



THE INFLUENCE OF UNEVEN BLADE SPACING ON THE SPL AND NOISE SPECTRA RADIATED FROM RADIAL FANS

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The influence of irregular blade spacing of car alternator radial fans on the total sound pressure level (*SPL*) and the noise spectrum has been investigated. For this purpose, the *SPL* and spectra were computed theoretically and the values were compared to measured results for several types of fans with various blade spacing. As theoretical background, the theory describing discrete frequency sound radiated by axial fans in open space was adopted. This was done in order to model the *SPL* and the spectra of alternator radial fans, placed inside a casing. Furthermore, due to the low blade tip Mach number, blades were modelled as dipole point sources. It was found, in accordance with previous results from known literature, that alterations in blade spacing do not significantly alter the total *SPL* nor the cooling capacity of the alternator radial fan. However, significant dispersion of the sound power over several harmonics was found with irregular fan blade spacing, thus allowing for a reduction of the siren effect. This phenomenon was predicted theoretically and confirmed experimentally.

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1. INTRODUCTION

An alternator is an electric power supply for vehicles that transforms mechanical energy into electricity [1]. One undesirable by-product of the energy transformation is sound radiation composed of mechanical, aerodynamic and magnetic noise [2, 3]. Demand for smaller and more efficient alternators with enhanced cooling capacity forced designers to place two radial cooling fans inside the casing. Due to this modification, the aerodynamic noise, which is dominant at higher rotating speeds, increased.

In order to reduce the total sound pressure level (*SPL*) and/or siren effect, available methods for reducing the aerodynamic noise of radial cooling fans have been examined [2–5]. Considering design and cost limitations for radial fans, the use of a fan with irregular blade spacing was found to be an appropriate method for favourably altering radiated noise. With irregular blade spacing, sound energy at the blade passage frequency (BPF) of a fan with even blade spacing is spread over several prominent tones while the total unweighted *SPL* remains practically unchanged. The goal is therefore to design a fan with prominent tones at as many discrete frequencies with similar sound pressure magnitude as possible. For such a fan, tonal noise can disappear in the range of unavoidable broadband noise [3].

So far, much experimental and theoretical research has been undertaken on the problem of fan noise radiation. One method of reducing the siren effect has been to design a fan with simple harmonically based blade spacing [2, 6]. Using this blade spacing model, Krishnappa [6] expressed radiation in terms of a modulated sound pressure field for the whole fan and conducted several experiments. But most detailed studies of irregular blade spacing have been done for axial fans, due to their aeronautical applications [10–13]. The majority of these analyses and computer programs are based on an acoustic analogy developed by Lighthill [7], Curle [8] and Ffowcs Williams and Hawkings [9]. In one important study [11], a theoretical analysis of loading noise radiated from rotors and stators was presented. Blades were modelled as point sources and fluctuating forces were included. An extension of this work can be found in reference [12], where a theory of open supersonic rotor noise was developed that also includes distributed blade forces and calculations of blade thickness noise. In both papers [11, 12] equations for the calculation of sound pressure at discrete frequencies radiated by a single blade were derived. The sound radiated from a fan was expressed as the interference of phase shifted sound fields radiated from each blade. Such an approach was also used in one recent study [13], describing the possibility of reducing propeller noise through irregular blade spacing. By shifting prominent tones into the frequency region of greater attenuation, due to the nature of the A-filter, considerable reduction in total A-weighted *SPL* was achieved [13].

Since the basic sound-generating mechanisms are the same for axial and radial fans, references [11, 12] offered solid background for our analysis of the influence of asymmetric alternator fan blade spacing on the total sound pressure level and spectra of radiated noise.

By following a similar approach as in references [11, 12], formulas for calculating discrete frequency noise of radial fans mounted inside a casing were derived, which also account for fluctuating blade forces unique to the case of housed alternators. In order to evaluate calculated results, measurements of the total *SPL* and the spectra of noise radiation were taken for several different alternator fans with irregular blade spacing.

2. THEORY

The major part of sound emitted by alternators is of aerodynamic origin [2–4, 14]. In car alternators, air flow driven by two radial fans mounted inside the casing is used to cool the stator and rotor (see Figure 1). Experience proves that obstacles placed in the air stream produce sound, therefore a primary source of aerodynamic noise is the fan itself [10]. Fan noise consists of sound caused by turbulence, sound due to the unsteady forces exerted by solid surfaces on the flow of air, and sound due to fluid volume displacement effects caused by moving surfaces [5]. The latter is often called blade thickness noise.

Due to the casing, the flow of air through the radial cooling fan involves unsteady flow components which consist of wake fields behind inlet obstructions and turbulence. Unsteady air flow causes fluctuating blade forces, which results in sound radiated from each blade. The interference of noise radiated from

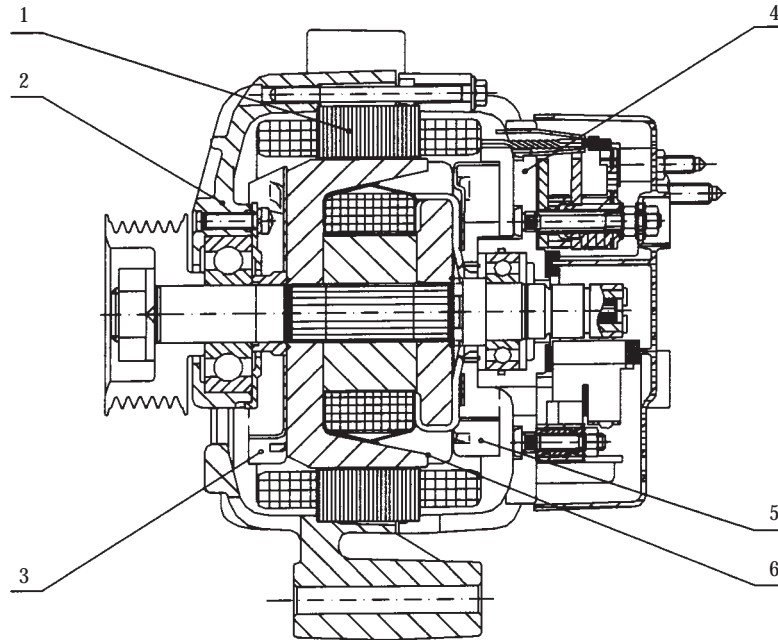


Figure 1. Car alternator with two radial cooling fans, (1) stator with winding; (2) front cover; (3) front axial-radial fan; (4) rear cover; (5) rear radial fan; (6) rotor with winding.

individual blades comprises the radial fan noise output. The wake interactions give rise to discrete frequency noise radiation by the radial fan, while turbulent flows give rise to broadband noise, which is generally of a lower level. Discrete frequency noise radiated by fans commonly ranges from 1000 to 5000 Hz and can thus be rather disturbing [10].

