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LETTERS TO THE EDITOR

PREDICTION OF THE FUNDAMENTAL TRANSVERSE NATURAL FREQUENCY OF TALL BUILDING STRUCTURES USING MAXIMUM STATIC DEFLECTION

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1. INTRODUCTION

In a recent article by Bert [1], a very simple relationship between fundamental natural frequency and maximum static deflection for linear systems has been proposed. The formula is

$$\omega_1 = C(\mathbf{g}/\delta)^{1/2}, \tag{1}$$

where C is a dimensionless constant depending on the parameters of the system, $\mathbf{g}$ is the acceleration due to gravity, and δ is now the maximum static deflection under gravity. Xie [2] extended this simple relationship to several types of building structures namely rigid frame, wall-frame and bracing frame in his investigation. He carried out static analyses on these structures with various heights using the finite element method and applied a fraction of the gravity loads in the horizontal direction to find maximum static deflection. Xie obtained the dimensionless constant C, which varies between 1·12 and 1·21, by comparing his results with the finite element dynamic analyses. The purpose of this letter is to establish the relationship between fundamental transverse natural frequency and structural parameters by combining the coupled-wall theory and equation (1) to cover a wide range of structures with uniform and symmetrical plans throughout their height.

2. ANALYSIS

Wall-frame, rigid frame, braced frame, coupled shear walls and a combination of them in a uniform symmetrical structure can be considered as shear–flexure cantilevers whose deflection can be estimated accurately by coupled-wall theory.

The differential equation governing the translational motion of shear–flexural cantilevers can be approximated as following [3]

$$\mathrm{d}^4y/\mathrm{d}z^4 - (k\alpha)^2\,\mathrm{d}^2y/\mathrm{d}z^2 = (1/EI)[w(z) - \alpha^2(k^2-1)M(z)]; \tag{2}$$

where, y is the lateral deflection, $w(z)$ is the intensity of the static lateral distributed loading, $M(z)$ is the accumulated external moment at the co-ordinate z from the base of the structure.

The structural characteristic parameters: α^2 and k^2 are as follows

$$\alpha^2 = GA/EI, \qquad k^2 = 1 + EI/\Sigma(EAc^2)_j = 1 + EI/EAc^2. \tag{3, 4}$$

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The parameter k^2 accounts for the effect of the axial deformations of the columns and walls on the overall flexure. The racking shear rigidity GA, and the flexural rigidity EI contributed by the bents are

$$GA = \Sigma(GA)_j, \qquad EI = \Sigma(EI)_j. \tag{5, 6}$$

$(EAc^2)_j$ for a bent is the flexural rigidity of the column and wall sectional areas acting about their common centroid.

In the case of a shear–flexure cantilever on a rigid base subjected to a uniformly distributed gravity load of intensity $m\mathbf{g}$ per unit height acting horizontally, the maximum lateral deflection on the top of the structure determined from differential equation (2) and boundary condition is given by

$$y_H = (mgH^4/8EI)F_{yH}, \tag{7}$$

where

$$F_{yH} = 1 - \frac{1}{k^2}\left[1 - \frac{4}{(k\alpha H)^2} + \frac{8}{(k\alpha H)^4 \cosh k\alpha H}(1 + k\alpha H \sinh k\alpha H - \cosh k\alpha H)\right]. \tag{8}$$

Substituting the maximum static deflection y_H into equation (1), the fundamental transverse frequency can be represented as

$$f_{y1} = (D'/\sqrt{F_{yH}})(1/H^2)\sqrt{EI/m} \tag{9a}$$

in which $D' = \sqrt{2}C/\pi$. When GA approaches zero, F_{yH} approaches 1 and the shear–flexural cantilever becomes a pure flexural cantilever. Equation (9a) will have the same form as the fundamental natural frequency of a cantilever beam in transverse vibration with an exact solution of the factor $D' = 0{\cdot}5595$ [4]. This can be written as

$$f_{y1} = (0{\cdot}5595/\sqrt{F_{yH}})(1/H^2)\sqrt{EI/m}. \tag{9b}$$

Equation (9b) can be rewritten as

$$f_{y1} = (0{\cdot}5595 D_y/H^2)\sqrt{EI/m}, \tag{10}$$

where

$$D_y = 1/\sqrt{F_{yH}} \tag{11}$$

The variation of the fundamental frequency coefficient, D_y, may be expressed most conveniently in terms of the parameters k and $k\alpha H$, as shown in Figure 1.

3. NUMERICAL EXAMPLES

The application of the proposed method is demonstrated in the following examples: (1) coupled shear wall building structure; (2) rigid frame building structure. The elevations of the structures are shown in Figure 2, and their properties are given in Tables 1 and 2 respectively.

