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EFFECT OF FLEXIBILITY ON LOW VELOCITY IMPACT RESPONSE

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The nature of the response of compact and flexible bodies subject to low velocity impact is studied. The key parameters which govern the response for such impacts are identified and their effects on impact response are examined through numerical simulations. It is shown that three non-dimensional parameters are sufficient to completely govern the low velocity impact response of such structures. By knowing the impact conditions the nature of the response as well as the maximum impact force can be predicted without running a simulation or conducting an experiment. It is also shown that the impact response of different structures with arbitrary boundary conditions can be scaled in a similar way. It is anticipated that the results will be useful in designing for impact loading, in providing guidelines for choosing appropriate simple models and in scaling experimental results between different structures.

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1. INTRODUCTION

Impact is a fairly complex event which has been studied extensively. From a materials point of view, research has been concentrated on the effects of the large forces generated during impact on material behavior and damage which in turn affects the load carrying capacity of the structure. For metals, damage involves plastic deformation and wear in the contact zone [1]. For fiber reinforced composite materials, damage is in the form of fiber failure, matrix cracking and delamination [2]. From a dynamic analysis point of view, the interest has been focussed on the response of the system after impact and ways to control it. In both areas, an accurate prediction of the impact forces and structural deflections is essential.

The character of impact between flexible bodies has long been known to differ from that of impact between compact bodies [3]. Impact models for these systems have traditionally used Hertzian type contact laws as an input to the dynamic analysis [4]. In cases where the local damage is significant, elastic–plastic contact laws have been shown to yield more realistic results [5]. Alternative impact models such as spring–dashpot and the momentum balance methods utilize the coefficient of restitution as an input to the dynamic analysis to characterize the local energy loss due to impact [6–8]. It has been shown that both models are essentially similar to the elastic–plastic contact law provided that an adequate coefficient of

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restitution is used [9, 10]. The effect of flexibility is in reducing the severity of impact at the contact zone. This is due to the fact that part of the impact energy is transferred to the flexible body in the form of structural vibrations.

Since a rigorous solution of impact problems is generally quite complex, approximate methods which omit one or more basic characteristics of the impact phenomenon are invariably employed [11]. For example, in many impact situations involving flexible bodies, it is desirable to use computationally efficient models since the use of the impact models mentioned above can be computationally extensive depending on the nature of impact response [12]. Furthermore, a simplified model may produce substantially incorrect results though for certian cases it may be adequate [13]. Therefore, the type of impact response must be known *a priori* without running a full simulation in order to choose an adequate impact model. In addition, predicting the response correctly is also very useful in designing experimental set-ups and planning the experimental studies dealing with impact of flexible bodies.

Depending on the characteristics of the impacting bodies, the impact event may be localized in nature with no structural response, or global: i.e., although the impact forces are local the response is mostly structural. It is of interest to identify key parameters which affect the response and consequently be able to characterize the type of impact response based on these parameters. There have been some traditional rule of thumb approaches regarding some extreme cases. For example, it has been known that a "heavy" impactor may yield a quasi-static impact [11]. Swanson [14] gave a more precise account for the limits of the quasi-static approximation in terms of the ratio of impactor mass to a lumped equivalent of the structural mass. Recently Yigit and Christoforou [15] proposed a method to characterize impact of beams through normalization of the governing equations. It was shown that a single non-dimensional parameter completely governs the initial response, and can be used to predict whether the response is locally or globally dominated in nature. This work was based on the initial response, however (i.e., before the waves generated by impact are reflected back from the boundaries), and the results are of limited use and valid only when the size effects can be neglected. In a more recent study on impact of plates [16], it was shown that another parameter called relative plate stiffness also plays an important role in determining the nature of impact response and accounts for the size effects. Limited experimental data were shown to confirm these findings in the case of composite plates [17]. The current paper extends the findings of references [16] and [17] to beams and presents a unified methodology for characterizing the nature of impact response of compact bodies as well as beams and plates with different support conditions and material systems. It is shown that three non-dimensional parameters are sufficient to govern completely the low velocity impact response of such structures. By knowing the impact conditions the nature of response as well as the maximum impact force can be predicted without running a simulation or conducting an experiment. Thus, the results may be useful in designing for impact loading, in providing guidelines for choosing appropriate simple models and scaling experimental results between different structures.

The results of this study are limited to low velocity impacts where the rate effects and perforation are not considered. Furthermore, the effects of various structural damages, such as global plastic deformation, delaminations and matrix cracking are beyond the scope of this paper.