Our theoretical analysis is based on an analytical result concerning the noise radiation by moving bodies. Due to the low blade tip Mach number ($M < 0.3$) of a car alternator fan, the radial fan blade thickness noise and the noise caused by turbulence were neglected [5, 10].

By using a similar approach as in references [11, 12], formulas for calculating the discrete frequency noise radiated by radial fans were developed. Furthermore, the alternator casing was included in the model as an obstruction to in-flowing air that causes fluctuating blade forces.

The starting point of the analysis is an equation describing pressure fluctuations at point \mathbf{x} and time t due to the fluctuating blade forces and is written as [12]

$$4\pi(p(\mathbf{x}, t) - p_0) = -\frac{\partial}{\partial x_j} \int_S \left[\frac{f_j}{r|1 - M_r|} \right] dS. \quad (1)$$

With the same assumptions as in reference [12] S represents the mean planform area, $\mathbf{f} = f_j$ denotes for net force per unit area exerted on the fluid by the blade, r is the source to observer distance and the term $1 - M_r$ is the Doppler amplification factor.

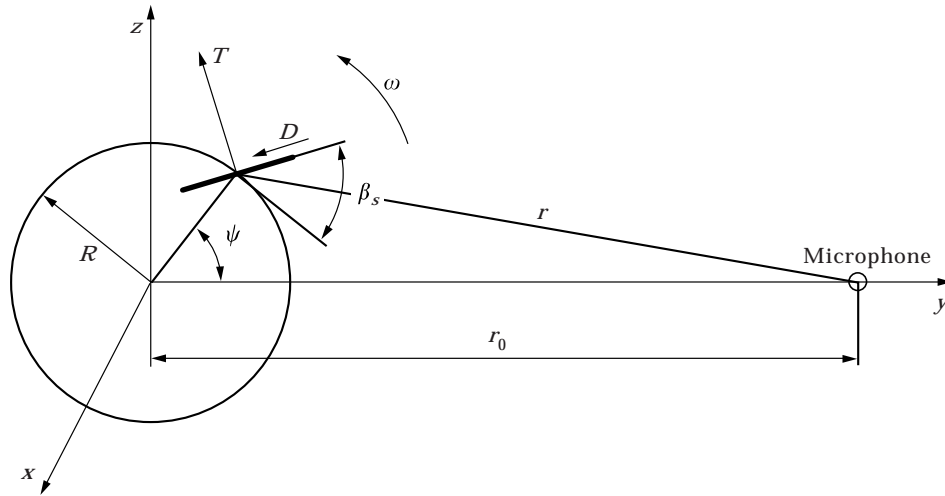


Figure 2. Radial fan blade model.

In Figure 2, the radial fan blade model developed in our study is presented. Due to low subsonic speeds, fan blades can be modelled as moving point dipoles [9, 11]. Therefore, the distributed forces on each blade were reduced to point forces $\mathbf{F} = F_j = (T, D)$ acting at the aerodynamic center of the blade. It could be assumed that at a constant angular velocity ω the blade force is periodic, with a period of at least $2\pi/\omega$. Furthermore, the noise radiated due to fluctuating blade forces is also periodic and could be represented by a Fourier series [11]. Upon applying a far field approximation ($r \gg R$), the complex magnitude of sound pressure at the n th harmonic radiated by a single dipole source, moving along a circular path, is then given by [11, 12].

$$c_n(\mathbf{x}) = a_n(\mathbf{x}) + ib_n(\mathbf{x}) = -\frac{in\omega^2}{4\pi^2 a_0 r_0} \int F_r e^{in\omega(\tau + r/a_0)} d\tau \quad (2)$$

where a_0 is the speed of sound, τ is source time and F_r is the component of a point force in the direction of the observer. The integral is over any period $2\pi/\omega$. In this paper, expression (2) was used as a basis to derive the equation for calculation of the sound pressure as discrete frequencies radiated by a radial fan mounted inside a casing.

According to Figure 2, the blade force \mathbf{F} is composed of component D , acting on the plane of the blade, called drag, and component T , acting perpendicular to this plane, called thrust. As drag is usually of a much lower level than thrust, the former was neglected. Hence,

$$\mathbf{F} = (0, T \cos(\beta_s + \psi), T \sin(\beta_s + \psi)). \quad (3)$$

In connection with measurements performed in this study, where a microphone was placed in the plane of the fan, the radiation vector and the component of the blade force in the direction of the observer are given by, respectively,

$$\mathbf{r} = (0, r_0 - R \cos \psi, -R \sin \psi), \quad F_r = T \cos(\beta_s + \psi), \quad (4, 5)$$

where the latter reflects the geometrical approximation ($r \gg R$); see Figure 2.

Furthermore, the blade angle ψ defined as $\psi = \omega\tau + \phi$ where ϕ is the blade angle at $\tau = 0$, was substituted as a new variable of integration in equation (2):

$$c_n = -\frac{i n \omega}{4\pi^2 a_0 r_0} e^{in(\omega r_0/a_0 - \phi)} \int_0^{2\pi} T \cos(\beta_s + \psi) e^{i(n\psi - nM \cos \psi)} d\psi, \quad (6)$$

where $M = \omega R/a_0$ is the rotational Mach number.

