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LETTERS TO THE EDITOR

NON-STATIONARY CUTTING

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1. INTRODUCTION

The presence in a work piece of material inhomogeneities, the occurrence of the breakaway of built-up cutting tool edges, and chip breakage [1] may lead to rapid transitions between cutting states. The requirement that a machine control system follow and correct such departures necessitates the use of state identification algorithms which are effective for short time intervals in non-stationary time series. The methods of time–frequency analysis are of particular interest in this context.

These range from the short-time Fourier spectrum through the Wigner, Wigner–Ville, Page, Choi–Williams and Cohen distribution [2, 3]. Extensions to higher-order spectra are given in reference [4].

The identification of cutting states, associated with the orthogonal cutting of stiff cylinders, was realized in reference [5] through an analysis of the behavior of the singular values of a Toeplitz matrix, $\mathbf{R}$, equation (5), of third order cumulants of acceleration measurements. It was shown in references [5, 6] that the ratio, R-ratio, of the dominant pairs of singular values of $\mathbf{R}$ for a lag of 100 differentiates between light-cutting, medium-cutting, pre-chatter and chatter states. The cumulant elements of $\mathbf{R}$ were estimated as the average of cumulants for one second intervals over time series with durations of 20 to 60 s except for the chatter state for which the duration of associated time series was limited to 3 s.

The detection of transitions between cutting states is predicated in the following on the adaptation of the narrow moving window of the short-time Fourier spectrum to the calculation of what could be termed short-time R-ratios. The window length over which cumulants for one second intervals of data are averaged was reduced from 20 to 60 s to 1 to 6 s. Test data was generated numerically through concatenation of three trigonometric functions with random phase and experimentally by concatenating orthogonal cutting data for light and medium cutting and chatter. Transitions in the resulting time series were detected by all windows of 1 to 6 s duration.

2. EXPERIMENTAL APPARATUS

The experimental apparatus consists of a Hardinge CNC lathe, a specially designed force dynamometer utilizing three Kisthler 9068 force transducers and

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its associated electronics, and a Hewlett Packard 3566A digital spectrum analyzer for data acquisition and real-time analysis.

All experiments involved only right-handed orthogonal cutting. Kennametal TPMR 322 positive rake tool inserts were employed and were supported by Kennametal KT-GPR 123B tool holders. The tool holder-inserted combination resulted in a rake angle of 5° and a clearance angle of 4°. Cylindrical work pieces of 1020 steel were machined under a wide range of cutting conditions. Since all work pieces were stubby, work piece modal characteristics did not affect the turning dynamics. The sampling rate was 4096 Hz and the cutoff frequency was 1100 Hz. Record lengths were from 20 to 60 s except for chatter records, which had a duration of 3 s.

3. THE R-MATRIX

The **R** matrix arises in connection with the determination of the coefficients, $a(i)$, in an auto-regressive process [4, 7]. A pth order AR process is described by

$$X(k) + \sum_{i=1}^{P} a(i)X(k-i) = W(k), \tag{1}$$

where $X(k)$, $k = 0, \pm 1, \pm 2, \ldots$ is a real third order stationary random process. Assume that $W(k)$ is non-Gaussian, $E(W(k)) = 0$ and $E(W^3(k)) \equiv \beta$, where $E(\ ) \equiv$ the expected value of ( ). Multiplying through equation (1) and summing gives

$$c_3^x(-k, -\ell) + \sum_{i=1}^{P} a(i)c_3^x(i-k, i-\ell) = \beta\delta(k, \ell), \tag{2}$$

where k, $l > 0$ and the third order cumulant $c_3^x(\tau_1, \tau_2)$ is

$$c_3^x(\tau_1, \tau_2) = (1/2n) \sum_{k=-n}^{+n} X(k)X(k+\tau_1)X(k+\tau_2). \tag{3}$$

Letting $k = \ell$ in equation (2) with $k = 0, \ldots, P$ yields $P + 1$ equations for the $P + 1$ unknowns $a(i)$ and β [4, 7]. In matrix notation,

$$\mathbf{R}a = \mathbf{b}, \tag{4}$$

where

$$\mathbf{R} = \begin{bmatrix} g(o, o) & g(1, 1) & \cdots g(P, P) \\ g(-1, -1) & g(0, 0) & \cdots g(P-1, P-1) \\ \vdots & \vdots & \vdots \\ g(-P, -P) & g(-P+1, -P+1) & \cdots g(o, o) \end{bmatrix}; \tag{5}$$

