Journal of Sound and Vibration (1998) **217**(2), 396–404
Article No. sv981789

OPTIMUM PLACEMENT OF BOLTS IN STRUCTURES BASED ON DYNAMIC SHEAR

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(*Received* 16 *February* 1998, *and in final form* 19 *June* 1998)

1. INTRODUCTION

Assemblies are often subjected to dynamic environments including vibration, shock, and/or thermal cycling. Such conditions can lead to fastener loosening and result in increased maintenance and failure. Previous work shows that transverse or shear loading of threaded products provides the most severe environment for loosening [1]. As a result, joints oriented such that fasteners are parallel to the direction of loading provide better resistance to dynamic failure. However, there are situations where this orientation cannot be achieved or, more often, loading is multi-directional.

Today, there exist numerous techniques for minimizing fastener loosening. Fasteners with locking features can be divided into four general groups: (1) free running pre-load independent locking fasteners, (2) free running pre-load dependent locking fasteners, (3) prevailing torque locking fasteners, and (4) chemical locking. In addition, there are a number of assembly design considerations which can lead to improved resistance to vibration-induced loosening. These include fastener orientation and joint shape details which minimize fastener movement, slip, and/or shear stress levels. A recent review [1] on vibration and shock induced loosening examines the existing literature, locking products, design considerations, and testing standards.

Kerley [2, 3] applied the principles of retroduction to the problem of vibration-induced loosening. He proposed that shear force calculations could be used to quantify loosening and specify fastener placement. His experiments were performed with an inertial loaded compound cantilever beam with one fastener. Since the focus of his work was to demonstrate the application of retroduction, no data was presented which quantified shear force levels that cause loosening for given fastener pre-loads. Also, the fastener position was not varied in any of the experiments.

The compound beam assembly apparatus was originally used by Haviland [4, 5] with direct loading to demonstrate loosening. This apparatus provides a realistic vibration test because it represents the most common type of structure that causes shear loading on a fastener. It includes joints and panels of most major buildings, aircraft, cars, homes, and household appliances. In addition, tests with an inertial loaded compound beam assembly apparatus provide more realistic and repeatable results than the more severe existing standard tests, such as MIL-STD-1312-7A [6] and NAS 1675 [7], which generate shock and impact loads.

0022–460X/98/420396 + 09 $30.00/0

This paper reports on experiments and analyses that were performed to investigate the effect of dynamic shear force on fastener loosening in assemblies. This work was initiated on the general premise that dynamic shear force calculations could be used to optimize fastener placement and orientation. This work is novel since placement and orientation of fasteners in assemblies is generally based on convenience or static load and strength considerations. The placement of fasteners with respect to vibratory mode shapes of a structure was also expected to be important since the vibration level and associated shear forces vary with position. This is an area of ongoing research [8], the goals of which are to develop general design guidelines and criteria which minimize maintenance and failure due to fastener loosening.

2. MECHANISM OF LOOSENING

At this point, it is worthwhile to briefly describe the mechanism of loosening from fluctuating shear loading. Figure 1 shows a cross-section of a bolt and nut assembly [4]. When a shear force acts on the clamped components or threaded products, the bolt and nut will loosen. To illustrate, imagine pushing and pulling the nut. As the nut is moved into the page, the right side tends to loosen and the left side tends to tighten. Since the left side will move with greater difficulty, it acts as a pivot point about which the nut rotates loose. When the nut is pulled from the page, the right side becomes the pivot about which the nut rotates loose. The net effect each time the nut is cycled sideways is a ratcheting loosening motion.

3. DYNAMIC SHEAR FORCE

Before looking at data from tests with the compound cantilever assembly, it is useful to review some simple mechanics and examine the shear force and the shear stress in a cantilever beam. Consider first a cantilever beam with a constant load P. The lateral load P creates internal shear forces and bending moments which result in deflection of the beam. Assuming an Euler–Bernoulli beam, the deflection of the beam is defined as

$$y(x) = -Px^2(3L - x)/6EI, \tag{1}$$

where L is the length of the beam, x defines the position along the beam, E is the modulus of elasticity, and I is the moment of inertia. From equilibrium

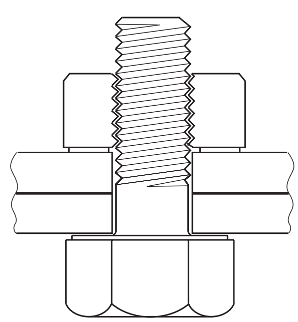

Figure 1. Cross-section of bolt and nut assembly.