Since the fan is mounted inside a casing, the flow over the blades is circumferentially unsteady, thus causing fluctuating thrust that can be described by a complex Fourier series as

$$T = \sum_{\lambda = -\infty}^{\infty} T_\lambda e^{-i\lambda\omega t}, \quad (7)$$

By referring to the shape of the front and rear covers of the alternator, periodical blade force history (as shown in Figure 3) was derived and used in equation (6). The following expression was taken from reference [15],

$$\int_0^{2\pi} e^{i(m\psi - z \cos \psi)} \sin \psi d\psi = -2\pi i^{-n} \frac{n}{z} J_n(z), \quad (8a)$$

and served as a basis for derivation of the following equation [14, and the references therein]:

$$\int_0^{2\pi} e^{i(m\psi - z \cos \psi)} \cos \psi d\psi = \pi i^{-n+1} (J_{n-1}(z) - J_{n+1}(z)). \quad (8b)$$

So the integrals for all of the λ s in equation (6) can be identified as Bessel functions of the first kind. By evaluating equation (6), the expression for

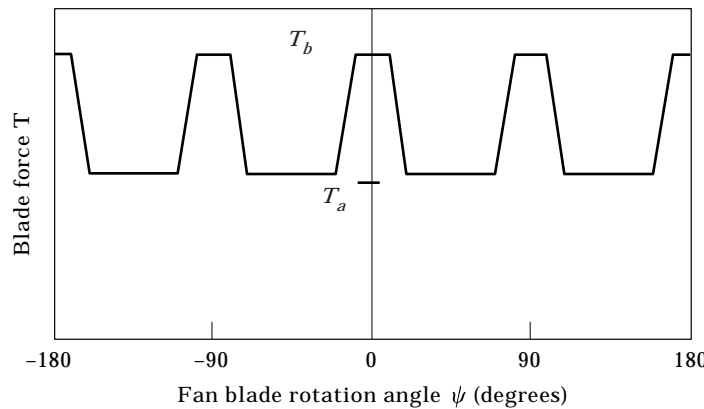


Figure 3. Periodic history of radial fan blade thrust.

calculating sound pressure at the n th harmonic of noise radiated by a single radial blade was calculated as [14]

$$c_n = \hat{c}_n e^{-in\phi} = -\frac{i n \omega}{4\pi r_0 a_0} e^{in(\omega r_0/a_0 - \pi/2)} e^{-in\phi} \sum_{\lambda=-\infty}^{\infty} T_\lambda(-i)^{-\lambda} \times \left[i \cos \beta_s (J_{n-\lambda-1}(nM) - J_{n-\lambda+1}(nM)) + 2 \sin \beta_s \frac{n-\lambda}{nM} J_{n-\lambda}(nM) \right]. \quad (9)$$

If variations in fan blade spacing angles ϕ are small, then blade size and shape changes will be negligible. It could be presumed that all blades radiated much the same but phase shifted sound fields. Therefore, the coefficients c_n for individual blades differ for each phase shifting term $\exp(-in\phi)$. The interference of sound fields radiated by each blade is simply the sum of phase shifting terms expressed as

$$G(\phi_j) = 1 + e^{-in\phi_2} + e^{-in(\phi_2 + \phi_3)} + \dots + e^{-in(\phi_2 + \phi_3 + \dots + \phi_j + \dots + \phi_B)}, \quad (10)$$

where ϕ_j is the angle between the j th and the $(j-1)$ th blade, B is the number of blades and j is an index that runs from 1 to B . For a fan with 12 equally spaced blades, $G(\phi_j)$ has a non-zero value only if n is a multiple of B . For such a fan, tonal noise occurs at blade passage frequency and at higher harmonics. When using a fan with irregular blade spacing, the term $G(\phi_j)$ is non-zero for most n . In this case, sound power at the BPF is spread over several harmonics of the rotating frequency but with a lower level than at the above BPF. Combining equations (9) and (10) yields the equation for calculating discrete frequency sound pressure radiated from a radial fan as

$$p(\mathbf{x}, t) = \sum_{n=-\infty}^{\infty} \hat{c}_n(\mathbf{x}) G(\phi_j) e^{-in\omega t}. \quad (11)$$

From equation (11) the expression for calculating the A-weighted total *SPL* can be derived. Estimation of blade force will be discussed later in section 4.

3. EXPERIMENTS

The measurements of the total *SPL* and the spectra of noise radiated from fans were taken in an anechoic chamber of the following dimensions: W 1.70 m, L 2.60 m and H 2.65 m. The alternator was belt driven by an electromotor placed inside the chamber (see Figure 4). With this set-up, a rotor speed produced was that ranged from 2000 to 12 000 r.p.m. A microphone was mounted in the plane of the fan at a distance of 0.50 m from the axis of rotation.

Radial cooling fans are made of 1-mm thick sheet-iron. Front and rear fans currently being manufactured have 12 equally spaced blades and are called F-REF and R-REF. In order to avoid an unbalance, the first studies of the influence of irregular blade spacing on total *SPL* and noise spectra were performed on fans

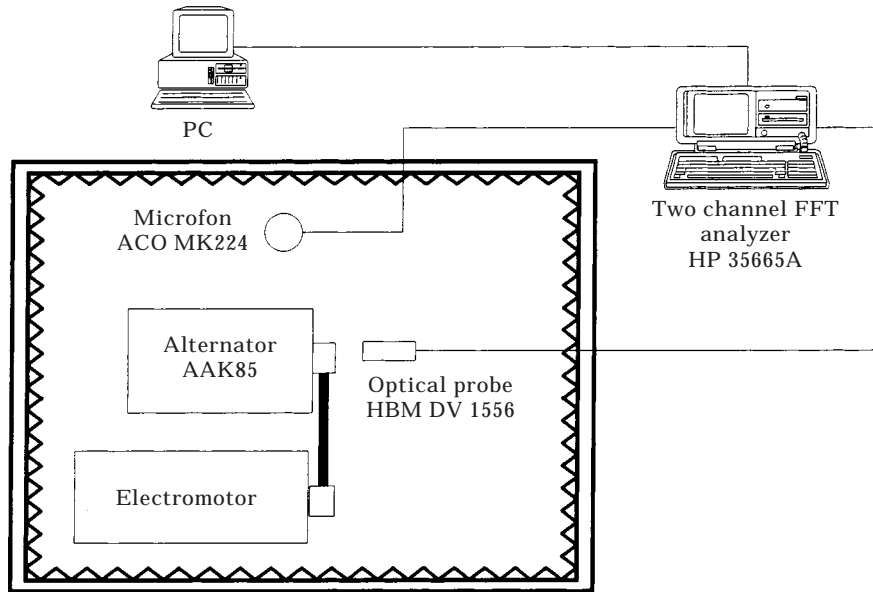


Figure 4. Scheme of the experimental set-up.

symmetrical with respect to the axis of rotation. One front (F-EQ) and one rear (R-EQ) fan were modelled following the directives for blade spacing angles found in references [2, 6]. In order to retain fan symmetry, the harmonic blade spacing modulation for these fans is repeated twice in one revolution of the fan. Another four front (F-1 . . . F-4) and rear (R-1 . . . R-4) fans were modelled in order to compare the theoretical results to experimental ones. Details of spacing angles of all fans, discussed in our analysis, are given in Table 1.