2. IMPACT OF COMPACT BODIES

In impact of compact bodies the vibrations of both the impactor and the target are negligible, and the deformations are confined in the vicinity of the contact region. Furthermore, for low velocity impacts the elastic wave motion is ignored and a static contact law (e.g., Hertzian contact) is used in a quasi-static type analysis [1]. If permanent deformation is present an elastic–plastic contact law should be used [5].

A representative example of impact of compact bodies is the impact of a rigid sphere with a half-space. In this case the motion of the impactor is described by

$$m_i \ddot{w}_i = -F(t) \tag{1}$$

where m_i and w_i are the mass and the displacement of the impactor, respectively, and F is the impact force given as

$$F(t) = \left\{ \begin{matrix} F_c & \text{if } F_c \geqslant 0 \\ 0 & \text{if } F_c < 0 \end{matrix} \right\}, \tag{2}$$

in which F_c is obtained from an appropriate contact law.

The initial conditions of the impact problem are

$$w_i(0) = 0, \qquad \dot{w}_i(0) = V_0, \tag{3}$$

where V_0 is the initial impact velocity.

The contact law used here was developed in reference [5] and accounts for permanent deformation. For completeness, a brief description follows.

phase I, elastic loading,

$$F_c(\alpha) = K_h \alpha^{3/2}, \quad 0 \leqslant \alpha \leqslant \alpha_y; \tag{4}$$

phase II, elastic–plastic loading,

$$F_c(\alpha) = K_y(\alpha - \alpha_y) + K_h \alpha_y^{3/2}, \quad \alpha_y \leqslant \alpha \leqslant \alpha_m; \tag{5}$$

phase III, elastic unloading and reloading,

$$F_c(\alpha) = K_h(\alpha^{3/2} - \alpha_m^{3/2} + \alpha_y^{3/2}) + K_y(\alpha_m - \alpha_y), \quad \alpha_f \leqslant \alpha \leqslant \alpha_m. \tag{6}$$

Here α is the indentation, and α_m and α_f are the maximum and permanent indentations, respectively. K_h is the Hertzian stiffness, α_y is the critical indentation for "local yield" to occur, and K_y is the linear stiffness of the elastic–plastic loading phase. The Hertzian stiffness K_h depends on the material properties and the contact geometry, and in the case of a spherical body of radius R in contact with a flat surface is given by Goldsmith [3] as

$$K_h = \tfrac{4}{3}\sqrt{R}E^*, \tag{7}$$

where E^* is given by

$$\frac{1}{E^*} = \frac{1 - \nu_i^2}{E_i} + \frac{1 - \nu_t^2}{E_t}, \tag{8}$$

in which ν_i, E_i and ν_t, E_t are the Poisson ratios and the Young's moduli of the impactor and the target, respectively.

The critical indentation α_y is given by

$$\alpha_y = 0{\cdot}68 S_y^2 \pi^2 R/E^{*2}, \tag{9}$$

where for metals S_y is the yield strength of the softer material, and for composites $S_y = 2S_u$, with S_u being the shear strength of the fibers. K_y is the linear stiffness given as

$$K_y = 1{\cdot}5 K_h \sqrt{\alpha_y}. \tag{10}$$

In impact of compact bodies it is of primary interest to characterize the energy loss during contact. The coefficient of restitution, e, which is defined as the ratio of relative velocities at the end and the beginning of contact has traditionally been used for characterizing the energy loss as well as an input parameter to determine the post-impact response [11]. It is clear that the use of the elastic–plastic contact law facilitates a consistent account for the energy loss during contact. Thus, the coefficient of restitution can be obtained as a function of all relevant impact parameters [5]. Figure 1 shows the variation of e with the normalized impact velocity, β, given as

$$\beta = V_0/\omega\alpha_y, \tag{11}$$

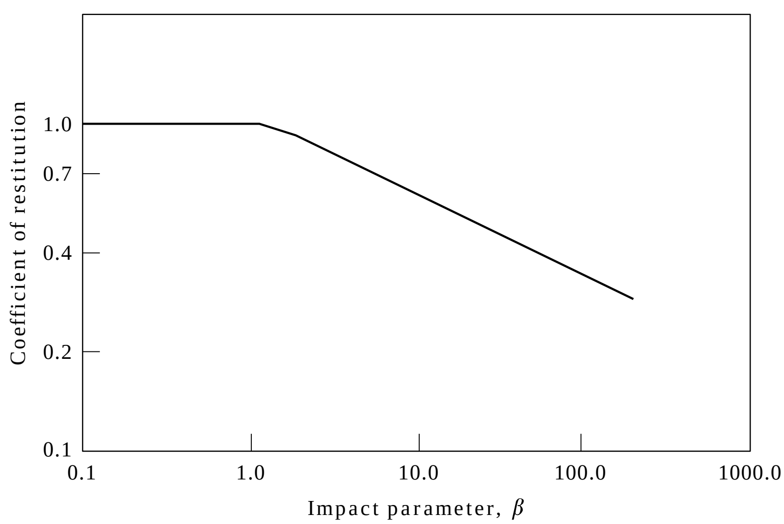

Figure 1. Variation of coefficient of restitution with normalized impact velocity.

