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LETTERS TO THE EDITOR

ACTIVE SUPPRESSION OF THE VIBRATION OF A FLEXIBLE BEAM USING AN EDDY CURRENT SENSOR

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1. INTRODUCTION

Normally, the increasing instability of structural control may result from two conditions. The first one is spill-over of the system due to unlimited structural freedom. A controller with limited freedom cannot be applied in unlimited freedom systems. The second one is the non-collocation [1, 2] of sensor and actuator. Due to spatial limits for a combined sensor and actuator, in a system such as a magnetic suspended bearing, the control force incurs a significant time delay resulting from energy wave propagation with respect to the sensed signal. Several interesting experiments on active vibration damping have been performed in flexible structures using different sensors and actuators [1–5].

By implementing the proposed eddy current sensor, the experiments are carried out by collocated control and non-collocated control in a cantilevered flexible beam. Both systems test the capability and performance of the eddy current sensor and the electromagnetic actuator. The control results can also be used to examine the non-minimum phase problem of this system using the non-collocated arrangement to increase system instability. In order to reduce the controller orders and simplify controller design, a curve fitting method is applied to reduce the flexible beam model into a fourth order transfer function. Then a pole placement method is used to design the PD controller for this test system. Finally, experimental results are obtained to compare the open loop and closed loop characteristics of the flexible beam system.

2. EDDY CURRENT SENSOR

The characteristics of eddy current sensors are applied to produce velocity and deflection feedback simultaneously. The eddy current sensor is composed of a permanent magnet, a copper plate and an electromagnetic coil, as shown in Figure 1. The detector circuit of this eddy current sensor is composed of an LC oscillator to generate a sine wave signal with oscillation frequency determined by L and C, and then amplified by an operational amplifier. Under dynamic conditions, a back-induced voltage can be detected from this electromagnetic coil due to flux variations. The amplitude of output voltage represents the changes of position and velocity. The variation flux through the inductor L shows three different effects on the eddy current sensor.

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(1) The oscillation of inductor L changes flux, and it in turn causes the copper plate to generate an eddy current, and thereby resisting changes in flux. The change of flux resulting from the eddy current reflects back to the inductor coil and causes the operational amplifier output voltage to change. By measurement, this change of flux offers the information of position change of the clamped–free beam.

(2) The permanent magnets on the flexible beam cause flux change due to its position variation, and will induce a voltage change across the inductor L. The change of flux on the inductor L has a close relationship with the position of the permanent magnet on the flexible beam and its velocity of movement.

(3) Theoretically, the flux variation will also cause the change of flux on the electromagnetic coil. However, compared to the former two components, this has only a small effect on the sensor L, and can be ignored.

However, due to the sensing range limit, the eddy current sensor is accurate only within a small gap. In this paper, a 30 mm range variation is tested. The electromagnetic force, in terms of proportional velocity feedback, can be used to construct mechanical dampers with a damping coefficient. The electromagnetic force being proportional to the deflection feedback can be used to adjust the mechanical stiffness. The detector circuit is designed with a capability to detect the system changes. Figure 2 shows different sensor gain resistance (RI value) with respect to the measured output voltage of the operation amplifier and the sensor distance.

3. EXPERIMENTAL SYSTEM DESCRIPTION

Figure 3 shows the mechanical configuration of a cantilever flexible beam system. Table 1 shows the pertinent data of the flexible beam.

Two eddy current sensors, made of 1000 turns of silver wire, are used to monitor the displacement and velocity of the two nodes in the transverse direction. Sensor 1 is applied for a collocated control, while Sensor 2 is applied for a non-collocated control. The electromagnetic actuator is made of an iron cored electromagnetic coil, fabricated with 7600 turns of 0·8 mm copper wire. As shown in Figure 3, the electromagnetic actuator located at the end of flexible beam is used as a collocated control. As shown in Figure 3, two permanent magnets are mounted on an

