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APPLICATION OF GREEN'S FUNCTION IN FREE VIBRATION ANALYSIS OF A SYSTEM OF LINE CONNECTED RECTANGULAR PLATES

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In this paper, the Green's function method is used to obtain an analytical solution of the free vibration problem of a system of two rectangular, orthotropic Levy plates. The plates of the system are elastically connected along straight lines perpendicular to the simply supported plate edges. The Green's functions for the orthotropic S–S–S–S and S–F–S–F plates are derived. The numerical calculations deal with a system of square plates connected along two lines. The results show the effect of the material orthotropy and the stiffness of translational connection on the eigenfrequencies of the combined system.

1. INTRODUCTION

The transverse vibrations of single rectangular plates have been thoroughly discussed in literature (see, e.g., references [1, 2]). The purpose of this paper is investigation of the free vibration of a combined system consisting of two plates, which are elastically line connected by means of translational springs. The use of Green's functions for the solution of this vibration problem is particularly profitable.

The natural frequencies of combined system under consideration depend on the vibration frequencies of the component plates as well as the method of their connection. If the stiffness of the elastic connection tends to zero, then the vibration frequencies tend to the frequencies of the isolated plates. Sometimes the spectrum of a combined system with non-zero connection stiffness included the frequencies of the isolated plates. These combined system frequencies are then called degenerate eigenfrequencies [3]. The natural frequencies of orthotropic rectangular plates for various boundary conditions are given in tabular form in papers [1, 2].

For the case of two identical, elastically line connected plates, the non-degenerate frequencies of the combined system are the same as for a single elastically line supported plate. The problems of vibrations of single rectangular plates which are line supported have been considered in references [4–8]. Kim and Dickinson in reference [4] and Zhou Ding in reference [5] deal with the free vibrations of uniform, orthotropic plates with straight line rigid supports which

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are parallel to the plate edges. The free vibration of plates with an arbitrary straight line support has been treated by Kim [6]. Two works by Gorman [7, 8] are devoted to the analysis of free vibrations of isotropic rectangular plates resting on lateral and rotational elastic edge supports. To solve these problems the authors have applied the Rayleigh–Ritz method [4–6] and the superposition method [7, 8]. The application of the method of Green's function synthesis to systems of layered structures was presented by Lueschen and Bergman [9]. The Green's function method was used in reference [10] to the free vibration problem of a system of isotropic, rectangular plates pointwise connected by translational springs.

In this paper a solution is presented for the free vibration problem of a system of two orthotropic, rectangular line connected plates which are simply supported at two opposite edges. The connection of the plates is achieved by means of translational springs distributed along straight lines which are orthogonal to the simply supported plate edges. The solution of the problem has been obtained using the properties of the Green's functions. The dynamic Green's functions for rectangular isotropic plates in reference [11] are given. These functions for orthotropic S–S–S–S and S–F–S–F plates are presented here. The numerical calculations of the frequencies refer to a system of square plates connected along two lines.

2. THEORY

Consider a system of two orthotropic, rectangular plates (Figure 1) which are connected by an elastic element distributed along the lines $y = y_j$, $j = 1, 2, \ldots, n$. Free vibration of the system is governed by the following equations (a list of notation is given in the Appendix B):

$$D_{x1}\frac{\partial^4 w_1}{\partial x^4} + 2H_1\frac{\partial^4 w_1}{\partial x^2\,\partial y^2} + D_{y1}\frac{\partial^4 w_1}{\partial y^4} + \rho_1\frac{\partial^2 w_1}{\partial t^2} = f_T(x, y, t), \tag{1}$$

$$D_{x2}\frac{\partial^4 w_2}{\partial x^4} + 2H_2\frac{\partial^4 w_2}{\partial x^2\,\partial y^2} + D_{y2}\frac{\partial^4 w_2}{\partial y^4} + \rho_2\frac{\partial^2 w_2}{\partial t^2} = -f_T(x, y, t), \tag{2}$$

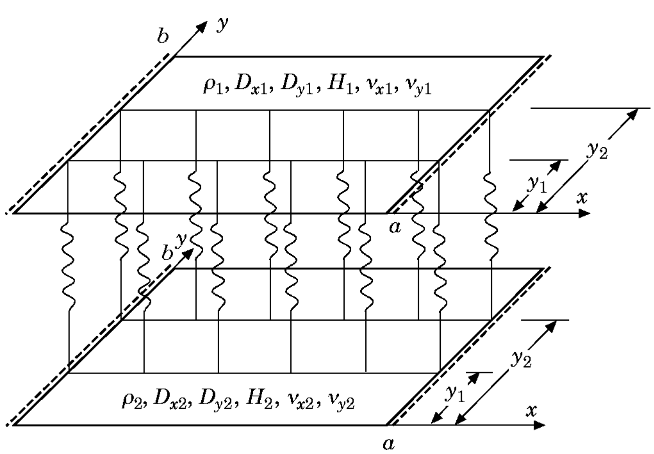

Figure 1. An example of the system of two elastically line connected rectangular plates.

where the function f_T represents the connections of the plates by translational springs. This function is assumed to take the form

$$f_T(x, y, t) = \sum_{j=1}^{n} k_j [w_2(x, y_j, t) - w_1(x, y_j, t)]\, \delta(y - y_j). \tag{3}$$

In order to consider free harmonic motion of the plates with frequency ω, the plate deflections are assumed in the form

$$w_1(x, y, t) = \bar{W}_1(x, y)\, e^{i\omega t}, \qquad w_2(x, y, t) = \bar{W}_2(x, y)\, e^{i\omega t}. \tag{4}$$

By substituting the equations (4) into equations (1–3) and introducing the non-dimensional co-ordinates (Appendix B) and quantities: $\Phi_r = (b/a)\sqrt[4]{D_{xr}/D_{yr}}$, $\Psi_r = H_r/\sqrt{D_{xr} D_{yr}}$, $\lambda_r^2 = \omega a^2 \sqrt{\rho_r/D_{xr}}$ for $r = 1, 2$, one obtains

$$\partial^4 W_1/\partial\eta^4 + 2\Psi_1 \Phi_1^2\, \partial^4 W_1/\partial\xi^2\, \partial\eta^2 + \Phi_1^4\, \partial^4 W_1/\partial\xi^4 - \Phi_1^4 \lambda_1^4 W_1 = F_T(\xi, \eta), \tag{5}$$

$$\partial^4 W_2/\partial\eta^4 + 2\Psi_2 \Phi_2^2\, \partial^4 W_2/\partial\xi^2\, \partial\eta^2 + \Phi_2^4\, \partial^4 W_2/\partial\xi^4 - \Phi_2^4 \lambda_2^4 W_1 = -\mu F_T(\xi, \eta), \tag{6}$$

where $\mu = D_{y1}/D_{y2}$, $\eta_j = y_j/b$ and

$$F_T(\xi, \eta) = \sum_{j=1}^{n} K_j [W_2(\xi, \eta_j) - W_1(\xi, \eta_j)]\, \delta(\eta - \eta_j). \tag{7}$$

