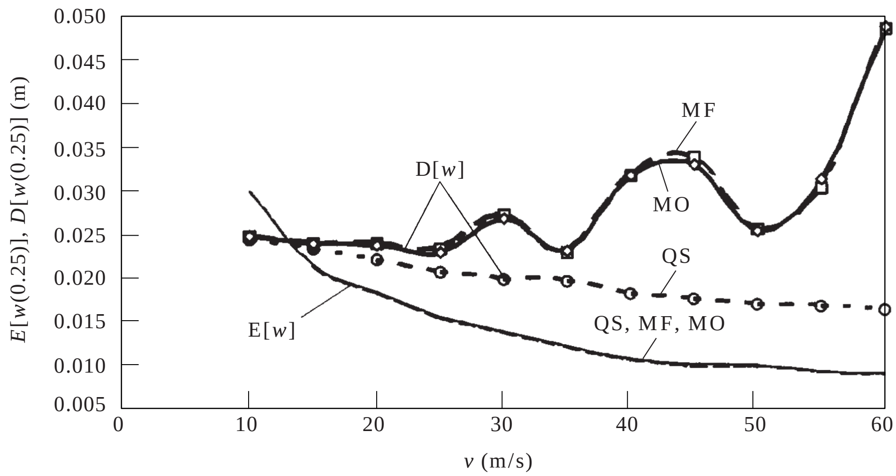

Figure 8. Expected value and standard deviation of the bridge deflection at a quarter of span; QS—random stream of moving forces, quasi-static response; MF—random stream of moving forces, dynamic response; MO—random stream of oscillators, dynamic response. -----, QS; – – –, MF; ——, MO; – –○– –, QS; – –□– –, MF; —◇—, MO.

derived from reference [15] and given in Table 1. Symbols M_{vf}, M_{vb} and M_f, M_b denote respectively sprung and unsprung vehicle masses divided between two axles (front and back). The other denotations correspond to the quantities which are shown in Figure 6. Masses and weights of two oscillators (front and back) modelling a vehicle should be calculated from the expressions

$$M_{of} = M_{vf} + M_t(1 - \varepsilon), \qquad M_{ob} = M_{vb} + M_t\varepsilon,$$
$$G_{of} = (M_{of} + M_f)g, \qquad G_{ob} = (M_{ob} + M_b)g, \tag{17}$$

where $g = 9{\cdot}81$ m/s^2.

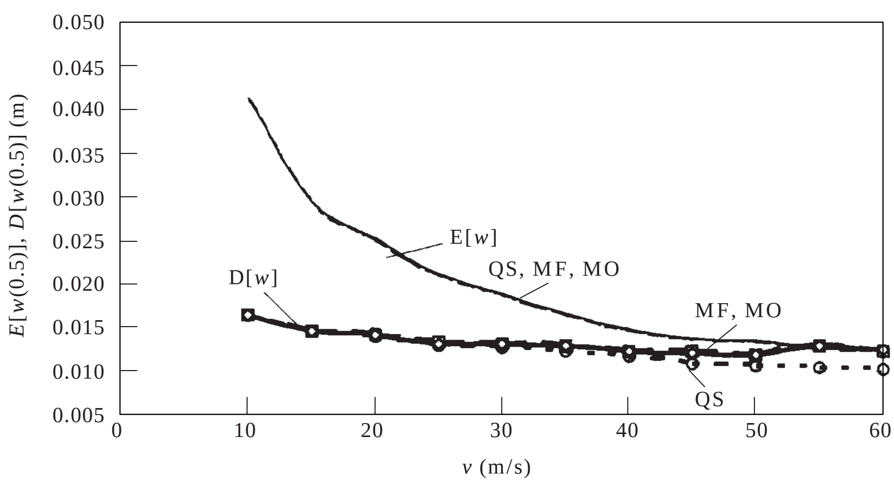

Figure 9. Expected value and standard deviation of the midspan bridge deflection; QS—random stream of moving forces, quasi-static response; MF—random stream of moving forces, dynamic response; MO—random stream of oscillators, dynamic response. Key as in Figure 8.