$g(i, j) \equiv c_3^x(i, j)$, $\mathbf{a} = [1, a(1), \ldots, a(P)]^T$ and $\mathbf{b} \equiv [\beta, o, \ldots, o]^T$. In general **R** is a non-symmetric Toeplitz matrix. In references [4, 7] the data set is segmented into

K records of M samples each and $c^x_{3,n}(i,j)$ is computed for the n record. $c^x_3(i,j)$ is then computed as an average of $c^x_{3,n}(i,j)$ over all K records. A sufficient but not necessary condition for the representation in equation (4) to exist is the symmetry and positive definiteness of **R**. A discussion of this and related conditions is given in reference [4].

4. THE *R*-RATIO

The R-ratio is defined in reference [5] as the ratio of the mean of the largest pair of singular values [8] to the mean of the second largest pair of singular values of the **R** matrix, equation (5). The cumulants in **R** were computed [5] as an average over one second intervals of stationary tool acceleration time series of from 20 to 60 s duration; $K = 20, \ldots, 60$.

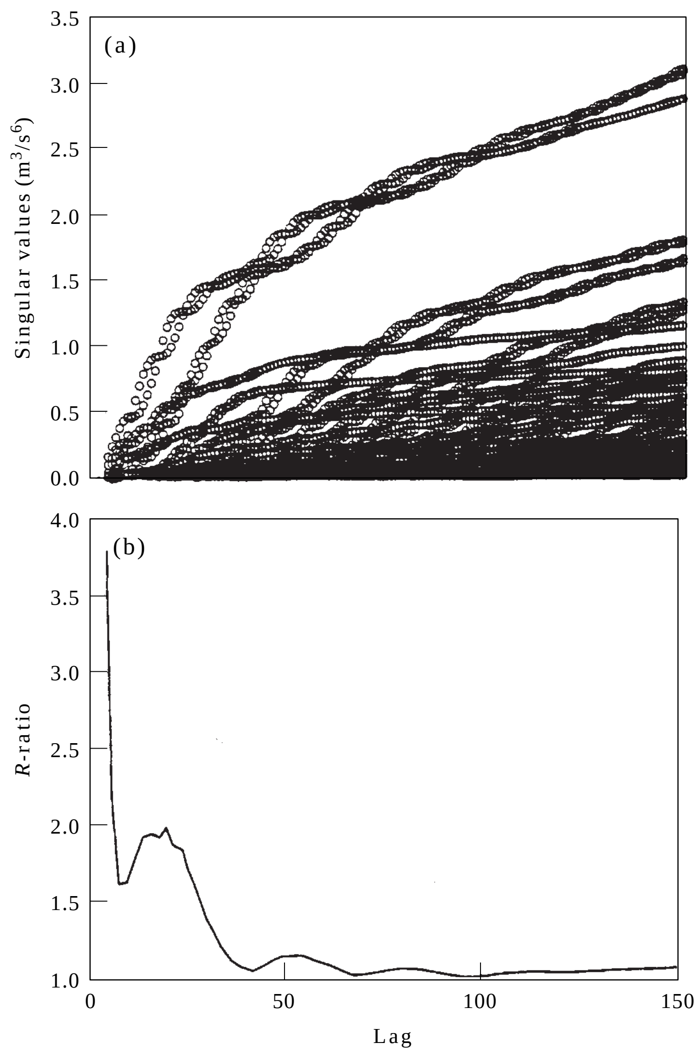

Figure 1. Data set s − 1, 2·5 mm: (a) singular values versus lag; (b) R-ratio versus lag.