The functions W_1 and W_2 satisfy homogeneous boundary conditions, which correspond to the attachments of the plate edges. The conditions can be written symbolically in the form

$$V_r[W_r]|_B = 0, \qquad r = 1, 2. \tag{8}$$

For determination of the vibration frequencies of the system the Green's functions G_r of the corresponding differential problems has been applied. The functions are solutions of the differential equation

$$\frac{\partial^4 G_r}{\partial\eta^4} + 2\Psi_r \Phi_r^2 \frac{\partial^4 G_r}{\partial\xi^2\, \partial\eta^2} + \Phi_r^4 \frac{\partial^4 G_r}{\partial\xi^4} - \Phi_r^4 \lambda_r^4 G_r = \delta(\xi - \zeta)\, \delta(\eta - \theta). \tag{9}$$

These functions with respect to variables ξ and η, satisfy the boundary conditions (8). Using the properties of the Green's function and equations (5) and (6), the following integral equations are obtained

$$W_1(\xi, \eta) = \sum_{j=1}^{n} K_j \int_0^1 [W_2(\zeta, \eta_j) - W_1(\zeta, \eta_j)] G_1(\zeta, \eta_j, \xi, \eta)\, d\zeta, \tag{10}$$

$$W_2(\xi, \eta) = \mu \sum_{j=1}^{n} K_j \int_0^1 [W_1(\zeta, \eta_j) - W_2(\zeta, \eta_j)] G_2(\zeta, \eta_j, \xi, \eta)\, d\zeta. \tag{11}$$

The functions W_r and G_r corresponding to plates, which are simply supported at the edges $\xi = 0$ and $\xi = 1$, may be written in the form

$$W_r(\xi, \eta) = 2 \sum_{m=1}^{\infty} Y_{rm}(\eta) \sin m\Pi\xi,$$

$$G_r(\xi, \eta, \zeta, \theta) = 2 \sum_{m=1}^{\infty} g_{rm}(\eta, \theta) \sin m\Pi\xi \sin m\Pi\zeta. \qquad (12, 13)$$

On the basis of equations (10–13), one obtains

$$Y_{1m}(\eta) = \sum_{j=1}^{n} K_j [Y_{2m}(\eta_j) - Y_{1m}(\eta_j)] g_{1m}(\eta, \eta_j), \qquad (14)$$

$$Y_{2m}(\eta) = \mu \sum_{j=1}^{n} K_j [Y_{1m}(\eta_j) - Y_{2m}(\eta_j)] g_{2m}(\eta, \eta_j). \qquad (15)$$

After subtracting both sides of equations (14) and (15), one has

$$\bar{Y}_m(\eta) = -\sum_{j=1}^{n} K_j [g_{1m}(\eta, \eta_j) + \mu g_{2m}(\eta, \eta_j)] \bar{Y}_m(\eta_j), \qquad (16)$$

where $\bar{Y}_m(\eta) = Y_{2m}(\eta) - Y_{1m}(\eta)$.

By substituting $\eta = \eta_j$, $j = 1, 2, \ldots, n$, successively into equation (16), one obtains a system of n equations (for each $m = 1, 2, \ldots$) with unknowns $\bar{Y}_m(\eta_j)$. For a non-trivial solution of the problem, the determinant of the coefficient matrix is set equal to zero, yielding the frequency equation:

$$|a_{ij}| = 0, \qquad (17)$$

where $a_{ij} = K_j[g_{1m}(\eta_i, \eta_j) + \mu g_{2m}(\eta_i, \eta_j)] + \delta_{ij}$, $|a_{ij}|$ denotes the determinant of the matrix $[a_{ij}]$ and δ_{ij} is the Kronecker delta. The equation (17), with the unknown ω, is then solved numerically.

3. THE GREEN'S FUNCTIONS FOR RECTANGULAR ORTHOTROPIC PLATES WITH TWO OPPOSITE EDGES SIMPLY SUPPORTED

The Green function as a solution of equation (9) for a plate with simply supported edges $\xi = 0$ and $\xi = 1$, may be written in the form given by equation (13). Substituting the function G_r into equation (9) and dropping the index r, one obtains

$$g_m^{IV} - 2\Psi\Phi^2(m\Pi)^2 g_m^{II} + \Phi^4[(m\Pi)^4 - \lambda^4] g_m = \delta(\eta - \theta), \qquad m = 1, 2, \ldots, \qquad (18)$$

The solution of equation (18) can be written in the form of a sum:

$$g_m(\eta, \theta) = g_m^0(\eta, \theta) + g_m^p(\eta - \theta)\mathrm{H}(\eta - \theta), \qquad (19)$$

where the first term of the sum denotes a general solution of the homogeneous equation, and the second is a particular solution of the non-homogeneous equation. If $\Psi \geqslant 1$, then for determination of the functions g_m^0 and g_m^p, two cases: $0 < \lambda < m\Pi$ and $\lambda > m\Pi$, should be considered. If $\Psi < 1$ then additionally in the interval $(0, m\Pi)$, two subintervals: $0 < \lambda < m\Pi\sqrt{1-\Psi^2}$ and $m\Pi\sqrt{1-\Psi^2} < \lambda < m\Pi$, must be distinguished. The general solution may be written in the form

$$g_m^0(\eta, \theta) =$$

$$\begin{cases} [C_1 \cos v_m\eta + C_2 \sin v_m\eta] \cosh u_m\eta + [C_3 \cos v_m\eta + C_4 \sin v_m\eta] \sinh u_m\eta \\ \text{for } 0 < \lambda < m\Pi\sqrt[4]{1-\psi^2} \text{ and } \Psi < 1, \\ \bar{C}_1 \sinh \beta_m\eta + \bar{C}_2 \cosh \beta_m\eta + \bar{C}_3 \sinh \gamma_m\eta + \bar{C}_4 \cosh \gamma_m\eta \\ \text{for } m\Pi^4\sqrt{1-\psi^2} < \lambda < m\Pi \text{ and } \Psi < 1 \quad \text{or} \quad 0 < \lambda < m\Pi \text{ and } \Psi \geqslant 1, \\ \bar{\bar{C}}_1 \sinh \beta_m\eta + \bar{\bar{C}}_2 \cosh \beta_m\eta + \bar{\bar{C}}_3 \sin \bar{\gamma}_m\eta + \bar{\bar{C}}_4 \cos \bar{\gamma}_m\eta \text{ for } \lambda > m\Pi \end{cases} \tag{20}$$