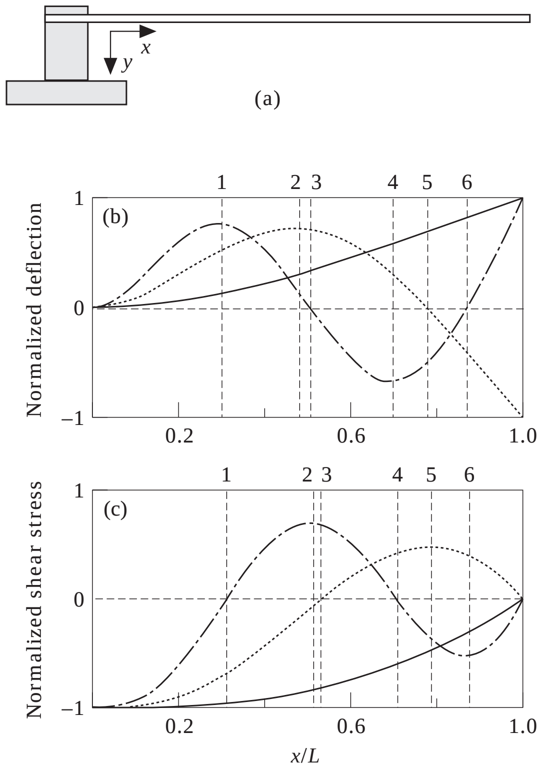

Figure 2. (a) A cantilever beam under inertial loading, (b) normalized deflection, and (c) normalized maximum shear stress for first three modes of vibration: ——, mode 1; · · · · , mode 2; –·–, mode 3.

considerations, the resulting shear force V in the beam is found to be constant and equal to P across the length of the beam. The distribution of shear stress τ for a cross-section of a uniform beam is parabolic, with a maximum value $\tau_{max} = 3V/2bh$ occurring at the neutral axis, where b is the width and h is the thickness of the beam. For an infinitesimal beam element to remain in equilibrium, the shear stress must act on all four surfaces of the element in the xy-plane, and the horizontal shear stress τ_{yx} must equal the vertical shear stress τ_{xy}.

Next, consider a cantilever beam subjected to inertial loading as shown in Figure 2(a). Assuming an Euler–Bernoulli beam, the flexural vibration is defined as

$$y(t, x) = \sum_{n=1}^{\infty} [A_n \sin(\omega_n t) + B_n \cos(\omega_n t)] X_n(x), \tag{2}$$

where ω_n are natural frequencies, $X_n(x)$ are mode shapes $= c_n\{\cosh(\beta_n x) - \cos(\beta_n x) - \sigma_n[\sinh(\beta_n x) - \sin(\beta_n x)]\}$, $\beta_n = (\rho bh/EI)^{1/4}(\omega_n)^{1/2}$, $\sigma_n = [\sinh(\beta_n L) - \sin(\beta_n L)]/[\cosh(\beta_n L) + \cos(\beta_n L)]$; A_n, B_n, c_n are constants; ρ is the density of the beam.

The deflection of the beam for the nth mode of vibration is

$$y_n(t, x) = c_1 \sin(\omega_n t + \phi)\{(\cosh(\beta_n x) - \cos(\beta_n x) - \sigma_n[\sinh(\beta_n x) - \sin(\beta_n x)]\}, \quad (3)$$

where

$$c_1 = c_n\sqrt{A_n^2 + B_n^2}, \qquad \phi = \operatorname{atan}(B_n/A_n). \quad (4)$$

The shear force is defined as

$$V(t, x) = -(\partial^3 y(t, x)/\partial x^3)EI. \quad (5)$$

The maximum shear stress at the neutral axis of the beam is

$$\tau_{max}(t, x) = 3V(t, x)/2bh. \quad (6)$$

Differentiating equation (3) three times with respect to x and substituting into equations (5) and (6), the maximum shear stress for the nth mode of vibration can be expressed as

$$\tau_{max}(t, x) = c_2 \sin(\omega_n t + \phi)\{\sinh(\beta_n x) - \sin(\beta_n x) - \sigma_n[\cosh(\beta_n x) + \cos(\beta_n x)]\}, \quad (7)$$

where

$$c_2 = -3c_1 EI\beta_n^3/2bh.$$

The normalized deflection and maximum shear stress for the first three vibration modes of the beam have been calculated and are shown in Figure 2. The vertical lines indicate the nodes and antinodes of the second and third modes. For the first mode of vibration the shear stress is largest at the fixed end and decreases to zero at the free end. For the second and third modes, the shear stress peaks at the nodes and is zero near the antinodes.