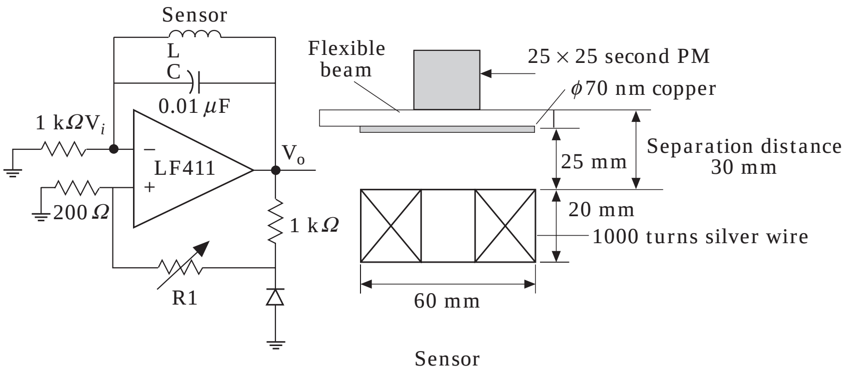

Figure 1. The eddy current sensor and circuit.

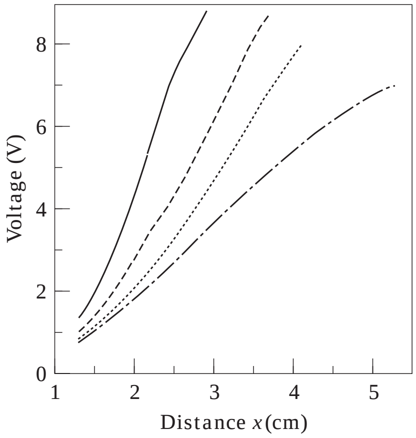

Figure 2. Relationship of sensor gain resistance (R1 value) to measured output voltage: Key for resistance (kΩ): ——, 4·7; - - - - , 4·5; · · ·, 4·2; — · — · — , 4.

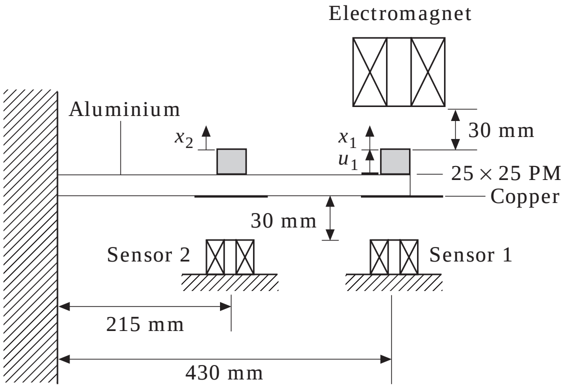

Figure 3. A cantilever flexible beam configuration.

aluminium flexible beam with distances to the fixed point of 215 mm and 430 mm. In this cantilever configuration of the flexible beam, these two permanent magnets are 25 mm in diameter and length, and a 70 mm diameter copper plate is placed beneath the permanent magnet to induce eddy current.

4. DYNAMIC MODEL OF THE SYSTEM

For controller design, a real model construction and measurement of the proposed cantilever flexible beam system is required. Using fast Fourier transform

TABLE 1

Main data of the flexible beam

Length (mm)	Thickness (mm)	Width (mm)	E (Pa)	Density (km/m^2)
430	5	24	2×10^{11}	$7{\cdot}8 \times 10^3$

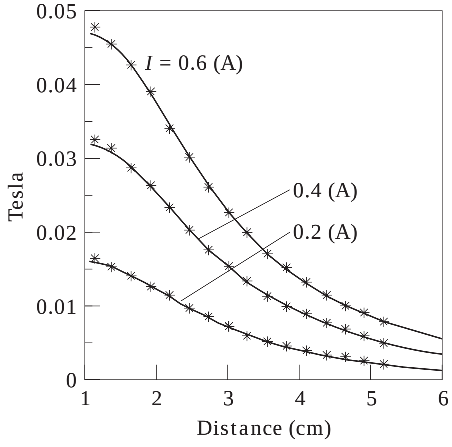

Figure 4. The fitted curves of magnetic field.