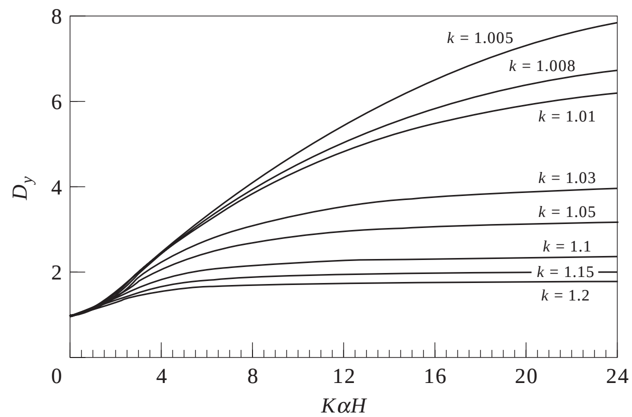

Figure 1. Fundamental frequency coefficients of shear-flexure cantilevers.

The racking rigidity (GA) for coupled shear walls and for rigid frames are expressed as follows:

For coupled shear walls

$$(GA) = 12EI_b l^2/b^3 h, \tag{12}$$

where l is the distance between centroid axes of the walls, h is the storey height, b is the clear span of connecting beam, I_b is the moment of inertia of connecting beam and $I = I_1 + I_2$ the sum of second moment of areas of wall 1 and wall 2.

For rigid frames

$$(GA) = 12E/h[1/\Sigma(I_c/h)_i + 1/\Sigma(I_b/l)_j] \tag{13}$$

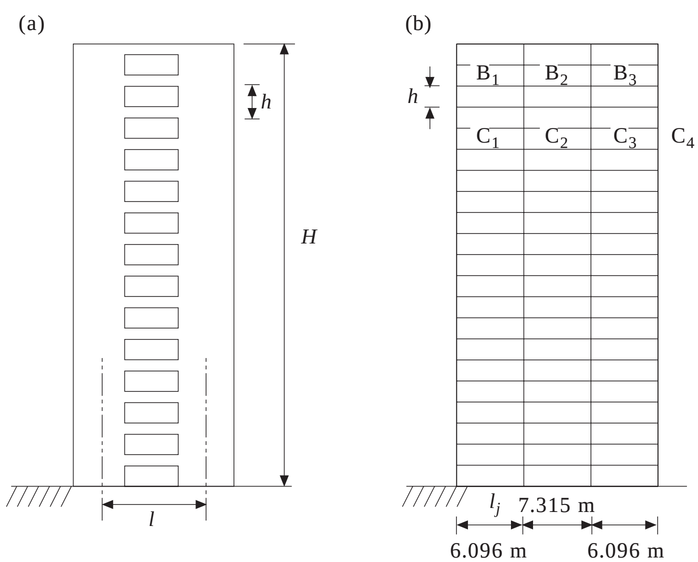

Figure 2. Example structures: (a) Coupled shear walls; (b) Rigid frame.

TABLE 1

Details of the properties of coupled shear walls

Dimensions	
Height of building H (m)	60·96
Storey height h (m)	3·048
Depth of each wall, $d_1 = d_2$ (m)	6·096
Thickness of each wall t (m)	0·3048
Span between wall b (m)	2·4384
Second moment of connecting beam I_b (m^4)	0·008631
Elastic modulus E (N/m^2)	2·876E + 10
Density of material (kg/m^3)	2403

TABLE 2

Details of the properties of a frame structure

Properties	
Height of building H (m)	60·96
Storey height h (m)	3·048
Cross sectional area of beam B_1, B_2, B_3 (m^2)	0·2478
Second moment of beam B_1, B_2, B_3 (m^4)	0·007672
Cross-sectional area of column C_1, C_4 (m^2)	0·2478
Second moment of column C_1, C_4 (m^4)	0·007672
Cross sectional area of column C_2, C_3 (m^2)	0·3304
Second moment of column C_2, C_3 (m^4)	0·018185
Elastic modulus E (N/m^2)	2·395E + 10
Density of material (kg/m^3)	2403

where I_{ci} is the moment of inertia of the column on line i; h = storey height; I_{bj} and l_j represent the moment of inertia and span of the beam in bay j. The girders and columns are taken across on storey level of the bent.

A comparison of the fundamental transverse natural frequency of the two example structures obtained using the proposed method and those obtained using the finite element method (ABAQUS software package) is shown in Table 3. It can be seen that good agreement between the results has been achieved.

TABLE 3

Comparison between results of proposed analysis and those from finite element analysis

	Proposed method		ABAQUS FEM	
Methods	Coupled walls	Frame	Coupled walls	Frame
Fundamental freq. (Hz)	2·07	0·8889	2·03	0·8833

The proposed method is also applicable to multi-bent structures which may be comprised of different types of structural components.

4. CONCLUSIONS

The relationship between fundamental transverse natural frequency and structural parameters of uniform high rise building structures has been established by using Bert's formula (Equation (1)) and coupled wall theory. This enables an accurate prediction of the fundamental transverse natural frequency by hand calculation for different types of building structures. The proposed method is not recommended for the determination of the fundamental natural frequency for structures with less than 6 storeys. This is because the governing differential equation, equation (2), has been found to give good approximation for tall buildings.

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