where ω is the contact frequency given as

$$\omega^2 = K_h \alpha_y^{1/2} / m_i . \tag{12}$$

It was also shown in reference [5] that for this type of impact, β alone characterizes the impact response: i.e., the coefficient of restitution and the impact response can be determined for various impact situations with this single parameter. In other words, Figure 1 covers all possible impact situations such as different material pairs and geometries. Thus, for impact models such as the momentum balance method [6, 8], where the coefficient of restitution is used as an input parameter, Figure 1 is of great value [10].

3. IMPACT OF FLEXIBLE BODIES

The impact of flexible bodies has long been known to differ from that of compact bodies [3]. The nature of impact response depends on the impactor as well as the target characteristics. In general, the impact of flexible bodies is characterized by relatively short impact durations, the possibility of multiple collisions and significant energy transfer into vibrations. The impact of a compact spherical impactor with flexible plates and beams are considered as representative examples for the impact of flexible bodies. In these cases in addition to the motion of the impactor, the motion of the target has to be considered.

The governing equations for a specially orthotropic plate subject to lateral loading including transverse shear deformation are given as

$$D_{11} \frac{\partial^2 \psi_x}{\partial x^2} + D_{66} \frac{\partial^2 \psi_x}{\partial y^2} + (D_{12} + D_{66}) \frac{\partial^2 \psi_y}{\partial x \, \partial y} - kA_{55}\left(\psi_x + \frac{\partial w}{\partial x} \right) = \frac{\rho h^3}{12} \frac{\partial^2 \psi_x}{\partial t^2}, \tag{13}$$

$$(D_{12} + D_{66}) \frac{\partial^2 \psi_x}{\partial x \, \partial y} + D_{66} \frac{\partial^2 \psi_y}{\partial x^2} + D_{22} \frac{\partial^2 \psi_y}{\partial y^2} - kA_{44}\left(\psi_y + \frac{\partial w}{\partial y} \right) = \frac{\rho h^3}{12} \frac{\partial^2 \psi_y}{\partial t^2}, \tag{14}$$

$$kA_{55}\left(\frac{\partial \psi_x}{\partial x} + \frac{\partial^2 w}{\partial x^2} \right) + kA_{44}\left(\frac{\partial \psi_y}{\partial y} + \frac{\partial^2 w}{\partial y^2} \right) + p(x, y, t) = \rho h \frac{\partial^2 w}{\partial t^2}, \tag{15}$$

where D_{ij} and A_{ij} are the bending and shear stiffnesses defined as usual, h is the plate thickness, ρ is the material density, w is the transverse deflection, ψ_x and ψ_y are the shear rotations, p is the lateral load per unit area, k is the Mindlin shear correction factor, x and y are the space variables, and t is the time.

The governing equations for a specially orthotropic beam subject to lateral loading including transverse shear deformation are given as

$$bD_{11} \frac{\partial^2 \psi}{\partial x^2} - kbA_{55}\left(\psi + \frac{\partial w}{\partial x} \right) = \rho I \frac{\partial^2 \psi}{\partial t^2}, \tag{16}$$

$$kbA_{55}\left(\frac{\partial \psi}{\partial x} + \frac{\partial^2 w}{\partial x^2} \right) + p(x, t) = \rho A \frac{\partial^2 w}{\partial t^2}, \tag{17}$$

where b, A and I are the width, cross-sectional area, and moment of inertia of the beam, respectively, and p now is the lateral load per unit length. The motion of the impactor is described by equations (1)–(3).

In the case of impact of flexible bodies, a linearized form of the elastic–plastic contact law given in the previous section is used,

$$F_c(\alpha) = K_y \alpha, \tag{18}$$

where α is the local indentation defined as the difference between the displacement of the impactor and the deflection of the target at the impact point, i.e.,

$$\alpha(t) = w_i(t) - w_t(t). \tag{19}$$

As will be shown later, the use of a linear contact stiffness is useful in the normalization procedure and in the identification of the key impact parameters.