The first measurements of total *SPL* were conducted for alternators with and without electrical loading. In accordance with references [2, 4], it was found that aerodynamic noise is most prevalent at rotational speeds above 5000 r.p.m. (see Figure 5).

TABLE 1

The list of the spacing angles for all discussed fans in degrees

Fan	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	ϕ_6	ϕ_7	ϕ_8	ϕ_9	ϕ_{10}	ϕ_{11}	ϕ_{12}
R-Ref	30	30	30	30	30	30	30	30	30	30	30	30
F-Ref	30	30	30	30	30	30	30	30	30	30	30	30
R-Eq	26	30	34	34	30	26	26	30	34	34	30	26
F-Eq	26	30	34	34	30	26	26	30	34	34	30	26
R-1	40	28	28	28	28	28	40	28	28	28	28	28
F-1	51	23	24	25	27	30	51	23	24	25	27	30
R-Opt	24	24	24	26	32	35	36	36	24	35	36	28

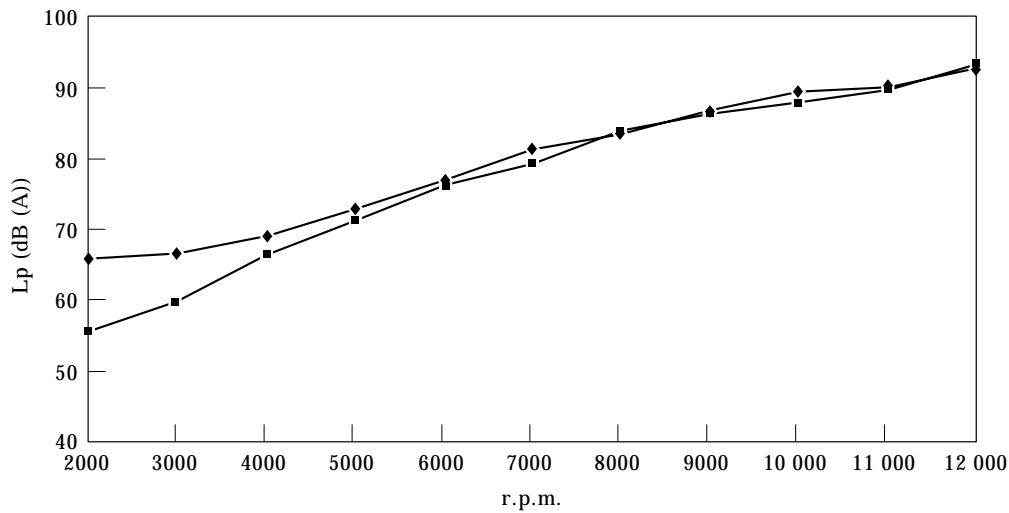


Figure 5. Comparison of total *SPL* radiated from alternator with (◆) and without (■) electrical loading.

During further measurements it was observed that a rotor spinning without a fan attached radiates noise at the same level as a radial cooling fan. Therefore, the rotor was replaced by cylinder whose total *SPL* is 15 dB (A) lower than the total *SPL* of the original rotor. Finally, all measurements of the total *SPL* and spectra of radiated noise were taken under the following conditions: with one fan only; without brushes; without electrical loading; in the speed range from 2000 to 12 000 r.p.m.

In addition to noise measurements, the cooling capacity of tested fans was measured in a heat-sealed chamber. The temperature was measured at several points on the stator. For all combinations of front and rear fans, stator temperature remained practically the same. Therefore, it was concluded that irregular fan-blade spacing has no significant influence on cooling capacity, a result which correlates with the experiments described in references [6, 13].

4. COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL RESULTS

Present fan manufacturing technology—cutting and bending—provides low cost fans but with several shape limitations. Thus, during the theoretical analysis, fan blade spacing angles ϕ were varied in the range from 24° to 36° for the rear fan and from 20° to 36° for the front fan.

In reviewing our calculated results, it was found that sound pressure magnitude depends primarily on the amplitude of the blade force fluctuation— $\frac{1}{2}(T_b - T_a)$ —whereas the blade force mean value— $\frac{1}{2}(T_b + T_a)$ —is of lesser importance (see Figure 3). Moreover, the calculated sound pressure for various fans remain the same over a wide blade-force amplitude range. Thus, similar fans with irregular blade spacing can be analytically compared, and optimal fan blade spacing can be achieved if an approximate blade force time history is known [14]. The amplitudes of the rear and front fan blade forces were not determined by means

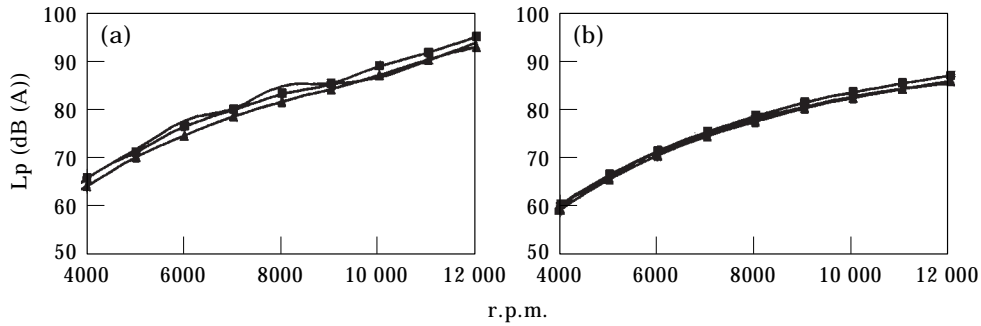


Figure 6. (a) Measured and (b) calculated total *SPL* for fans R-REF (—), R-EQ (—■—), R-1 (—▲—).

of pure theoretical considerations. They were estimated by comparing the measured and calculated values of sound pressure at BPF for the reference fans at a rotor speed of 10 000 r.p.m. and further used in theoretical analysis. Since blade force is proportional to planform blade area, the amplitude of blade force fluctuations $\frac{1}{2}(T_b - T_a)_j$ for the *j*th blade was evaluated by scaling the $\frac{1}{2}(T_b - T_a)_{ref}$ for the reference fan blade by the corresponding planform area quotient S_j/S_{ref} .