This simple analysis clearly shows how shear stress varies with position for an Euler–Bernoulli cantilever beam. In the next section, data from experiments on the effect of placement of fasteners on a compound beam will be presented. Even though the variation in stress in a compound cantilever beam assembly is somewhat different than the Euler–Bernoulli beam examined here, this simple analysis serves to illustrate that shear stress peaks at nodes and essentially vanishes near anti-nodes. This suggests that, for a particular mode of vibration, fasteners placed near nodes in a structure will experience larger shear forces and will loosen more readily than fasteners placed near anti-nodes.

4. EXPERIMENTS

The apparatus used in this work consists of a compound beam with one fastener as shown in Figure 3. The compound cantilever beam consists of two pieces of 316 stainless steel which are 330·2 mm long, 25·4 mm wide and 1·59 mm thick. The combined mass of the two beams is 0·25 kg. The fixed end of the beam is attached to an aluminum test fixture with four 10-32 UNF-3A socket heat cap screws with a torque of 5·7 Nm. The length of the cantilever is 304·8 mm because 25·4 mm of

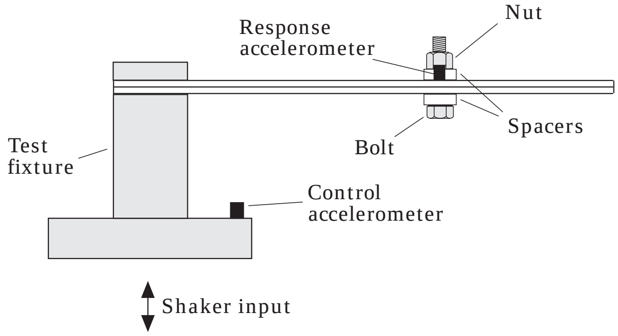

Figure 3. Test apparatus.

the steel pieces is secured in the fixture. The fixture is attached on a shaker with four 10-32 UNF-3A socket head cap screws with a torque of 8·5 Nm.

The fastener consists of a 25·4 mm long, grade 5, 0·25-20 UNC-2A hex head carbon steel bolt with a mating 0·25-20 UNC-2B nut, and two 316 stainless steel spacers. Each spacer has a 6·35 mm inside diameter, a 12·7 mm outside diameter and is 4·76 mm thick. The combined mass of the bolt, nut and spacers is 0·018 kg, which is 7% of the mass of the compound beam.

Prior to testing, the fastener components and the compound beam are cleaned with acetone. WD-40 lubricant is applied to the fastener components and in between the two pieces of the beam. The lubricant is used to reduce galling and variations in friction, and to obtain more repeatable data. Use of lubricant with threaded product provides more consistency between torque and pre-load.

The dynamics of the beam without the fastener is assessed using an impact hammer test. The first three natural frequencies of the beam are found to be 15, 109 and 295 Hz, respectively. The positions of the nodes and antinodes for the first three modes of vibration of the beam are summarized in Table 1.

Six 6·35 mm diameter holes are drilled at each of the six positions listed in Table 1. Since the beam dynamics will change when the fastener is added and the assembly dynamics will vary as the fastener is moved from one position to another, the first three natural frequencies of the assembly are determined for each fastener position. The fastener is tightened, using a dial type torque wrench, to 3·4 Nm for each position, and the first three natural frequencies of the assembly are determined from impact hammer tests. The average value and range of variation

TABLE 1

Fastener positions on beam assembly

Position	1	2	3	4	5	6
Distance from fixed end (mm)	66·8	107·2	149·4	206·5	223·8	265·2
Dynamic property	anti-node (mode 3)	anti-node (mode 2)	node (mode 3)	anti-node (mode 3)	node (mode 2)	node (mode 3)

TABLE 2
Test data for first mode of vibration (2 $\boldsymbol{g}$ *at* 20 Hz)