where

$$u_m = (\Phi/2)\sqrt{2(\sqrt{(m\Pi)^4 - \lambda^4} + (m\Pi)^2\Psi)},$$
$$v_m = (\Phi/2)\sqrt{2(\sqrt{(m\Pi)^4 - \lambda^4} - (m\Pi)^2\Psi)},$$
$$\beta_m = \Phi\sqrt{(m\Pi)^2\Psi + \sqrt{\lambda^4 + (m\Pi)^4(\Psi^2 - 1)}},$$
$$\gamma_m = \Phi\sqrt{(m\Pi)^2\Psi - \sqrt{\lambda^4 + (m\Pi)^4(\Psi^2 - 1)}},$$
$$\bar{\gamma}_m = \Phi\sqrt{-(m\Pi)^2\Psi + \sqrt{\lambda^4 + (m\Pi)^4(\Psi^2 - 1)}}$$

and

$$C_1, \ldots, \bar{\bar{C}}_4, \quad \text{are the integral constants.}$$

The function g_m^p is evaluated after substitution of the expression $g_m^p(\eta - \theta)\mathrm{H}(\eta - \theta)$ into equation (18). This function runs as follows

$$g_m^p(\eta - \theta) =$$

$$\begin{cases} ([1/[2u_m v_m(u_m^2 + v_m^2)]) [u_m \cosh u_m(\eta - \theta) \sin v_m(\eta - \theta) \\ \quad - v_m \sinh u_m(\eta - \theta) \cos v_m(\eta - \theta)] \text{ for } 0 < \lambda < m\Pi\sqrt{1-\Psi^2} \text{ and } \Psi < 1, \\ (1/[\beta_m^2 - \gamma_m^2])[(1/\beta_m) \sinh \beta_m(\eta - \theta) - (1/\gamma_m) \sinh \gamma_m(\eta - \theta)] \\ \text{for } m\Pi\sqrt{1-\Psi^2} < \lambda < m\Pi \text{ and } \Psi < 1 \text{ or } 0 < \lambda < m\Pi \text{ and } \Psi \geqslant 1 \\ (1/[\beta_m^2 + \bar{\gamma}_m^2])[(1/\beta_m) \sinh \beta_m(\eta - \theta) - (1/\bar{\gamma}_m) \sin \bar{\gamma}_m(\eta - \theta)] \text{ for } \lambda > m\Pi \end{cases} \tag{21}$$

The Green's function for the rectangular plate simply supported at the edges $\eta = 0$ and $\eta = 1$, satisfies the following conditions: $G = M_\eta = 0$, where

$M_\eta = \partial^2 G/\partial\eta^2 + v_x(b^2/a^2)(\partial^2 G/\partial\xi^2)$. By taking into consideration the form of the function G (equation (13)), it can be seen that the functions g_m, $m = 1, 2, \ldots,$ satisfy the following conditions: $g_m = \partial^2 g_m/\partial\eta^2 = 0$ for $\eta = 0$ and $\eta = 1$. On the basis of these conditions for $\eta = 0$, the function g_m^0 may now be written in the form:

$$g_m^0(\eta, \theta) = \begin{cases} C_1 \sin v_m\eta \cosh u_m\eta + C_2 \cos v_m\eta \sinh u_m\eta \\ \quad \text{for} \quad 0 < \lambda < m\Pi\sqrt[4]{1-\psi^2} \quad \text{and} \quad \Psi < 1 \\ \bar{C}_1 \sinh \beta_m\eta + \bar{C}_2 \sinh \gamma_m\eta \\ \quad \text{for} \quad m\Pi\sqrt[4]{1-\psi^2} < \lambda < m\Pi \quad \text{and} \quad \psi < 1 \quad \text{or} \\ \quad 0 < \lambda < m\Pi \quad \text{and} \quad \psi \geqslant 1 \\ \bar{\bar{C}}_1 \sinh \beta_m\eta + \bar{\bar{C}}_2 \sin \bar{\gamma}_m\eta \quad \text{for} \quad \lambda > m\Pi \end{cases} \tag{22}$$

For determination of the constants C_1, C_2, $\bar{C}_1$, $\bar{C}_2$, $\bar{\bar{C}}_1$, $\bar{\bar{C}}_2$, the conditions at boundary $\eta = 1$, are used. The constants are:

$$C_1 = \frac{-q_{1m} \sin v_m(1-\theta) \cosh u_m(1-\theta) + q_{2m} \cos v_m(1-\theta) \sinh u_m(1-\theta)}{u_m v_m(u_m^2 + v_m^2)(\cos 2v_m - \cosh 2u_m)},$$

$$C_2 = \frac{q_{1m} \cos v_m(1-\theta) \sinh u_m(1-\theta) + q_{2m} \sin v_m(1-\theta) \cosh u_m(1-\theta)}{u_m v_m(u_m^2 + v_m^2)(\cos 2v_m - \cosh 2u_m)}, \tag{23}$$

$$\bar{C}_1 = -\frac{\sinh \beta_m(1-\theta)}{\beta_m(\beta_m^2 - \gamma_m^2) \sinh \beta_m}, \qquad \bar{C}_2 = \frac{\sinh \gamma_m(1-\theta)}{\gamma_m(\beta_m^2 - \gamma_m^2) \sinh \gamma_m}, \tag{24}$$

$$\bar{\bar{C}}_1 = -\frac{\sinh \beta_m(1-\theta)}{\beta_m(\beta_m^2 + \bar{\gamma}_m^2) \sinh \beta_m}, \qquad \bar{\bar{C}}_2 = \frac{\sin \bar{\gamma}_m(1-\theta)}{\bar{\gamma}_m(\beta_m^2 + \bar{\gamma}_m^2) \sin \bar{\gamma}_m}. \tag{25}$$

and

$$q_{1m} = v_m \cos v_m \sinh u_m + u_m \sin v_m \cosh u_m,$$

$$q_{2m} = v_m \sin v_m \cosh u_m - u_m \cos v_m \sinh u_m.$$

Finally the Green's function for the orthotropic, rectangular, simply supported plate (S–S–S–S) is given by the equations (13) and (19), where the functions $g_m^p(\eta - \theta)$ and $g_m^0(\eta, \theta)$ are designated by equations (21) and (22), respectively. Similarly, the Green's functions for other homogeneous conditions at the edges $\eta = 0$ and $\eta = 1$, may be obtained. The Green's function for the simply supported–free–simply supported–free orthotropic plate (S–F–S–F) is given in Appendix A.