The number of harmonics *N* that have to be included in equation (11) is directly related to the rotational Mach number *M*. At low *M*, as in our case, the value of the Bessel function in equation (9) drops rapidly with an increasing *N*. Therefore, sound pressure at the first 30 harmonics was included in the calculation of the total *SPL* and sound pressure spectra of radiated noise.

4.1. TOTAL SOUND PRESSURE LEVEL

Comparison of the measured values (a) and the calculated values of tonal noise (b) for various rear (Figure 6) and front (Figure 7) fans shows that irregular fan-blade spacing has no significant influence on the total *SPL* of radiated noise, a result which matches experimental results from reference [4] and confirms the viability of the theoretical approach [16]. When using fans with irregular blade spacing, the sound power at BPF is spread over several harmonics of rotating frequency, but the unweighted total *SPL* remains constant [4, 14, 16]. The BPF of reference fans as well as discrete frequencies at which the sound level is significant

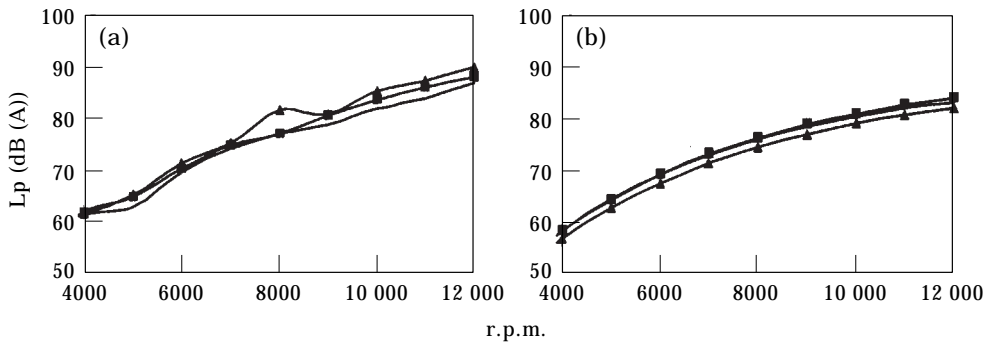


Figure 7. (a) Measured and (b) calculated total *SPL* for fans F-REF (—), F-EQ (—■—), F-1 (—▲—).

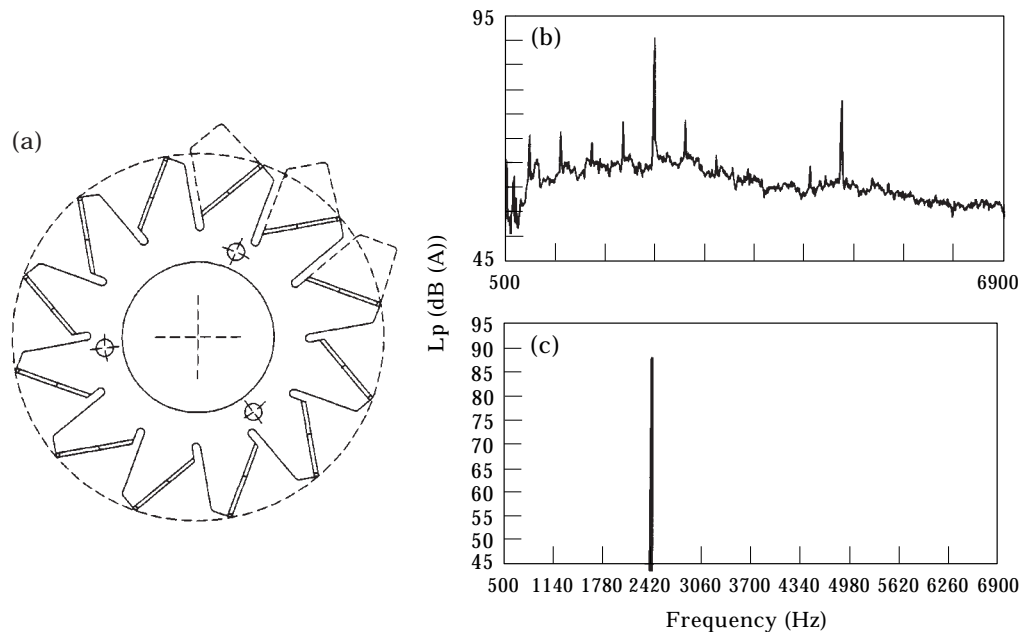


Figure 8. (a) Drawing, (b) measured and (c) calculated spectra for R-REF at 12 000 r.p.m.

for analyzed fans, lie between 1000 and 3000 Hz for rotor speeds above 5000 r.p.m. In this frequency range, attenuation due to the A-weighted filter is almost constant, meaning that the A-weighted total *SPL* remained practically equal for all tested fans.

In the theoretical analysis, a lower total *SPL* was obtained for fans having a large space between two blades (fan F-1). This is probably due to the aerodynamic interaction between blades and noise reflection from the casing not encountered in our analysis.

4.2. SOUND PRESSURE SPECTRUM

As shown in Figures 8–13, calculated sound pressure spectra correspond well to measured ones. Almost all harmonics that are represented in measured sound pressure spectra are also visible in the calculated spectra. Furthermore, the calculated relative magnitudes of sound pressure at discrete frequencies with respect to the sound pressure peak also correspond to measured values. Since our study was performed with fans having axisymmetrically spaced blades, only sound pressure at even harmonics of the rotating frequency is represented in the spectra.