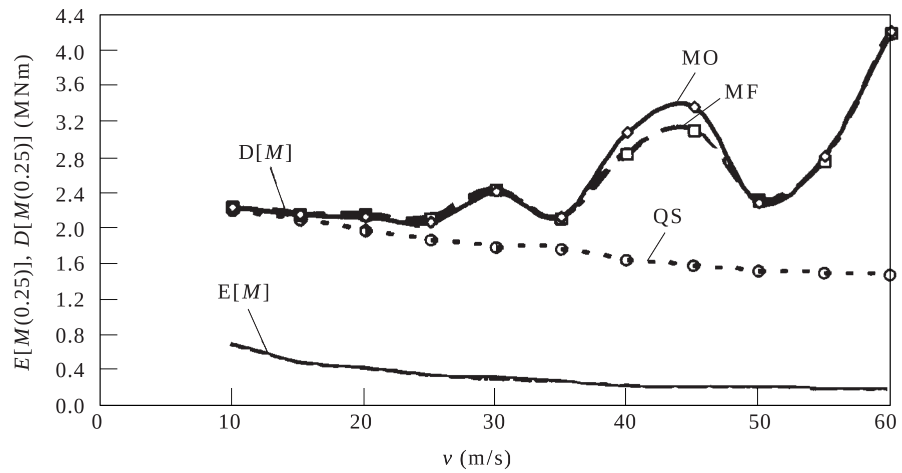

Figure 10. Expected value and standard deviation of the bending moment at a quarter of span of the bridge beam; QS—random stream of moving forces, quasi-static response; MF—random stream of moving forces, dynamic response; MO—random stream of oscillators, dynamic response. Key as in Figure 8.

The response of the bridge due to the described load is supposed to be a stationary ergodic random process for $t \gg 0$. Figure 7 shows exemplary stabilization histories of the first and second order moments calculated from equations (15) and (16) in the interval [0; 3000 s] for the passage speed of vehicles $v = 30$ m/s. It could be seen that moments get constant. So, from a practical point of view the hypothesis concerning the response process can be acceptable.

The steady state values of the mean ($E[\,]$) and standard deviation ($D[\,]$) for different types of the bridge response are plotted against the passage speed in

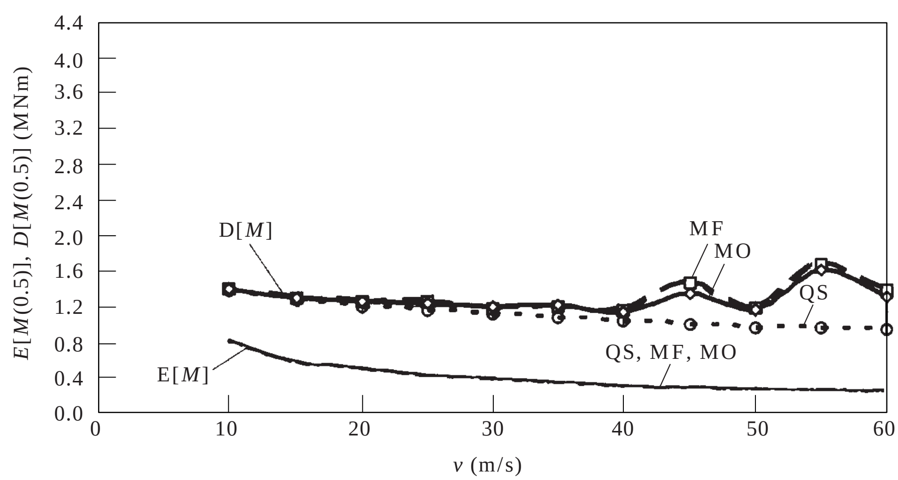

Figure 11. Expected value and standard deviation of the bending moment at midspan of the bridge beam; QS—random stream of moving forces, quasi-static response; MF—random stream of moving forces, dynamic response; MO—random stream of oscillators, dynamic response. Key as in Figure 8.

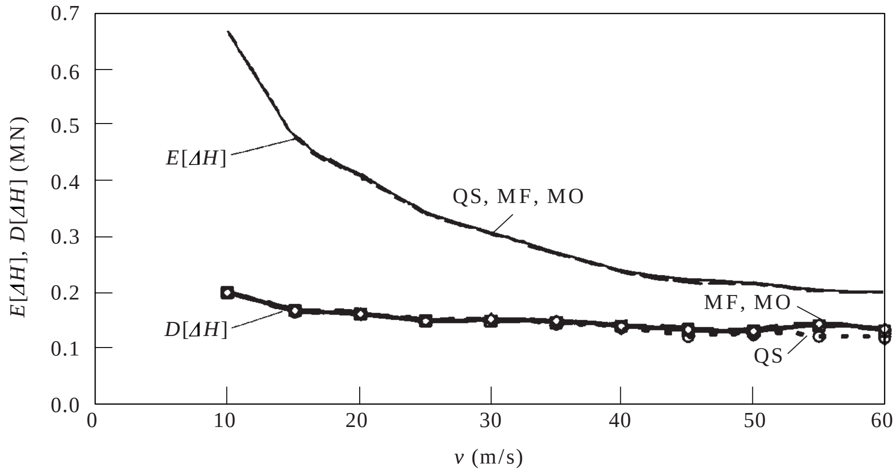

Figure 12. Expected value and standard deviation of the cable tension increment; QS—random stream of moving forces, quasi-static response; MF—random stream of moving forces, dynamic response; MO—random stream of oscillators, dynamic response. Key as in Figure 8.