4. RESULTS AND DISCUSSION

Consider a system of two plates elastically connected along two lines $\eta = \eta_1$ and $\eta = \eta_2$. On the basis of equation (17), the frequency equation corresponding to the considered system with $K_1 = K_2 = K$, has the form:

$$[\Psi_m(\eta_1, \eta_2) + 1][\Psi_m(\eta_2, \eta_2) + 1] - \Psi_m(\eta_1, \eta_2)\Psi_m(\eta_2, \eta_1) = 0, \tag{26}$$

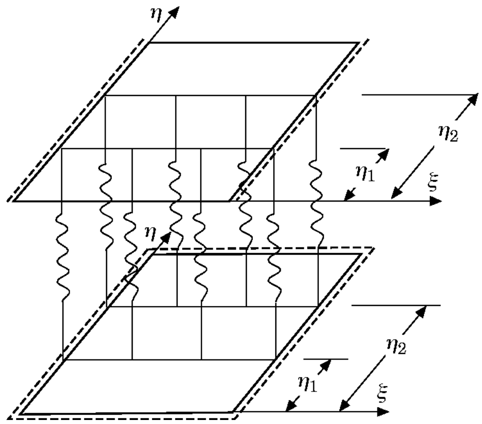

Figure 2. Sketch of a system of S–F–S–F plates elastically connected along two lines.

where $\Psi_m(\eta, \theta) = K[g_{1m}(\eta, \theta) + \mu g_{2m}(\eta, \theta)]$. The frequency equation (26) for a system of identical plates connected by translational springs can be rewritten as follows

$$[g_{1m}(\eta_1, \eta_1) + 1/2K][g_{1m}(\eta_1, \eta_2) + 1/2K] - g_{1m}(\eta_1, \eta_2)g_{1m}(\eta_2, \eta_1) = 0. \qquad (27)$$

The equation (27) is also the frequency equation of an isolated plate elastically supported by translational springs, which are distributed along the lines $\eta = \eta_1$ and

TABLE 1

Frequency parameter values $\Omega_{mn} = \omega a^2\sqrt{\rho_2/D_{x2}}$ *for the system of two isotropic, square plates shown in Figure 2*

(m, n)	$\eta_1 = 0{\cdot}2$, $\eta_2 = 0{\cdot}8$: $K = 1$	$K = 100$	$K = 1000$	$K = \infty$	$\eta_1 = 0{\cdot}4$, $\eta_2 = 0{\cdot}6$: $K = 1$	$K = 100$	$K = 1000$	$K = \infty$
(1, 1)	9·7339	14·2548	16·0448	16·2458	9·7251	12·5985	13·4820	13·6006
(1, 1)†	19·7744	24·3879	36·1281	36·3620	19·8309	26·7669	31·5461	32·0287
(1, 2)	16·2027	20·9982	29·2800	31·7669	16·1428	16·8336	19·3128	21·3144
(1, 2)†	49·3847	52·9917	67·5117	70·6238	49·3620	50·6286	56·0308	60·2163
(1, 3)	36·7265	36·8404	47·4806	63·4191	36·7584	41·5073	77·5536	90·7851
(1, 3)†	98·7144	100·4912	111·3353	120·2311	98·7030	99·4476	111·5618	129·1507
(2, 1)	38·9701	40·9492	44·3275	45·0353	38·9679	40·4111	41·9766	42·3126
(2, 1)†	49·3620	50·9426	64·2640	69·7859	49·3847	52·9149	61·9350	63·8098
(2, 2)	46·7621	48·8432	55·8760	59·8670	46·7412	47·0205	48·4263	50·4187
(2, 2)†	78·9797	81·2613	96·6707	105·5191	78·9656	79·7855	84·3502	90·7181
(2, 3)	70·7407	70·8072	72·0852	87·9547	70·7582	72·8511	98·3758	119·6568
(2, 3)†	128·3189	129·6957	139·5847	153·4374	128·3102	128·8712	137·1803	164·8997
(3, 1)	87·9977	88·9818	92·4103	93·9363	87·9968	88·7955	90·5724	91·2319
(3, 1)†	98·7030	99·4383	102·5468	120·1388	98·7144	100·5291	109·5043	113·7484
(3, 2)	96·0523	97·1428	98·6960	108·1940	96·0420	96·1884	97·0850	99·1436
(3, 2)†	128·3189	129·7226	141·4380	158·5210	128·3102	128·8246	132·1897	139·9484
(3, 3)	122·0405	122·0883	122·6188	133·8740	122·0509	123·2140	138·5311	168·2850
(3, 3)†	177·6631	178·6623	186·6149	206·0100	177·6568	178·0550	183·1440	219·9049

† S–S–S–S plates.

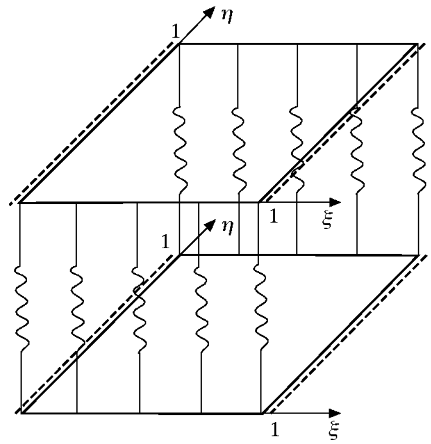

Figure 3. Sketch of a system of two S–F–S–F plates elastically connected along the free edges.

$\eta = \eta_2$. This means, that the non-degenerate frequencies of a system of two identical plates, which are connected by an elastic element with stiffness modulus K, are the same as for a single plate on an elastic foundation with stiffness coefficient $2K$.

The effect of stiffness of the elastic connection and material orthotropy of the plates on the natural frequencies $\Omega_{mn} = \lambda_{2mn}^2$ of the combined system was numerically investigated. In all examples one (bottom) plate is an iostropic ($D_{x2}/D_{y2} = 1$ and $H_2/D_{y2} = 1$) and the second (top) is an orthotropic or isotropic plate whereas $\mu = D_{y1}/D_{y2} = 1$. For both plates it is assumed that $\nu_x = 0{\cdot}3$.