In the power spectrum of fans with equally spaced blades (R-REF and F-REF), there is an exaggerated sound pressure at the 12th harmonic (Figures 8, 9(b) and (c)), which corresponds to the blade passing frequency. For the fans R-EQ or F-EQ, two additional prominent tones on the lower and upper end of the BPF were obtained in the calculated results. The same was found in measured spectra of these fans around the BPF as well as around the second harmonic of the BPF (see Figures 10 and 11). The theoretically predicted and experimentally confirmed redistribution of sound energy in the case of fans with modulated blade spacing

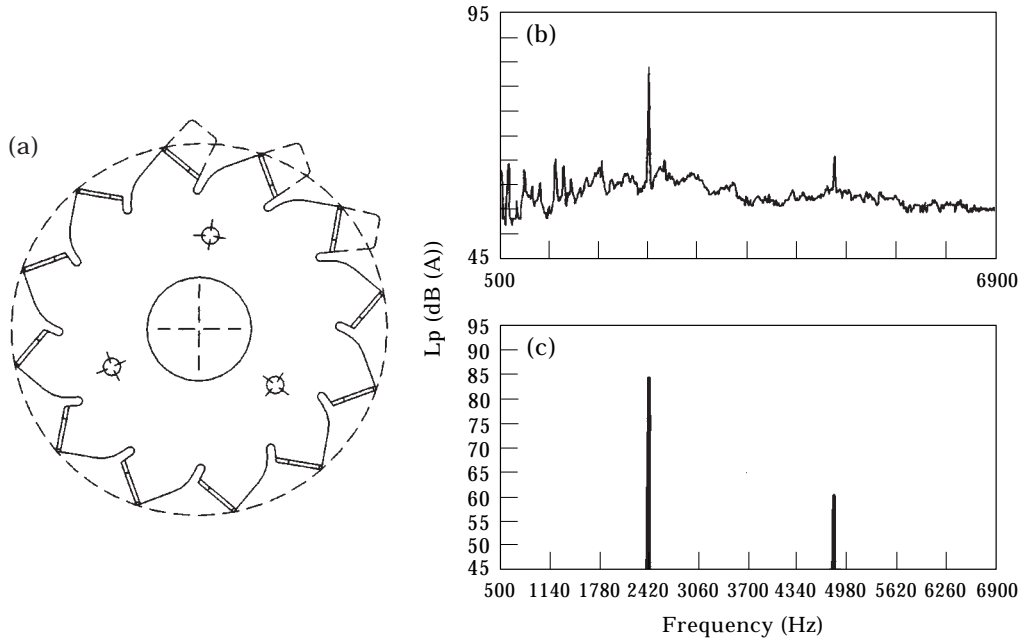


Figure 9. (a) Drawing, (b) measured and (c) calculated spectra for F-REF at 12 000 r.p.m.

corresponds well with the results for normal flow conditions given in reference [6]. An even greater reduction in the tonal noise at BPF was achieved with other tested fans. Sound pressures of similar magnitude at six discrete frequencies were

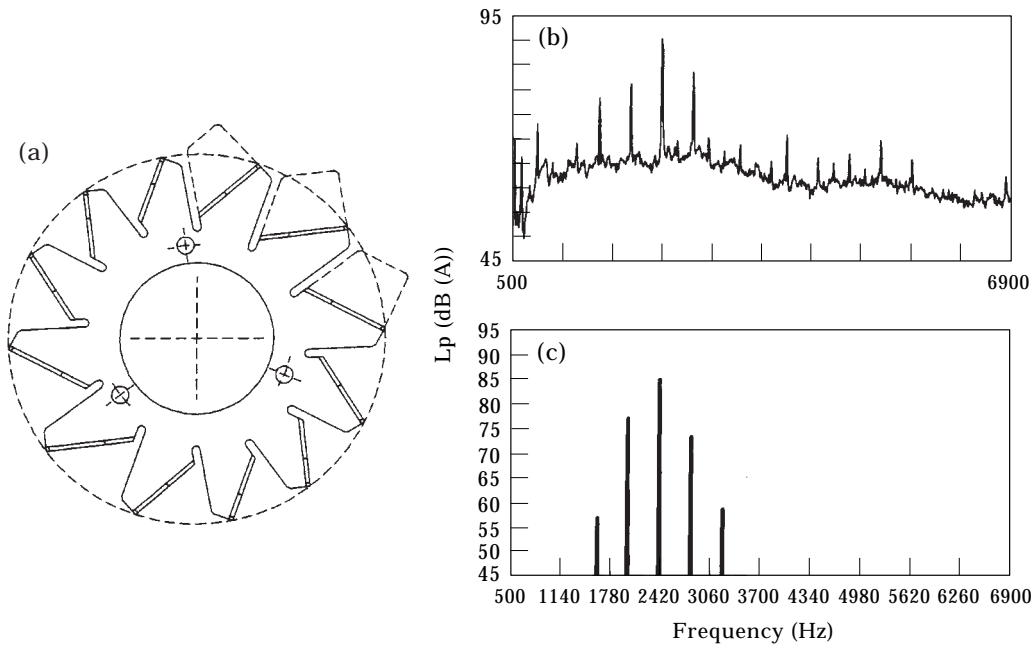


Figure 10. (a) Drawing, (b) measured and (c) calculated spectra for R-EQ at 12 000 r.p.m.

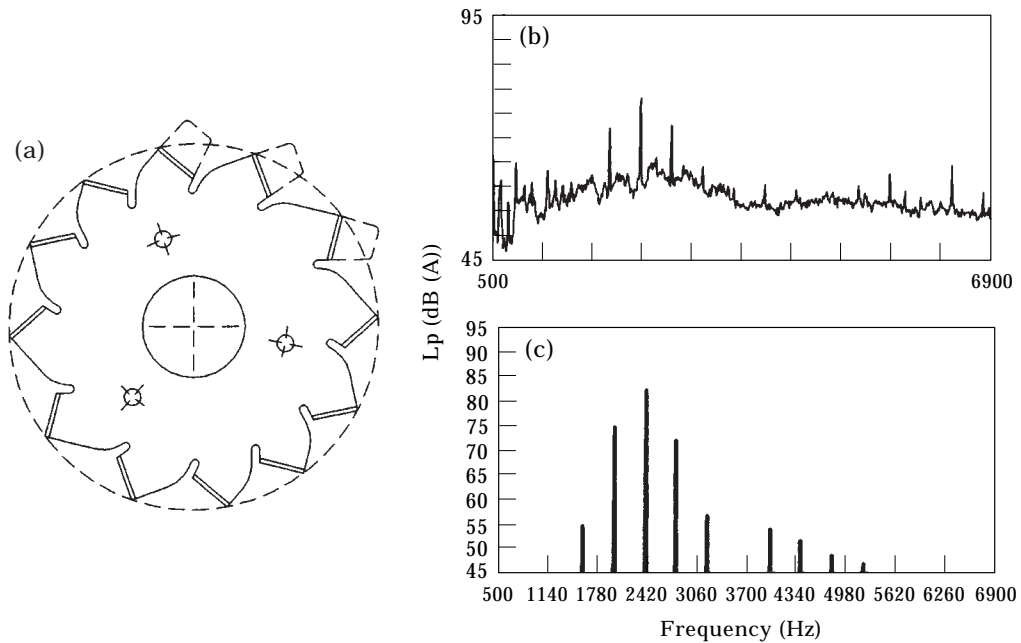


Figure 11. (a) Drawing, (b) measured and (c) calculated spectra for F-EQ at 12 000 r.p.m.

obtained for fans R-1 and F-1. The sound pressure peak for the latter occurs at the 14th harmonic of the rotating frequency.