Figures 8–12. The bold and fine continuum lines denote results determined for the random stream of moving oscillators modelling the traffic flow (MO-model). For comparison purposes, two simplified models of the traffic load have been considered: the first one in the form of a semi-infinite stream of forces G_{oi}, $i = 1, 2, \ldots$ moving in the dynamic conditions (MF-model) and the second in which forces G_{oi} move in the quasi-static conditions (QS-model). The matrix equations of motion for these load models can be easily obtained from equation (9).

Table 2

Comparison of results for the non-linear and linear problems

$E[U(\xi)]$ or $D[U(\xi)]$, unit of measure	MODEL					
	QS[a]		MF[b]		MO[c]	
	Linear	Non-linear	Linear	Non-linear	Linear	Non-linear
$E[w(0{\cdot}25)]$, m	0·013688	0·013679	0·013690	0·013681	0·013695	0·013686
$E[w(0{\cdot}5)]$, m	0·018976	0·018957	0·018976	0·018957	0·018977	0·018958
$E[M(0{\cdot}25)]$, MNm	0·313583	0·313713	0·313764	0·313865	0·314607	0·314736
$E[M(0{\cdot}5)]$, MNm	0·383490	0·382557	0·383492	0·382551	0·382999	0·382082
$E[\Delta H]$, MN	0·306435	0·306213	0·306433	0·306210	0·306507	0·306283
$D[w(0{\cdot}25)]$, m	0·020704	0·020599	0·028558	0·028166	0·029074	0·028863
$D[w(0{\cdot}5)]$, m	0·012985	0·012957	0·013370	0·013337	0·013458	0·013423
$D[M(0{\cdot}25)]$, MNm	1·854669	1·845315	2·557026	2·521334	2·608602	2·587765
$D[M(0{\cdot}5)]$, MNm	1·124079	1·120856	1·197407	1·191951	1·212169	1·206059
$D[\Delta H]$, MN	0·150665	0·150463	0·152893	0·152649	0·154095	0·153835

[a] Random stream of moving forces, quasi-static response.
[b] Random stream of moving forces, dynamic response.
[c] Random stream of oscillators, dynamic response.

Curves concerning the expected values of bridge responses do not differ for three considered models of moving vehicles (see Figures 8–12). It means that the dynamic and quasi-static solutions are practically the same. However, for the standard deviations, the dynamic effects are substantial and increase with the speed, rapidly for $v > 30$ m/s. It should also be noted, that the last comment refers to the sensitive point of the structure—at a quarter of the span. For the cable tension, midspan deflection, and midspan bending moment, standard deviations in the dynamic and quasi-static problem differ only slightly.

The numerical results reached for two dynamic models (moving forces—MF and moving oscillators—MO) are illustrated in Figures 8–12 by curves flowing into one another. In other words, the effect of the vehicles' springing as well as vehicles' inertia forces is negligible. So, the simplified model of the traffic load (MF-model) is sufficient. This is a significant remark because it means that the usually applied numeric-analytical approach [6, 7] using the traffic model in the form of the train of concentrated forces is well founded. On the other hand, only the simulation technique makes investigation of the above-mentioned effects possible.

Figures 8–12 show that along with the increase of vehicle speed the expected value of the bridge responses decreases, especially for bridge deflections and the cable tension in comparison to the bending moments. The standard deviation of the quasi-static responses also decreases but in the dynamic conditions it grows. Expected values are greater than standard deviations only for the midspan deflection and for the increment of cable tension. This is the result of the slight sensitivity of the bridge responses mentioned above.

All the solutions presented above relate to the non-linear response of the bridge. The linear solutions can be easily determined in the same way, after the substitution of $\eta = 0$ into equation (7). Table 2 contains an exemplary comparison between results obtained for $v = 30$ m/s when the problem is considered as linear and non-linear. Practically there is no difference between them.

From the results presented in Table 2, it follows that the influence of the geometrical non-linearity of the structure can be omitted for the cases considered. A similar conclusion has been formulated by the authors in reference [6].

As a final note, it should be stated that the simulation method, applied in the stochastic problem of highway suspension bridge vibrations, enables the description of the vehicular traffic in a more adequate way. It means that some special properties of the traffic load, such as, inertia, springing or multiaxiality of vehicles can be taken into account. Moreover, the process of vehicle arrivals can be formulated in the form which corresponds better with reality than it does in the case when the analytical approach is developed. Obviously, the probabilistic analytical calculus in the range of non-linear problems is very troublesome. In the case presented, when the numerical simulation method was applied, determining the solutions with effects of structural non-linearity was not difficult. Nevertheless, when the higher order moments are necessary the efficiency of such a simulation approach decreases because of numerical difficulties. So, one can note that the simulation method should be preferred for the numerical estimation of special load effects or structure effects which can not be taken into account by analytical techniques.

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