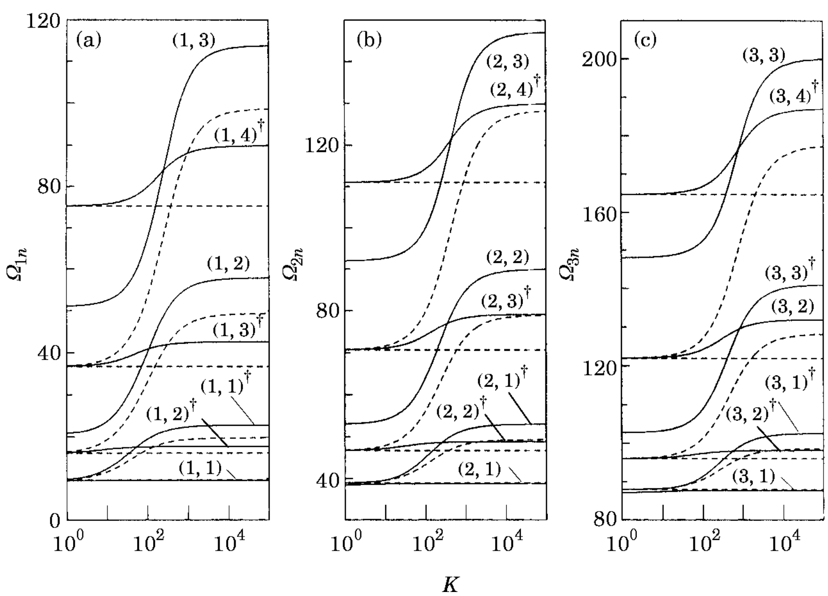

Figure 4. Frequency parameter values $\Omega_{mn} = \omega a^2\sqrt{\rho_2/D_{x2}}$ as functions of the stiffness of translational springs K connecting two square S–F–S–F plates along the free edges shown in Figure 3; ----, identical isotropic plates; ——, isotropic bottom plate and $D_{x1}/D_{y1} = 0{\cdot}5$, $H_1/D_{y1} = 1{\cdot}0$ for the top plate.

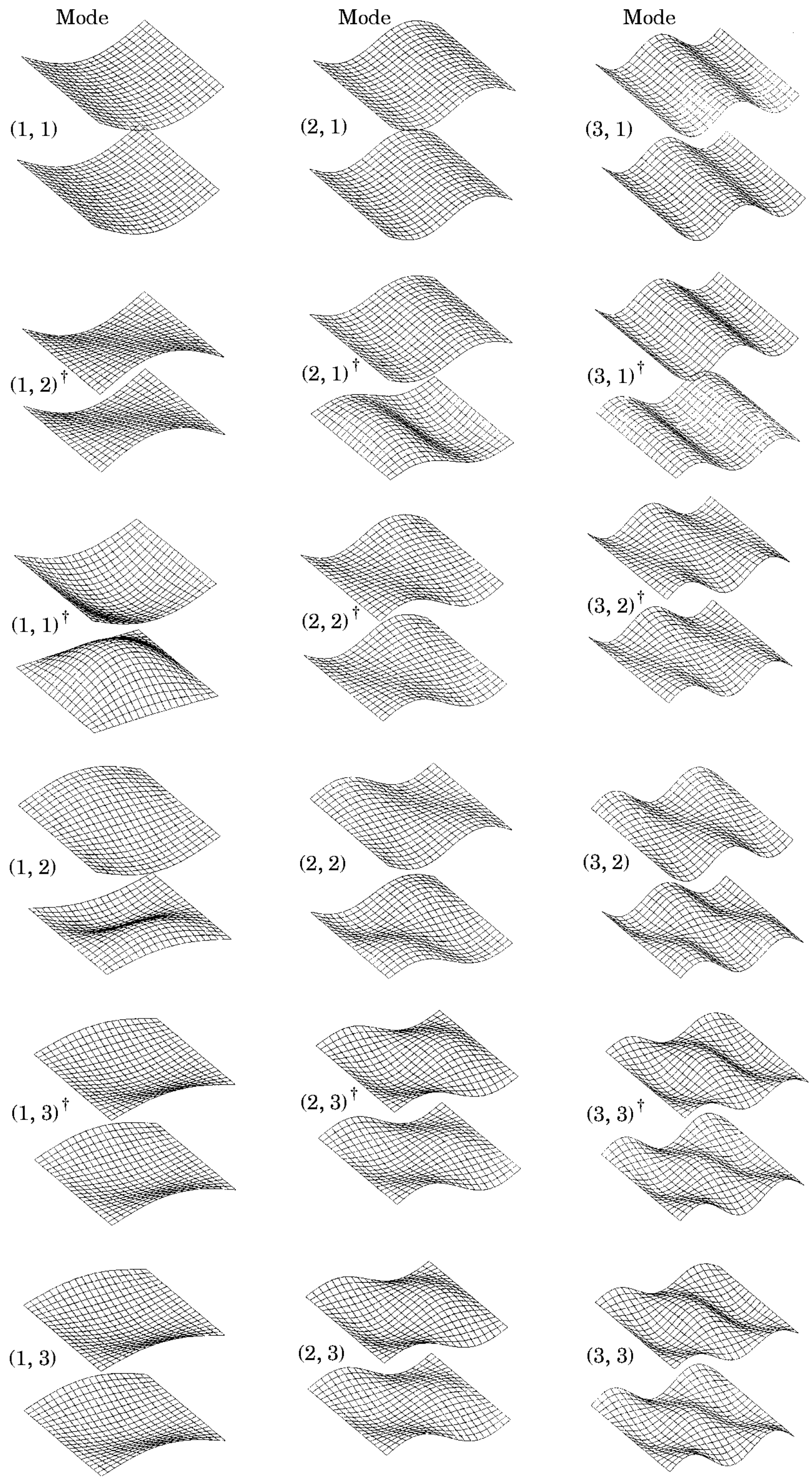

Figure 5. Mode shapes of the system shown in Figure 3 with the stiffness of translational springs $K = 100$.

The first example concerns the system of the S–S–S–S and S–F–S–F quadratic plates with the same isotropic material properties (Figure 2). The plates are intermediately elastically connected by means of translational springs distributed along two lines with $\eta_1 = 0{\cdot}2$, $\eta_2 = 0{\cdot}8$ or $\eta_1 = 0{\cdot}4$, $\eta_2 = 0{\cdot}6$. The eighteen non-dimensional vibration frequencies for $K = 1$; 100; 1000 and $K \to \infty$, are presented in Table 1. The modes (m, n) and $(m, n)^\dagger$ apply to the isolated plates (for $K \to 0$), with the dagger denoting that the frequency refers to the S–S–S–S plate. The comparison of the results for various values of K has shown that the increase of the stiffness causes the increase of the frequencies. Besides the effect is greater for lower frequencies when the connecting lines are closer to the plate edges.