On the basis of the above results, a fan with asymmetrical blade spacing (R-OPT) whose sound power is spread over many harmonics was designed (see

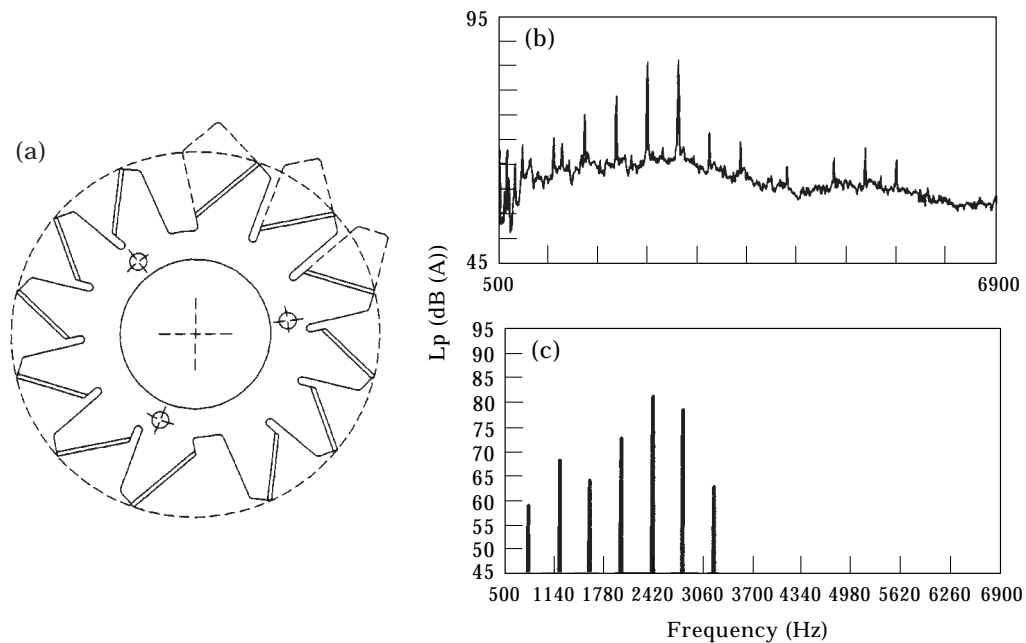


Figure 12. (a) Drawing, (b) measured and (c) calculated spectra for R-1 at 12 000 r.p.m.

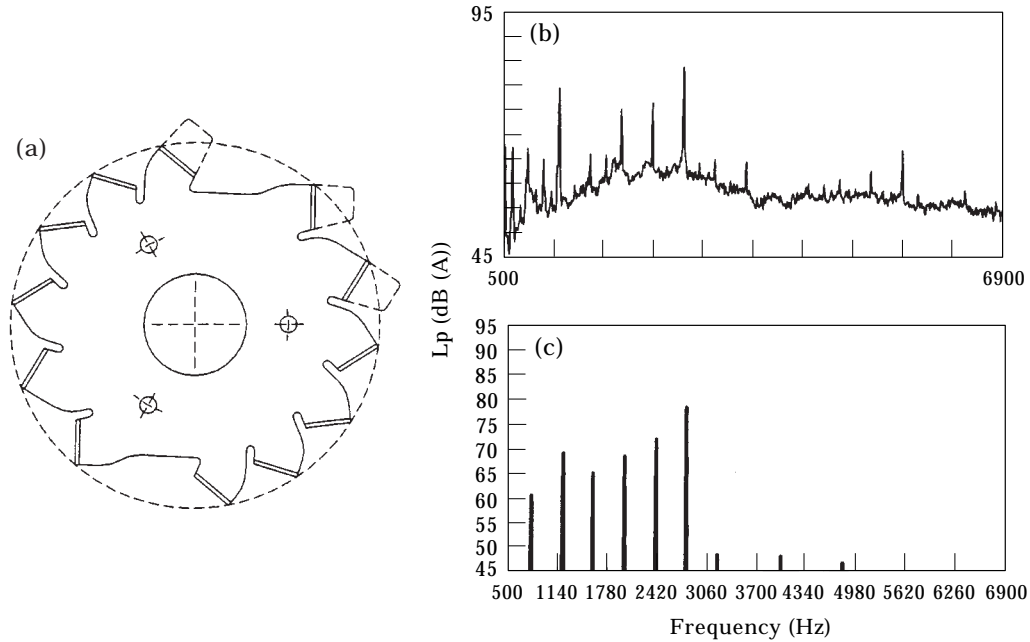


Figure 13. (a) Drawing, (b) measured and (c) calculated spectra for F-1 at 12 000 r.p.m.

Figure 14). Sound pressure at prominent tones for this fan is estimated to be 13 dB higher than broadband noise, compared to the 25 dB higher sound pressure at BPF for the reference fan. Therefore, the tonal character of such a fan will be greatly reduced by unavoidable broadband noise. Desired fan blade spacing also depends on the rotating speed of the rotor [11]. As the operating range of the rotor speed extends from 2000 to 15 000 r.p.m. [1] and noise of aerodynamic origin prevails at the upper rotor speed range, the optimal fan blade spacing was adjusted for 12 000 r.p.m. Since the optimal fan is asymmetric, the static imbalance of the fan was reduced with a minor decrease in the lengths of some blades.