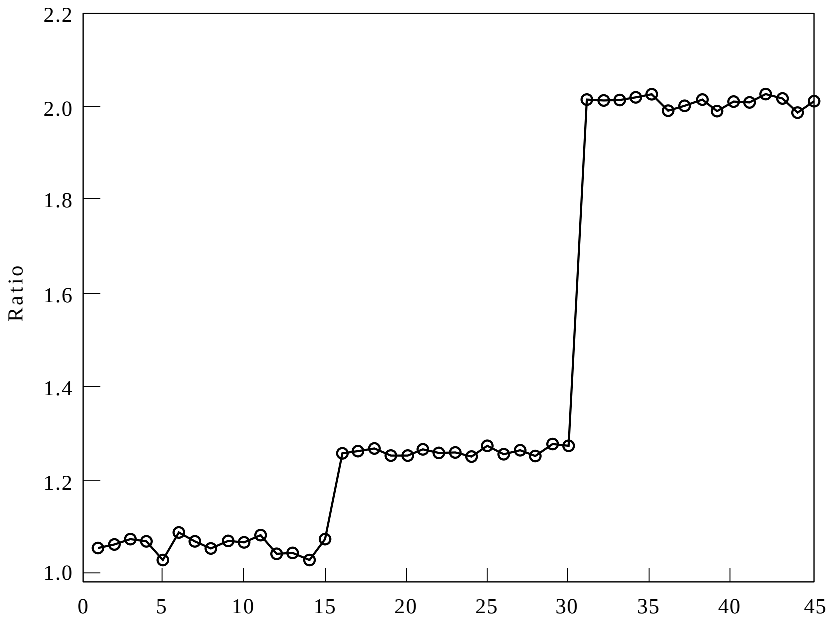

Figure 2. R-ratio versus time, s, for $f(t)$ with 1 s window.

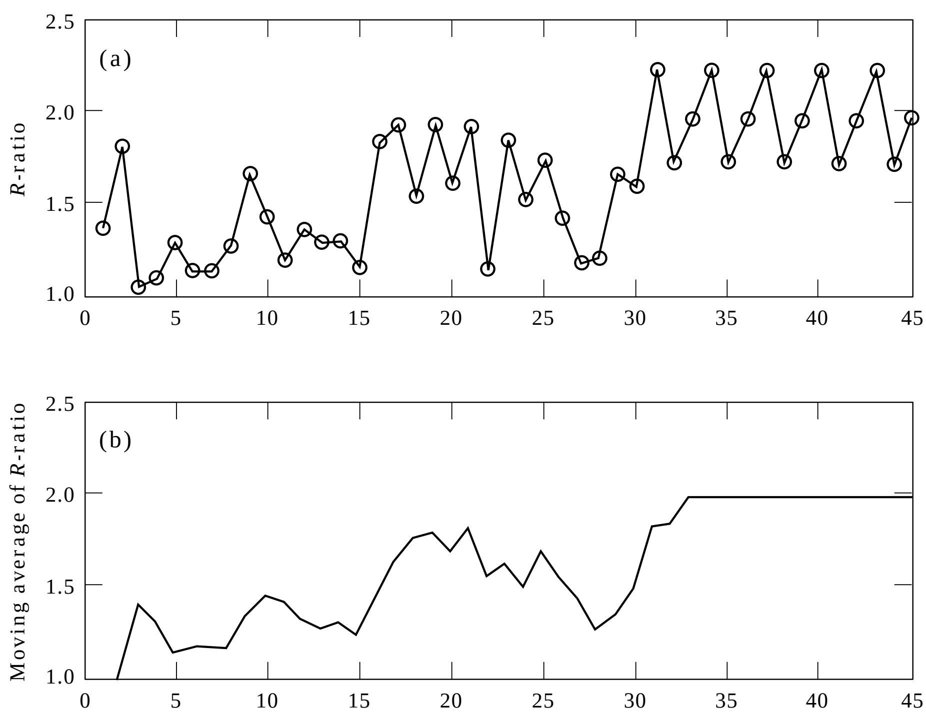

Figure 3. R-ratio versus time, s, for $g(t)$ with 1 s window. (a) unaveraged; (b) moving average.

A typical result [5] is shown in Figure 1 for data sets s − 1 with turning frequency = 460 r.p.m., rake angle = 5°, surface speed = 90 m/min, feed rate = 0·007 in/rev, resampling rate = 1024 Hz, frequency cut-off = 1100 Hz, and depth of cut = 2·5 mm. For this case of light cutting, singular values of **R**, equation (5), and *R*-ratio versus delay are shown in Figures 1(a) and (b), respectively. Four dominant singular values of **R** occur in two pairs which approach each other as the lag increases. The *R*-ratio $\simeq 1$ for lag $\geqslant 50$, which was found to be characteristic of light cutting. It was shown in reference [5] that the *R*-ratio evaluated for lag = 100 approximates one for light cutting, two or more for chatter and near chatter states and takes intermediate values for intermediate states, increasing monotonically from one to two as chatter is approached.