The changes of the frequency values of a combined system consisting with two S–F–S–F quadratic plates (Figure 3) versus the stiffness coefficient of the connecting springs, are presented in Figure 4. The plates are connected by means of translational springs along the free plate edges ($\eta_1 = 0$, $\eta_2 = 1$). Either both plates of the system are identical isotropic ones (dashed line) or the top is an orthotropic plate (solid line) with $D_{x1}/D_{y1} = 0{\cdot}5$ and $H_1/D_{y1} = 1{\cdot}0$. The K-axis is logarithmic. The curves shown in figures (a), (b) and (c) are obtained for $m = 1, 2$ and 3, respectively. The values of the frequencies of the combined system started at the frequencies of the isolated plates ($K = 0$). The modes of the isolated orthotropic plate are denoted by (m, n), and those of the isotropic plate by $(m, n)^\dagger$. These notations are preserved for the frequency curves (solid line) which are obtained for $K > 0$.

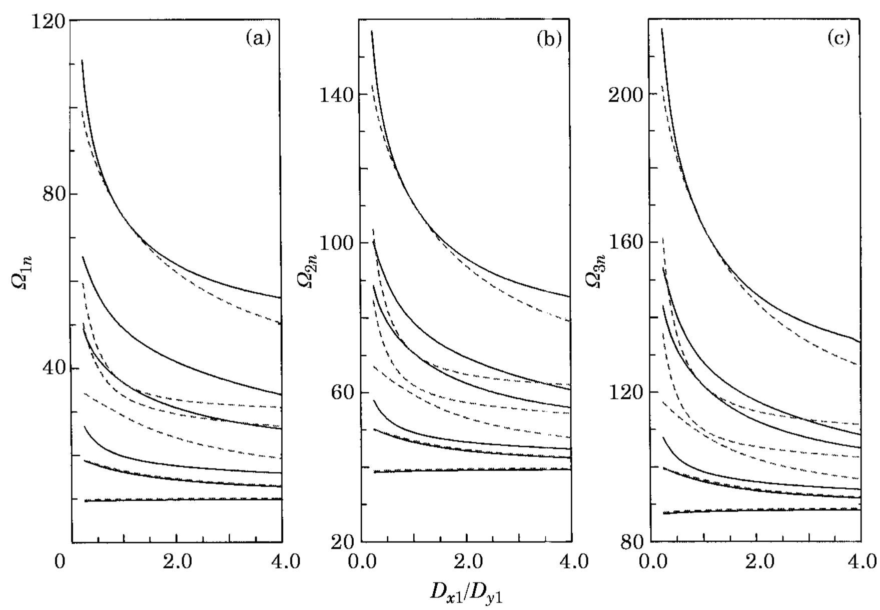

Figure 6. Frequency parameter values $\Omega_{mn} = \omega a^2\sqrt{\rho_2/D_{x2}}$ as functions of the ratio of material orthotropy of the top plate D_{x1}/D_{y1} for two square S–F–S–F plates rigidly connected along two lines: ----, $\eta_1 = 1/3$, $\eta_2 = 2/3$, ——, $\eta_1 = 0$, $\eta_2 = 1$.

TABLE 2

Frequency parameter values $\Omega_{mn} = \omega a^2 \sqrt{\rho_2/D_{x2}}$ for the system of two square S–F–S–F plates connected by translational springs along the free edges with bottom isotropic plate and $H_1/D_{y1} = 1{\cdot}0$ for the orthotropic top plate

	$K = 1$			$K = 100$			$K = 1000$			$K = \infty$		
(m, n)	$D_{x1} = 0{\cdot}5D_{y1}$	$D_{x1} = D_{y1}$	$D_{x1} = 2D_{y1}$	$D_{x1} = 0{\cdot}5D_{y1}$	$D_{x1} = D_{y1}$	$D_{x1} = 2D_{y1}$	$D_{x1} = 0{\cdot}5D_{y1}$	$D_{x1} = D_{y1}$	$D_{x1} = 2D_{y1}$	$D_{x1} = 0{\cdot}5D_{y1}$	$D_{x1} = D_{y1}$	$D_{x1} = 2D_{y1}$
(1, 1)	9·5115	9·8784	9·8755	9·5499	16·9670	15·4326	9·5502	19·3928	17·3008	9·5502	19·7392	17·5606
(1, 1)*	9·8813	9·6314	9·6922	19·4659	9·6314	9·7111	22·3881	9·6314	9·7112	22·7890	9·6314	9·7112
(1, 2)	20·8655	16·4937	13·4757	40·9271	33·1915	14·2784	55·5052	46·8757	14·3281	57·8565	49·3480	14·3338
(1, 2)*	16·3028	16·1348	16·3214	17·6164	16·1348	29·7694	17·6969	16·1348	39·8637	17·7061	16·1348	41·5017
(1, 3)	51·1359	36·9249	26·9578	67·5314	53·6348	29·3309	105·5779	88·6621	30·7973	113·8106	98·6960	30·9310
(1, 3)*	36·8244	36·7256	36·8259	40·2889	36·7256	48·2592	42·4210	36·7256	74·9803	42·6151	36·7256	80·7783
(2, 1)	38·4703	39·0151	39·2324	38·7564	43·3794	42·3894	38·7621	48·2497	46·0000	38·7627	49·3480	46·7731
(2, 1)*	38·9849	38·9450	38·9776	45·1130	38·9450	39·1191	51·7054	38·9450	39·1219	53·0745	38·9450	39·1222
(2, 2)	53·1231	46·8620	43·2937	63·9790	56·1252	44·1658	84·8193	73·6883	44·3723	89·9891	78·9568	44·3987
(2, 2)*	46·7990	46·7382	46·8007	48·4014	46·7382	53·1595	48·7776	46·7382	66·1546	48·8257	46·7382	69·4859
(2, 3)	91·9949	70·8338	57·3120	99·8808	79·7359	59·4157	134·3012	112·6362	62·0313	147·1342	128·3049	62·5557
(2, 3)*	70·7868	70·7401	70·7871	74·1749	70·7401	75·9431	78·3404	70·7401	103·4249	79·1700	70·7401	107·7156
(3, 1)	87·1720	88·0202	88·4162	87·6640	90·6584	90·1541	87·6945	96·4941	94·4566	87·6978	98·6960	95·9856
(3, 1)*	88·0041	87·9867	88·0031	91·4624	87·9867	88·2471	99·7696	87·9867	88·2623	102·6467	87·9867	88·2639
(3, 2)	102·8067	96·1016	92·5195	108·9540	101·3880	93·2202	131·2584	118·9543	93·5800	141·0682	128·3049	93·6348
(3, 2)*	96·0708	96·0405	96·0712	97·4160	96·0405	99·4010	98·1034	96·0405	112·0690	98·2081	96·0405	117·8778
(3, 3)	148·0196	122·0908	106·7476	152·3907	127·0636	108·0035	180·5971	154·3760	111·2940	200·1797	177·6529	112·4696
(3, 3)*	122·0654	122·0400	122·0654	124·2552	122·0400	124·7258	129·9330	122·0400	148·2518	131·9420	122·0400	154·8519