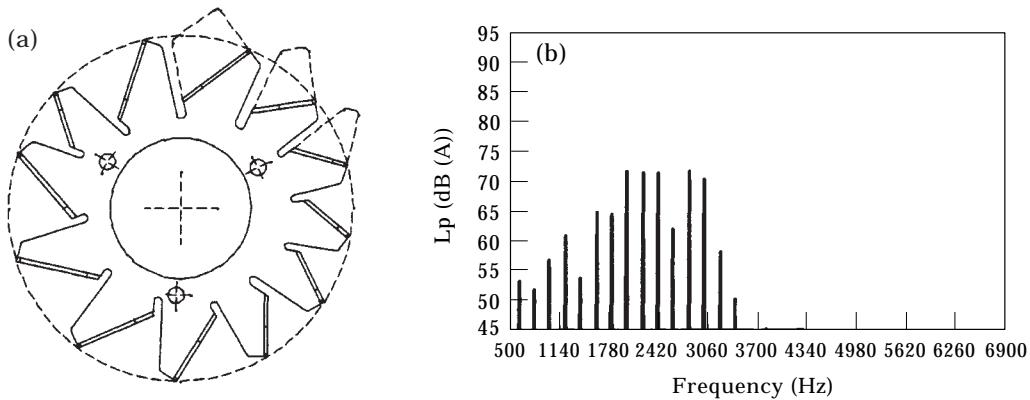


Figure 14. (a) Drawing, (b) measured and (c) calculated spectra for R-OPT at 12 000 r.p.m.

5. CONCLUSIONS

The aerodynamic noise of an alternator, which is dominant at higher rotating speeds of the rotor, can be especially disturbing due to its intensity and tonal quality. Primary sources of aerodynamic noise in alternators are the front and the rear cooling fans and rotor movement. In this paper, the influence of irregular radial fan-blade spacing on the total *SPL* and noise spectra was theoretically and experimentally analyzed.

The analytical approach is based on a theory describing noise at discrete frequencies radiated by moving bodies. This theory was adapted to alternator radial cooling fans placed inside casing. The front and rear covers of the alternator were included in the theory as an obstruction to in-flowing air. The wake field behind inlet obstructions causes fluctuating blade forces and contributes significantly to the noise radiated by the fan. Blade force, exerted by the blade on the flow of air was reduced to a point force. In the course of performing a theoretical analysis, it was found that sound pressure at discrete frequencies depends primarily on the blade-force amplitude, which was estimated by comparing calculated and measured sound pressure at BPF for reference fans.

In the theory, the number of blades and the angular velocity of the rotor are included in the analysis, whereas the contributions of sound reflection from the housing and irregularities in flow (due to highly uneven blade spacing) are not. As an extension of the afore-mentioned paper on radial fans [6], the essential part of our theoretical approach is providing a description of basic noise-generating mechanisms. Equations (1) and (2), which describe the sound pressure radiated by a moving dipole sound source, were taken from known theories for axial fans, but were altered to describe more accurately the model we designed for a radial fan inside a casing. As a further addition to reference [6] the sound field of the whole fan was theoretically described in terms of the superposition of individual blade effects. Key results of the presented theoretical model are equations (9–11), for calculating sound pressure emitted by a radial fan at discrete frequencies, and the resulting calculation of the total *SPL* excluding broadband noise. The correspondence between theoretical and experimental results proves that our model is sufficiently accurate.

A measurement of cooling capacity showed that the performance of all tested fans remained practically the same as those of the reference fans. Furthermore, it was observed that irregular alternator fan-blade spacing has no significant effect on the unweighted total *SPL*, which matches results found in the literature. As a consequence, an acoustic benefit can be derived only from a change in the shape of the radiated sound spectrum. When using alternator fans with uneven blade spacing, sound power at BPF was spread over several discrete frequencies at which the attenuation due to the A-weighted filter is almost constant. This implies a practically equal A-weighted total *SPL* for all tested fans. Nevertheless, the siren effect can be reduced by spreading tonal noise at blade passing frequency over several harmonics at a lower sound pressure level. As a result, the tonal aspect of noise radiated by a fan, which has been optimized for specific operating conditions, will generally disappear in unavoidable broadband noise.

REFERENCES

1. U. ADLER and H. BAUER 1988 *Automotive Electric/Electronic Systems*. VDI-Verlag GmbH, Düsseldorf. ISBN 0-89883-509-7.
2. P. L. TÍMÁR, A. FAZEKAS, J. KISS, A. MIKLÓS and S. J. YANG 1989 *Noise and Vibration of Electrical Machines*. Akadémiai Kiadó, Budapest. ISBN 963-05-4672-8.
3. G. H. HUEBNER 1961 *ETZ-A* **82**, 771–781. Entstehung und Bekämpfung der Geräusche elektrischer Maschinen.
4. W. NEISE 1982 *Journal of Engineering for Industry* **104**, 151–161. Review of noise reduction methods for centrifugal fans.
5. W. NEISE 1992 *CETIM* 1–3 September, 45–56. Review of fan noise generation mechanisms and control methods.
6. G. KRISHNAPPA 1980 *Proceedings of Inter-Noise'80*, 215–218. Effect of modulated blade spacing on centrifugal fan noise.
7. M. J. LIGHTHILL 1952 *Proceedings of the Royal Society of London* **211**, 564–587. On sound generated aerodynamically.
8. N. CURLE 1955 *Proceedings of the Royal Society of London* **231**, 505–513. The influence of solid boundaries upon aerodynamic sound.
9. J. F. FLOWCS WILLIAMS and D. L. HAWKINGS 1969 *Philosophical Transactions of the Royal Society of London* **A264**, 321–342. Sound generation by turbulence and surfaces in arbitrary motion.
10. M. V. LOWSON 1968 *Journal of the Acoustical Society of America* **43**, 37–50. Reduction of compressor noise radiation.
11. M. V. LOWSON 1970 *Journal of the Acoustical Society of America* **47**, 371–385. Theoretical analysis of compressor noise.
12. D. L. HAWKINGS and M. V. LOWSON 1974 *Journal of Sound and Vibration* **36**, 1–20. Theory of open supersonic rotor noise.
13. W. DOBRZYNSKI 1993 *Journal of Sound and Vibration* **163**, 123–136. Propeller noise reduction by means of unsymmetrical blade spacing.
14. M. MESARIC, M. BOLTEZAR and A. KUHELJ 1996 *Proceedings of Inter-Noise'96*, Book 1, 207–210. The influence of the unsymmetric alternator fan blade spacing on the total sound pressure level and spectra of radiated noise.
15. N. W. MCLACHAN 1955 *Bessel Functions for Engineers*. Oxford University Press.
16. S. LÉWY 1992 *Journal of the Acoustical Society of America* **92**, 2181–2185. Theoretical study of the acoustic benefit of an open rotor with uneven blade spacings.