(FFT) to process and analyze some 2000 test data, a frequency response characteristics of both collocated and non-collated control can be obtained to design the cantilever flexible beam system. It is obvious that the cantilever flexible beam has its first natural frequency of about 12 Hz, and its second natural frequency of about 23 Hz. The curve fitting method is applied to obtain a fourth order approximate transfer function to represent this test system.

$$x_1/u_1 = 83/(s^2 + 4{\cdot}7s + 5500)/ + 308/(s^2 + 2{\cdot}44s + 20\,900), \tag{1}$$

$$x_2/u_1 = 50{\cdot}8/(s^2 + 3{\cdot}75s + 5800) + 250/(s^2 + 1{\cdot}4s + 22\,000), \tag{2}$$

where u_1 is the input force (N) from the end of the flexible beam, x_1 and x_2 (mm) represent the displacements of the flexible beam at the end and at the middle, where collocated and non-collocated sensors are correspondingly applied.

5. ELECTROMAGNETIC ACTUATORS

The applied force F on the permanent magnet and the electromagnet contains two components [6], one is the acting force from the electromagnetic coil to the permanent magnet, F_1, and the other is the attraction force from the permanent magnet to the iron core, F_2. From the electromagnetic field measurement of Figure 4, the acting force F_1 can be calculated proportional to the coil current. The following assumption is made:

$$F(x, I) = F_1(x, I) + F_2(x) = I/(c_1 + c_2x^2) + 1/(c_3 + c_4x^2 + c_5x^4), \tag{3}$$

From model identification tests, the force model of the active suspension system can be identified as:

$$F(x, I) = F_1(x, I) + F_2(x) = 100I/(1 + 1{\cdot}1x^2) + 50x/(-1 - 3x^2 + 44x^4), \tag{4}$$

where F is the applied force in newtons, I is the coil current in amperes, and x is the separation distance of electromagnetic actuator to the permanent magnet

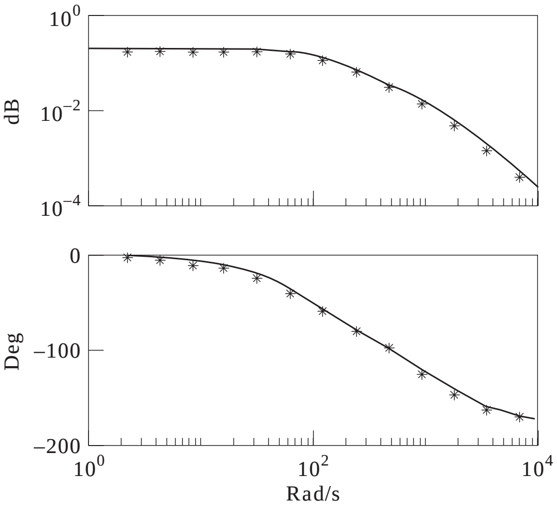

Figure 5. Frequency response of actuator.

in cm. By linearization, the force variation around the operating point can be formulated as:

$$F(x, I) = F(x_0, I_0) + \frac{100}{1 + 1{\cdot}1x_0^2}(I - I_0) + \left[\frac{-220I_0x_0}{(1 + 1{\cdot}1x_0^2)^2} + \frac{50(-1 + 3x_0^2 - 132x_0^4)}{(-1 - 3x_0^2 + 44x_0^4)^2}\right](x - x_0) \tag{5}$$

where $F(x_0, I_0)$ is the magnetic force resulting from the driving current I_0, with the permanent magnet at an equilibrium point X_0.

6. DYNAMIC CHARACTERISTIC OF THE MAGNETIC COIL

After the force model has been established, a frequency analyzer is used to examine the frequency spectrum of the magnetic actuator from the magnetic coil

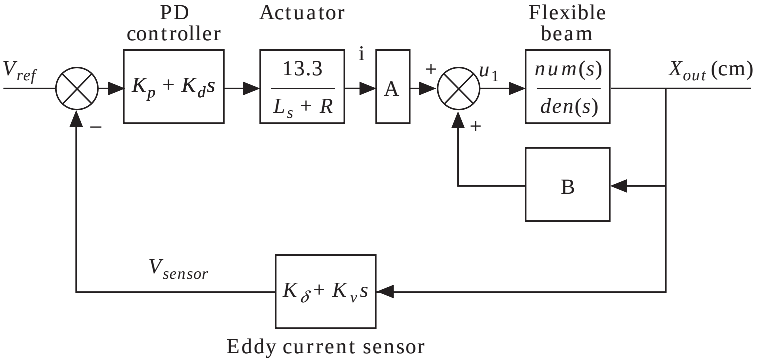

Figure 6. Block diagram of linearized feedback control.