The eigenfrequencies of the system considered, increase with increasing the stiffness coefficient, except the case of the system of two identical plates when the degenerate frequencies do not change. The non-degenerate eigenfrequencies of the system of identical plates increase from frequency values of the S–F–S–F isolated plate (when $K = 0$) to frequency values of the S–S–S–S plate (when $K \to \infty$). The greater increase of the frequencies appears for K between 10 and 1000. The corresponding points of the plates during the free vibration of the system, are moving in the same or in opposite direction. The mode shapes corresponding to the frequencies evaluated in this example for $K = 100$, are shown in Figure 5.

The vibration frequencies of the combined systems as functions of the ratio of material orthotropy D_{x1}/D_{y1}, are presented in Figure 6. The calculations were performed for $1/4 < D_{x1}/D_{y1} < 4$ and $H_1/D_{y1} = 1$. The results are obtained for the system of two S–F–S–F plates rigidly connected ($K \to \infty$) along the free edges (solid lines) and along the lines $\eta = 1/3$, $\eta = 2/3$ (dashed line). It follows, that the change of the orthotropy of one plate affects significantly the alteration in the frequencies of the combined system.

The eighteen non-dimensional vibration frequencies presented in Table 2 have been evaluated for the combined systems with various values of the stiffness coefficient of the translational springs connecting two S–F–S–F plates along the free edges. The results are calculated for an isotropic bottom plate and the material orthotropy of the top plate: $H_1/D_{y1} = 1$ and $D_{x1}/D_{y1} = 0{\cdot}5$; $1{\cdot}0$; $2{\cdot}0$. The modes (m, n) and $(m, n)\dagger$ apply to the isolated plates, and the dagger denotes that the mode refers to the bottom isotropic plate.

5. CONCLUSIONS

The solution of the free vibration problem of the system of line connected orthotropic, rectangular plate by applying the Green's function method was obtained. The theoretical investigations comprise the systems of plates connected by translational springs distributed along the lines perpendicular to the two opposite simply supported edges of the plates. Although the system considered in the given examples consist of one isotropic and one orthotropic square plates elastically connected along two lines, the solution can be used for a system of two orthotropic rectangular plates connected along arbitrary number of lines.

The spectrum of the combined system of two identical plates included the eigenfrequencies of the isolated plates. These degenerate frequencies do not depend on the stiffness of the connection. In this case the non-degenerate frequencies of the combined system are the same as for a single, elastically line supported plate. The numerical examples have shown that the stiffness of the elastic connections as well as the material orthotropy of the plates significantly affect the vibration frequencies of the combined system.

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APPENDIX A

A.1. THE GREEN'S FUNCTION FOR RECTANGULAR, ORTHOTROPIC S–F–S–F PLATE

The Green's function G, which corresponds to an orthotropic, rectangular S–F–S–F plate with free edges $\eta = 0$ and $\eta = 1$, satisfies the following conditions: $M_\eta = V_\eta = 0$, where $M_\eta = \partial^2 G/\partial\eta^2 + v_\xi\, \partial^2 G/\partial\xi^2$, $V_\eta = \partial^3 G/\partial\eta^3 + R_\xi\, \partial^3 G/\partial\eta\, \partial\xi^2$ and $R_\xi = 2(b^2/a^2)(\mathrm{H}/D_y) - v_\xi$. Taking into consideration the form of the function G (equation (13)), the boundary conditions for the function $g_m(\eta, \theta)$ can be obtained. The conditions are:

$$\frac{\partial^2 g_m}{\partial\eta^2} - v_\xi (m\Pi)^2 g_m = 0, \qquad \frac{\partial^3 g_m}{\partial\eta^3} - R_\xi (m\Pi)^2 \frac{\partial g_m}{\partial\eta} = 0 \quad \text{for } \eta = 0 \quad \text{and} \quad \eta = 1. \tag{A1}$$

On the basis of equations (19), (20) and (A1), the function $g_m^0(\eta, \theta)$ may be written in the following form:

$$g_m^0(\eta, \theta) = f_{1m}(\eta) C_1 + f_{2m}(\eta) C_2, \tag{A2}$$

where

$$C_1 = (a_{12} r_2 - a_{22} r_1) D^{-1}, \qquad C_2 = (a_{21} r_1 - a_{11} r_2) D^{-1}. \tag{A3}$$

The functions $f_{1m}(\eta)$, $f_{2m}(\eta)$ and the quantities a_{11}, a_{12}, a_{21}, a_{22}, r_1, r_2 and D are designated in the three cases as follows:

A.1.1. *Case* 1: $0 < \lambda < m\Pi\sqrt[4]{1-\psi^2}$ *and* $\Psi < 1$

$$f_{1m}(\eta) = c_{vm}\cosh u_m\eta \cos v_m\eta - b_{vm}\sinh u_m\eta \sin v_m\eta,$$

$$f_{2m}(\eta) = c_{Rm}\cosh u_m\eta \sin v_m\eta - b_{Rm}\sinh u_m\eta \cos v_m\eta,$$

$$a_{11} = -(b_{vm}^2 + c_{vm}^2)\sinh u_m \sin v_m, \qquad a_{22} = (b_{Rm}^2 + c_{Rm}^2)\sinh u_m \sin v_m,$$

$$a_{12} = (b_{vm}c_{Rm} + b_{Rm}c_{vm})\cosh u_m \sin v_m + (b_{vm}b_{Rm} - c_{vm}c_{Rm})\sinh u_m \cos v_m,$$