5. SHORT TIME *R*-RATIOS

A numerically generated test function, $f(t)$, ws formed through the concatenation of 15 s intervals of the phase-coupled trigonometric functions $f_i(t)$, equations (6)–(8), where the ϕ_i are mutually independent and uniformly distributed over $[0, 2\pi]$:

$$f_1(t) = 0{\cdot}9\cos(2\pi \cdot 90t + \phi_1) + \cos(2\pi \cdot 100t + \phi_2) + 0{\cdot}2\cos(2\pi \cdot 100t + \phi_1 + \phi_2), \tag{6}$$

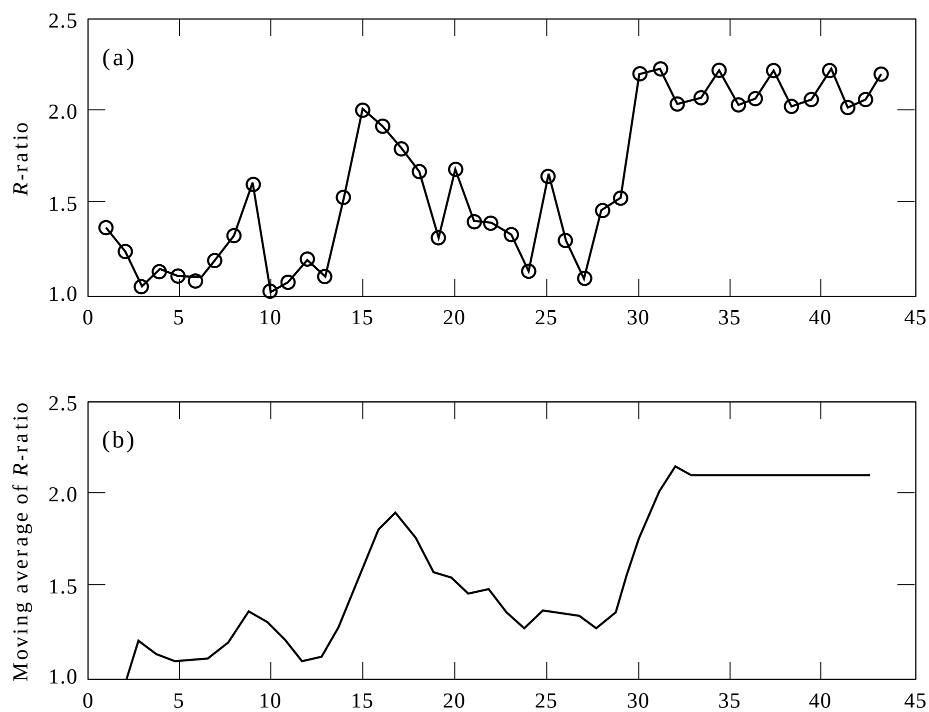

Figure 4. *R*-ratio versus time, s, for $g(t)$ with 2 s window: (a) unaveraged; (b) moving average.