Fastener position	Tightening torque (Nm)	Completely loosened	Time to loosen (s)	Breakaway torque (Nm)	Response acceleration ($\boldsymbol{g}$)
2	1·1	yes	<2	–	
	2·3	yes	<2	–	4·2 ± 1·1
	3·4	yes	5 ± 2	–	
	4·5	no	–	3·6 ± 0·2	
4	1·1	yes	<2	–	
	2·3	yes	<2	–	7·3 ± 1·6
	3·4	yes	5 ± 1	–	
	4·5	no	–	3·4 ± 0·0	
6	1·1	yes	<2	–	
	2·3	yes	<2	–	8·2 ± 2·0
	3·4	yes	6 ± 3	–	
	4·5	no	–	3·7 ± 0·1	

of the first three natural frequencies of the assembly are 20 ± 1, 120 ± 9, and 292 ± 10 Hz, respectively. In an effort to excite primarily one mode of vibration at a time, the average values for the natural frequencies are used to drive the assembly in the tests, regardless of the fastener position.

The tests consist of tightening the fastener at a particular position to a torque of 1·1 Nm, vibrating the assembly for up to one min, recording either the time to completely loosen or the breakaway torque after one min, and recording the acceleration amplitude of the beam near the fastener prior to loosening. If the fastener completely loosens, the tightening torque is increased by 1·1 Nm and the test is repeated. This process is repeated until the fastener does not completely loosen within 60 s. Then, the fastener is moved to another position and the process is repeated until all six positions, or at least all positions near the assembly nodes and antinodes, have been tested. All of these tests are performed for 20, 120 and 292 Hz sinusoidal vibration inputs. All tests are performed for three different bolt and nut test specimens, and all tests are performed twice.

A control accelerometer mounted on the test fixture is used to control the vibration level input to the assembly. A low vibration level of 2 $\boldsymbol{g}$ is used for the 20 Hz sine wave tests to avoid excessive beam deflection. A higher 30 $\boldsymbol{g}$ input is used for the 120 Hz and 292 Hz sine wave tests. A miniature 0·001 kg accelerometer is used to measure the acceleration of the beam near the fastener position.

The maximum tightening torque used in the testing is 6·8 Nm. This corresponds to a pre-load of about 6008 N as measured using a load cell, and is equivalent to about 46% of the fastener yield strength. For the dynamic test conditions used in these experiments, this level of pre-load is sufficient to prevent loosening for all cases (see Tables 2–4).

Complete fastener loosening is easily detected both visually and audibly. When complete loosening occurs, the time to loosen is recorded. When complete loosening does not occur after 60 s exposure to vibration, the breakaway torque is measured with a dial type torque wrench on the nut.

The test data is summarized in Tables 2–4. Average values and range of variation of time to completely loosen, breakaway torque, and beam acceleration near the fastener are listed. Table 2 summarizes the data from all tests with a 20 Hz vibration input. This driving frequency excites predominately the first mode of vibration of the assembly because it is near the first natural frequency of the assembly, regardless of the fastener position. Complete loosening occurs within several seconds for all cases for a tightening torque of 3·4 Nm or less. Complete loosening does not occur for any case for a tightening torque of 4·5 Nm.

The break-away torque in threaded fasteners is usually about 70% of the tightening torque without any loosening, due to the mechanics of threaded fasteners. The breakaway torque measurements in this work were found to be within 51–92% of the tightening torque. This variation is due to limitations in torque measurement, especially for torque values less than 1·1 Nm, as well as partial dynamic loosening or tightening.

Table 3 summarizes the data from all tests with a 120 Hz vibration input. The response acceleration is minimum when the fastener is at position 5. This is expected since position 5 is near a nodal line in the beam assembly. The data in Table 3 show that complete fastener loosening occurs at position 5 for tightening torques as high as 5·7 Nm. However, when the fastener is at position 2, 3 or 4,

TABLE 3

Test data for second mode of vibration (30 **g** *at* 120 Hz)