$$a_{21} = -(b_{vm}c_{Rm} + b_{Rm}c_{vm})\cosh u_m \sin v_m + (b_{vm}b_{Rm} - c_{vm}c_{Rm})\sinh u_m \cos v_m,$$

$$r_1 = u_m(u_m^2 + v_m^2 - v_\xi(m\Pi)^2)\cosh(1-\Theta)u_m \sin(1-\Theta)v_m$$
$$- v_m(u_m^2 + v_m^2 + v_\xi(m\Pi)^2)\sinh(1-\Theta)u_m \cos(1-\Theta)v_m,$$

$$r_2 = (u_m^2 + v_m^2)((u_m^2 - v_m^2 - R_\xi(m\Pi)^2)\sinh(1-\Theta)u_m \cos(1-\Theta)v_m)$$
$$+ 2u_m v_m \cosh(1-\Theta)u_m \sin(1-\Theta)v_m,$$

$$D = 2u_m v_m(u_m^2 + v_m^2)(a_{11}a_{22} - a_{12}a_{21}), \qquad b_{vm} = 2u_m v_m,$$

$$c_{vm} = u_m^2 - v_m^2 - v_\xi(m\Pi)^2,$$

$$b_{Rm} = v_m[3u_m^2 - v_m^2 - R_\xi(m\Pi)^2], \qquad c_{Rm} = u_m[u_m^2 - 3v_m^2 - R_\xi(m\Pi)^2].$$

A.1.2. Case 2: $m\Pi\sqrt[4]{1-\psi^2} < \lambda < m\Pi$ *and* $\Psi < 1$ *or* $0 < \lambda < m\Pi$ *and* $\Psi \geqslant 1$

$$f_{1m}(\eta) = c_{vm}\cosh\beta_m\eta - b_{vm}\cosh\gamma_m\eta, \qquad f_{2m}(\eta) = c_{Rm}\sinh\beta_m\eta - b_{Rm}\sinh\gamma_m\eta,$$

$$a_{11} = b_{vm}c_{vm}(\cosh\beta_m - \cosh\gamma_m), \qquad a_{12} = b_{vm}c_{Rm}\sinh\beta_m - b_{Rm}c_{vm}\sinh\gamma_m,$$

$$a_{21} = b_{Rm}c_{vm}\sinh\beta_m - b_{vm}c_{Rm}\sinh\gamma_m, \qquad a_{22} = b_{Rm}c_{Rm}(\cosh\beta_m - \cosh\gamma_m),$$

$$r_1 = \frac{b_{vm}}{\beta_m}\sinh(1-\Theta)\beta_m - \frac{c_{vm}}{\gamma_m}\sinh(1-\Theta)\gamma_m,$$

$$r_2 = \frac{b_{Rm}}{\beta_m}\cosh(1-\Theta)\beta_m - \frac{c_{Rm}}{\gamma_m}\cosh(1-\Theta)\gamma_m,$$

$$D = (\beta_m^2 - \gamma_m^2)(a_{11}a_{22} - a_{12}a_{21}), \qquad b_{vm} = \beta_m^2 - v_\xi(m\Pi)^2,$$

$$c_{vm} = \gamma_m^2 - v_\xi(m\Pi)^2,$$

$$b_{Rm} = \beta_m[\beta_m^2 - R_\xi(m\Pi)^2], \qquad c_{Rm} = \gamma_m[\gamma_m^2 - R_\xi(m\Pi)^2].$$

A.1.3. *Case* 3: $\lambda > m\Pi$

$$f_{1m}(\eta) = c_{vm}\cosh\beta_m\eta + b_{vm}\cos\bar{\gamma}_m\eta, \qquad f_{2m}(\eta) = c_{Rm}\sinh\beta_m\eta + b_{Rm}\sin\bar{\gamma}_m\eta,$$

$$a_{11} = b_{vm}c_{vm}(\cosh\beta_m - \cos\bar{\gamma}_m), \qquad a_{12} = b_{vm}c_{Rm}\sinh\beta_m - b_{Rm}c_{vm}\sin\bar{\gamma}_m,$$

$$a_{21} = b_{Rm}c_{vm}\sinh\beta_m + b_{vm}c_{Rm}\sin\bar{\gamma}_m, \qquad a_{22} = b_{Rm}c_{Rm}(\cosh\beta_m - \cos\bar{\gamma}_m),$$

$$r_1 = \frac{b_{vm}}{\beta_m}\sinh(1-\Theta)\beta_m + \frac{c_{vm}}{\bar{\gamma}_m}\sin(1-\Theta)\bar{\gamma}_m,$$

$$r_2 = \frac{b_{Rm}}{\beta_m} \cosh(1-\Theta)\beta_m + \frac{c_{Rm}}{\bar{\gamma}_m} \cos(1-\Theta)\bar{\gamma}_m,$$

$$D = (\beta_m^2 + \bar{\gamma}_m^2)(a_{11}a_{22} - a_{12}a_{21}), \qquad b_{vm} = \beta_m^2 - v_\xi (m\Pi)^2,$$

$$c_{vm} = \bar{\gamma}_m^2 + v_\xi (m\Pi)^2,$$

$$b_{Rm} = \beta_m[\beta_m^2 - R_\xi (m\Pi)^2], \qquad c_{Rm} = \bar{\gamma}_m[\bar{\gamma}_m^2 + R_\xi (m\Pi)^2].$$

Thus the Green's function for the S–F–S–F orthotropic rectangular plate expresses the equations (13) and (19), where the functions $g_m^p(\eta - \theta)$ and $g_m^0(\eta, \theta)$ are given by equations (21) and (A2), respectively.

APPENDIX B: NOTATION

a, b	length dimensions of rectangular plates in the x and y directions, respectively
$D_{x1}, D_{x2}, D_{y1}, D_{y2}$	flexural rigidities of plates
H_1, H_2	coefficients containing the torsional rigidities of plates
M_y	bending moment in a plate, $M_\eta = M_y b^2/aD_y$
t	time
V_y	plate vertical edge reaction, $V_\eta = V_y b^3/aD_y$
(x, y)	Cartesian co-ordinates, $(\xi, \eta) = (x/a, y/b)$
w_1, w_2	transverse plate deflections
$\bar{W}_1, \bar{W}_2$	amplitude of transverse plate deflections, $W_1 = \bar{W}_1/a$, $W_2 = \bar{W}_2/a$
ρ_1, ρ_2	masses of plates per unit area
v_x	Poisson ratio of an orthotropic plate, $v_\xi = v_x b^2/a^2$
k_j	stiffness of the translational springs, $K_j = b^3 k_j/D_{y1}$