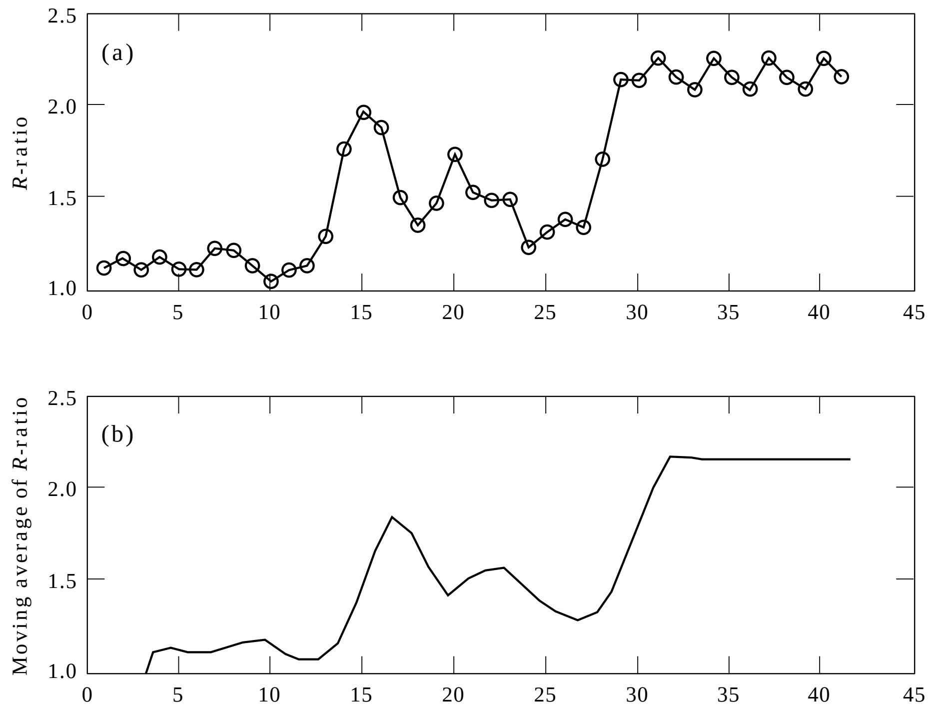

Figure 5. R-ratio versus time, s, for $g(t)$ with 4 s window: (a) unaveraged; (b) moving average.

$$
\begin{aligned}
f_2(t) = {} & \cos(2\pi \cdot 90t + \phi_1) + \cos(2\pi \cdot 100t + \phi_2) \\
& + \cos(2\pi \cdot 190t + \phi_1 + \phi_2) + 0{\cdot}5\cos(2\pi \cdot 100t + \phi_1) \\
& + 0{\cdot}5\cos(2\pi \cdot 110t + \phi_3) \\
& + 0{\cdot}6\cos(2\pi \cdot 210t + \phi_1 + \phi_3), \qquad (7) \\
f_3(t) = {} & \cos(2\pi \cdot 100t + \phi_1) + \cos(2\pi \cdot 100t + \phi_2) \\
& + 0{\cdot}2\cos(2\pi \cdot 200t + \phi_1 + \phi_2), \qquad (8)
\end{aligned}
$$

$f_1(t)$ was sampled at 1024 Hz. R-ratios, computed for 100 lags, averaging cumulants for one second intervals over $0 \leqslant t \leqslant 15$ s, were 1·01, 1·34 and 2·00 for $f_i(t)$, $i = 1, 3$, respectively.

Setting both the window length and the interval length over which the R-ratio is computed equal to 1 s and finding the R-ratio over the window for each second of $f(t)$ gives the R-ratio versus time plot shown in Figure 2. The location of discontinuities between the concatenated $f_i(t)$ functions is correctly found and values of the R-ratios at interior points are in good agreement with those found for the individual $f_i(t)$ through averaging cumulants for 1 s intervals over $0 \leqslant t \leqslant 15$ s. Apparently, noise associated with the random ϕ_i is suppressed by the singular value decomposition.

Calculations identical to the foregoing were carried out for the concatenation, $g(t)$, of three times series, $g_i(t)$, $i = 1, 3$, of cutting tool accelerations associated with the orthogonal cutting of stiff metal cylinders. For $g_i(t)$, the rake angle $= 5^\circ$, feed rate = 0·007 in/rev, surface speed = 90 m/min, resampling rate = 1024 Hz and frequency cutoff = 1100 Hz. $g_1(t)$ and $g_2(t)$ of 15 s duration are associated with light and medium cutting, respectively. $g_3(t)$, of 3 s duration is associated with chatter. The R-ratios computed for $g_i(t)$, $i = 1, 3$, by long time averaging of R-ratios of 1 s intervals over time series of 20 to 40 s duration were 1·0, 1·5, 2·2, respectively.

The R-ratio, computed over the 1 s interval, versus time for a 1 s window, no averaging, is showing in Figure 3(a). The R-ratio for the nth window is assigned an abscissa of n. A moving average of three consecutive values of the R-ratio from Figure 3(a) is shown in Figure 3(b). The moving average is assigned an abscissa corresponding to that of the central point of three averaged values. Three seconds of chatter data have been continued into a 10 s interval through repetition to simplify calculations based on large windows for $t > 30$. Transitions at 15 s and 30 s between light, medium and heavy cutting are clearly indicated both for the averaged and unaveraged cases. The identification of the chatter state, R-ratio $\simeq 2·0$, is made at $t = 31$ s in Figure 3(a) and $t = 33$ s in Figure 3(b).