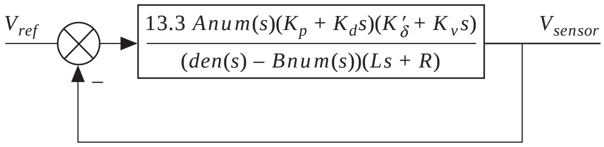

Figure 7. The unit feedback block diagram.

power supply. The frequency response of the actuator is shown in Figure 5. It can be found that the dominant pole is located at 1400 rad/s with 0·01 s time delay.

$$I(s)/E(s) = \frac{0{\cdot}2}{[(s/1400) + 1]}\,\mathrm{e}^{0{\cdot}01s} \approx 0{\cdot}2/(0{\cdot}01s + 1). \tag{6}$$

7. CONTROLLER DESIGN

The linearized feedback control scheme of the cantilever flexible beam system is shown in Figure 6. The parameters shown in this figure are measured from the readings of the experiment and the curve fitting method.

The overall transfer function of the cantilever flexible beam is:

$$\frac{X_{out}(s)}{V_{ref}(s)} = \frac{13{\cdot}3\mathrm{Annum}(s)(K_p + K_d s)}{(Ls + R)(\mathrm{den}(s) - \mathrm{Bnum}(s)) + 13{\cdot}3\mathrm{Anum}(s)(K_\delta + K_v s)(K_p + K_d s)}, \tag{7}$$

where K_p and K_d are controller gains of the PD controller.

Regarding the control problem of the proposed flexible beam system, an appropriate distance from sensor coil (L) and electromagnetic actuator to permanent magnet should be selected to obtain proper velocity feedback, which might maintain a better damping effect for system control. In the experiment system design, the range of data selected is taken between 20 and 40 mm for the permanent magnet to electromagnetic actuator, and 15–35 mm for the eddy current copper plate to the permanent magnet, as shown in Figure 3. From experiments, the sensor gain resistance R1 is selected as 4·5 Ω as shown in Figure 2.

When the load is located at the equilibrium point, by substituting gap distance and current data, e.g., 30 mm and 0 A, into equation (5), one can find the operating condition for this flexible beam system. For small variations in current and position, one obtains the position feedback gain value to be 0·04, and the current feedback gain to be 9·18. The transfer functions of the collocated and non-collocated control for the flexible beam are shown in equations (1) and (2). According to the pole placement method [7], the collocated control roots are chosen at -3137, $-46{\cdot}4$, $-22{\cdot}1$, $-2.9 \pm 93{\cdot}6i$, and the non-collocated control roots are chosen at -3532, $-47{\cdot}5$, $-16{\cdot}7$, $-2{\cdot}3 \pm 92{\cdot}5i$. Therefore, the collocated PD controller can be determined as $K_p = 15{\cdot}5$, $K_d = 0{\cdot}31$ and the sensor gain as $K_\delta = 3{\cdot}2$, $K_v = 0{\cdot}21$; while the non-collocated PD controller is determined as $K_p = 15{\cdot}5$, $K_d = 0{\cdot}31$, and the sensor gain as $K_\delta = 3{\cdot}2$, $K_v = 0{\cdot}31$.

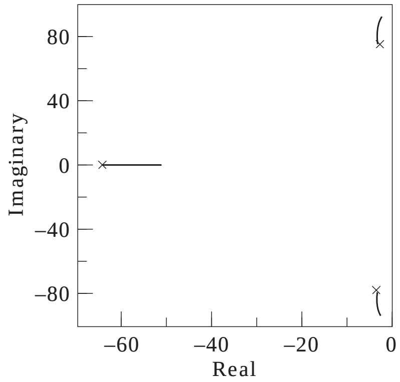

Figure 8. The root-locus without velocity feedback.