Fastener position	Tightening torque (Nm)	Completely loosened	Time to loosen (s)	Breakaway torque (Nm)	Response acceleration (**g**)
1	1·1	yes	<2	–	
	2·3	yes	<2	–	20·5 ± 3·0
	3·4	no	–	2·3 ± 0·2	
2	1·1	no	–	0·7 ± 0·1	36·5 ± 3·5
3	1·1	no	–	0·7 ± 0·1	30·7 ± 5·4
4	1·1	no	–	0·7 ± 0·1	40·4 ± 3·2
5	1·1	yes	<2	–	
	2·3	yes	<2	–	
	3·4	yes	<2	–	14·8 ± 1·0
	4·5	yes	5 ± 2	–	
	5·7	yes	5 ± 1	–	
	6·8	no	–	6·0 ± 0·2	
6	1·1	yes	<2	–	
	2·3	yes	<2	–	
	3·4	yes	<2	–	40·0 ± 15·2
	4·5	yes	<2	–	
	5·7	no	–	4·5 ± 0·0	

TABLE 4

Test data for third mode of vibration (30 g *at* 292 Hz)

Fastener position	Tightening torque (Nm)	Completely loosened	Time to loosen (s)	Breakaway torque (Nm)	Response acceleration (g)
1	1·1	no	–	0·7 ± 0·1	53·3 ± 1·9
3	1·1	yes	<2	–	
	2·3	yes	<2	–	1·5 ± 1·1
	3·4	no	–	2·6 ± 0·1	
4	1·1	no	–	0·7 ± 0·1	60·2 ± 1·6
6	1·1	yes	<2	–	
	2·3	yes	<2	–	28·3 ± 4·2
	3·4	no	–	2·6 ± 0·1	

complete loosening does not occur at the lowest tightening torque of 1·1 Nm. Positions 2 and 3 are near an antinode in the beam.

Table 4 summarizes the data from all tests driven at 292 Hz near the third mode of vibration. Data is presented with the fastener at positions 1, 3, 4 and 6. Positions 1 and 4 are near antinodal lines and positions 3 and 6 are near nodal lines. As before, fastener integrity is maintained at much lower tightening torques when the fastener is positioned near an antinode, compared to when the fastener is positioned near a node.

In addition, tests were performed with 316 stainless steel, 31·8 mm long, 0·25-20 UNC-2A hex head bolts with mating 0·25-20 UNC 2B nuts using a highly refined paraffinic oil for lubricant. Data from these tests showed the same trends.

Response acceleration measurements could be used to compute estimates of beam deflection and shear force acting on the fastener. Optimum placement could then be determined from estimates of shear. However, identification of vibration modes and associated nodes provides an indirect method for assessing shear in assemblies and optimum fastener placement to minimize failure by loosening.

5. CONCLUSIONS

Although shear loading of threaded fasteners is known to be the most severe environment for loosening, guidelines for placement of fasteners based on dynamic shear forces have not been explored. This paper examines this issue using an inertial loaded cantilever beam.

Calculations for an Euler–Bernoulli cantilever beam show that the dynamic shear stress is maximum near the nodes, and essentially vanishes near the anti-nodes. Experiments with a compound cantilever beam assembly with one fastener, support this simple analysis and demonstrate that loosening occurs more readily when the fastener is placed near a nodal line where shear forces are maximum.

The data shows that fastener integrity is maintained for longer periods of time and with lower tightening torques when the bolt and nut are positioned away from nodal lines where shear stresses are lower, even though acceleration levels are higher. Although further research is needed, this work is expected to lead to design guidelines for fastener placement in assemblies which minimize maintenance and loosening type failure.

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APPENDIX A: NOMENCLATURE

A_n	constants
b	width of beam
B_n	constants
c_n	constants
c_1	constants
c_2	$-3c_1 EI\beta_n^3/2bh$
E	modulus of elasticity
$\mathbf{g}$	acceleration due to gravity (9·81 m/s^2)
h	thickness of beam
I	moment of inertia
L	length of beam
P	constant load
t	time
V	shear force
x	defines position along beam
X_n	mode shapes
y	deflection of beam
y_n	deflection of beam for the nth mode of vibration
β_n	$(\rho bh/EI)^{1/4}(\omega_n)^{1/2}$
ϕ	atan (B_n/A_n)
ρ	density of beam
σ_n	$[\sinh(\beta_n L) - \sin(\beta_n L)]/[\cosh(\beta_n L) + \cos(\beta_n L)]$
τ_{max}	maximum shear stress
τ_{xy}, τ_{yx}	shear stress
ω_n	natural frequencies