The results of a calculation identical to the foregoing, except that its basis is a 2 s window, are shown in Figures 4(a) and (b). The R-ratio for the nth two second window is assigned an abscissa of n. This numbering convention shifts the resulting R-ratio versus time plot one second to the left. Values of the R-ratio $\simeq 2·0$, indicating chatter, occur for $t \geqslant 30$ s for the unaveraged case, Figure 4(a), and for $t \geqslant 31$ s for the averaged cases, Figure 4(b). Transitions between light and medium cutting and chatter are evident at 14 s and 29 s, respectively.

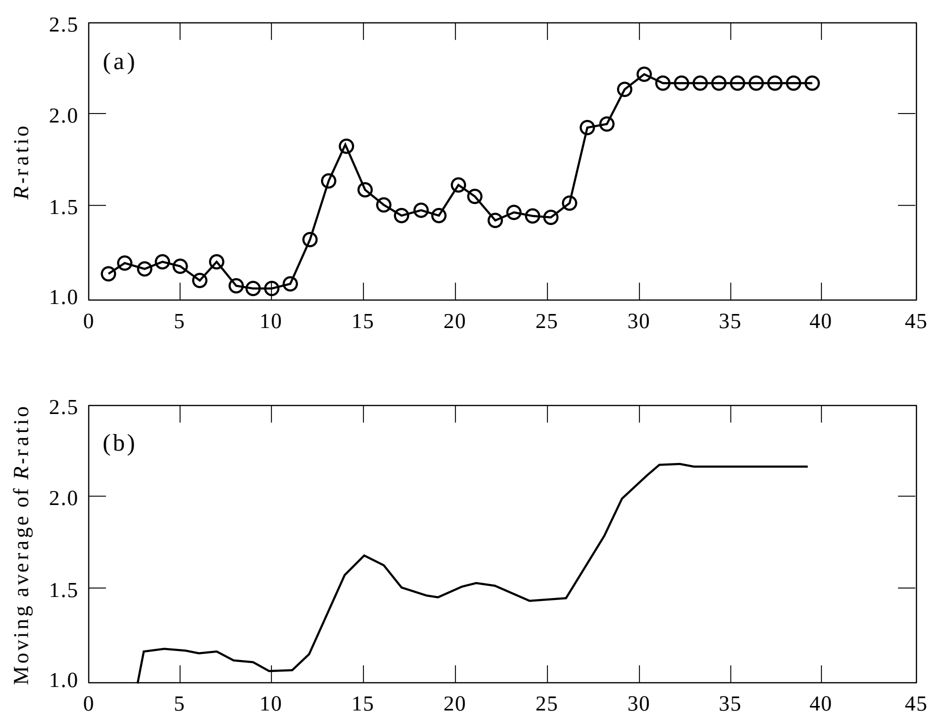

Figure 6. R-ratio versus time, s, for $g(t)$ with 6 s window: (a) unaveraged; (b) moving average.

Results for the 4 s window are shown in Figures 5(a) and (b). The *R*-ratio of the *n*th four second window is assigned an abscissa *n* producing a shift in the resulting *R*-ratio versus time plot of three seconds to the left. As before, the onset of the chatter state and the transition between light and medium cutting are identified.

Solutions for the 6 s window are shown in Figures 6(a) and (b). In this case the *R*-ratio versus time plot is shifted 5 s to the left. The onset of chatter and transitions are identified. Considerable smoothing of the time series is evident. Long time averages over each of the concatenated time series are approximated for windows in positions which only include points of a particular time series.

6. CONCLUSIONS

The *R*-ratio was defined in reference [5] and, for a lag $= 100$, shown to approximate one for light cutting, two or more for chatter or near chatter and intermediate values for intermediate states, increasing from one to two as chatter is approached. Third order cumulants, forming the elements of the **R** matrix were approximated by averages over stationary time series of from 20 s to 60 s duration.

The present study demonstrates that the short-time *R*-ratio effectively characterizes non-stationary cutting time series. Cumulants were approximated by averages over moving windows 1 s to 6 s in width. The resulting short time *R*-ratios detected the transitions between as well as characterized the cutting state.

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