8. EFFECT OF SENSOR ZERO

The block diagram of Figure 6 can be simplified into a unit feedback block diagram as shown in Figure 7. Since the conventional position sensor cannot measure and identify system velocity, say $K_v = 0$, the zeros of the PD controller have to be chosen to the right of the actuator poles for a stable control. The root locus will appear similar to Figure 8. However, since the eddy current sensor can detect the velocity, the open loop transfer function has one additional zero. If the controller zero and the sensor zero can be located to the right of the actuator pole, then the root locus becomes similar to Figure 9. This implies that the cantilevered flexible beam system has greatly been improved with better stiffness and damping characteristics. The control result suggests that this cantilevered flexible beam system can be operated in a stable condition.

9. EXPERIMENTAL RESULTS

In the collocated experiment, a step response of open loop and closed-loop performance at the end of flexible beam is shown in Figure 10. In the non-collocated experiments, the sensor is located in the middle of the cantilever

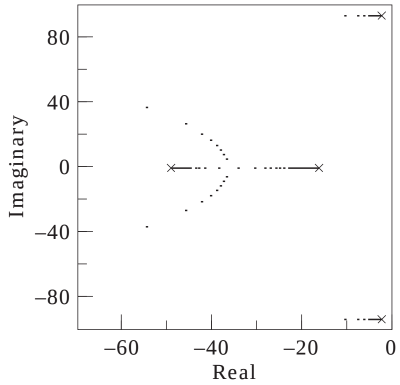

Figure 9. The root-locus with velocity feedback.

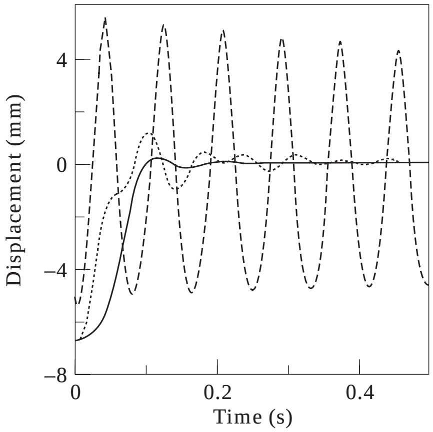

Figure 10. Collocated step response. Key: - - - - , open loop; ——, closed loop; · · · , simulation.

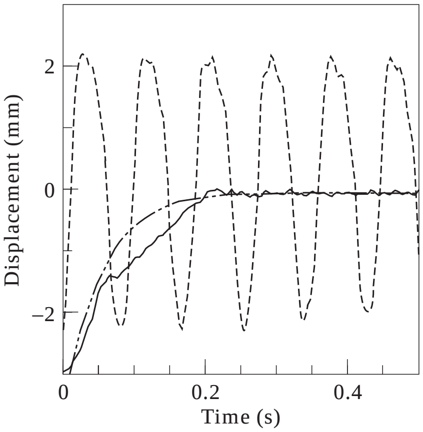

Figure 11. Non-collocated step response. Key as for Figure 10.

flexible beam, and the actuator is located at its end. The step response of the open loop and closed-loop control at the middle of the beam is shown in Figure 11. Comparing these two control results, one finds that the non-collocated control has a longer rise time and longer settling time. Since the actuator can offer a large damping force, both the collocated or non-collocated control can suppress system overshoot, and offer excellent vibration damping.

10. CONCLUSION

In this paper, an active vibration damper for a cantilevered flexible beam has been designed and implemented. This system uses an eddy current sensor to feedback both position and velocity signals into the control loop. Although the beam model is truncated into a fourth order transfer function, which incorporates some approximations, the control implementation and result are a welcome improvement to this study. Thanks to a successful velocity measurement using the proposed eddy current sensor, only a simple PD controller is able to achieve a remarkable operation in collocated and non-collocated control.

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