The equations of motion are discretized by a standard modal analysis procedure and numerically solved to obtain the response. In order to manage the various impact parameters and the simulations effectively, however, the governing equations are normalized by defining the following non-dimensional variables:

$$\bar{x} = x/a, \qquad \bar{y} = y/b, \qquad \tau = \omega t,$$

$$\bar{w} = w/a^*, \qquad \bar{\psi}_x = (a/\alpha^*)\psi_x, \qquad \bar{\psi}_y = (b/\alpha^*)\psi_y, \qquad \bar{\alpha} = \alpha/\alpha^*. \tag{20}$$

Here $\omega = \sqrt{K_y/m_i}$ is the linear contact frequency and $\alpha^* = v_0/\omega$ is the maximum indentation obtained from the half-space solution (i.e., with no structural response), and a and b are the length and the width of the plate, respectively. Note that for the beam case, the y dependence disappears and $a = l$. The normalized equations of motion and initial conditions are then given in general form as

$$\ddot{\mathbf{q}} + [\bar{\omega}^2]\mathbf{q} = \mathbf{f}, \qquad \ddot{\bar{\omega}}_i = -\bar{F}(\tau), \qquad \bar{F}_c = \bar{\alpha}, \tag{21–23}$$

$$\mathbf{q}(0) = 0, \qquad \dot{\mathbf{q}}(0) = 0, \tag{24}$$

$$\bar{\alpha}(0) = 0, \qquad \dot{\bar{\alpha}}(0) = 1, \tag{25}$$

where $\mathbf{q}$ is the vector of normalized modal co-ordinates, $\mathbf{f}$ is the vector of normalized modal forces, and $[\bar{\omega}]$ is the matrix of normalized natural frequencies with respect to the contact frequency. The normalized impact force $\bar{F}(\tau)$ is given as

$$\bar{F}(\tau) = F(t)/m_i v_0 \omega. \tag{26}$$

4. IDENTIFICATION OF KEY IMPACT PARAMETERS

The approach taken in this paper for identifying the key parameters in impact is to isolate and examine the physical phenomena involved through asymptotic cases. There are three basic phenomena which play a role in shaping the impact response: (i) the local contact behavior, (ii) wave propagation and (iii) the initial interaction of the impactor and the target dynamics. Impact on a half-space (compact bodies), quasi-static impact, and the impact on an infinitely large structure, respectively, are used to isolate the above mentioned phenomena and identify the key parameter involved.

The nature of impact response depends on the impactor as well as the target characteristics. If the impactor mass is very small, and the target is fairly stiff, impact does not produce significant structural response and can be approximated by impact on a half-space. In this case the flexibility of the structure is negligible and the impact response is localized in nature. As mentioned in section 2, the normalized impact velocity, β alone characterizes the impact response. In general, the local contact problem is non-linear, and therefore an analytical solution is not possible. However, a linear contact law, such as the one given in equation (18), facilitates analytical solution. Furthermore, this linear contact law results in a normalized impact velocity of unity for all cases. Thus, the impact force can be obtained from equation (22) by neglecting the structural response as

$$\bar{F}_{hs}(\tau) = \sin\tau. \tag{27}$$

Therefore, in general, for a locally dominated impact response the maximum normalized impact force will be close to unity.

On the other hand, if the impactor is quite heavy, its inertia will dominate and the response can be approximated by a quasi-static analysis. The quasi-static approximation involves treating the impact problem as an equivalent static problem with a static load applied to the impact site. By neglecting the mass of the structure the system can be thought of as a single-degree-of-freedom system with the contact stiffness K_y and the static stiffness K_{st} of the structure in series. The normalized impact force is then given as

$$\bar{F}_{qs}(\tau) = \sqrt{\lambda/(\lambda+1)}\,\sin\left(\sqrt{\lambda/(\lambda+1)}\,\tau\right). \tag{28}$$

As can be seen the maximum normalized impact force as well as the impact duration is a function of only the relative stiffness of the structure, λ, which is defined as

$$\lambda = K_{st}/K_y. \tag{29}$$

In cases where the size of the target is large such that the flexural waves due to impact are not reflected back from the boundaries before the contact is over, the structural deflection can be approximated by using infinite beam or plate solutions [18, 19].

For an infinite plate, the velocity at the impact point is related to the contact force as [19]

$$\dot{w}_t(t) = \frac{1}{8\sqrt{\rho h D^*}}\,F_c(t)\,\mathrm{d}t, \tag{30}$$

where D^* is the effective plate stiffness [20] given as

$$D^* = \tfrac{1}{2}\left(D_{12} + 2D_{66} + \sqrt{D_{11}D_{22}}\right). \tag{31}$$

Substitution of equation (30) into the equations of motion and normalization yield

$$\ddot{\bar{\alpha}} + 2\zeta_p \Omega_n \dot{\bar{\alpha}} + \Omega_n^2 \bar{\alpha} = 0, \tag{32}$$

where

$$\Omega_n = 1, \qquad \zeta_p = \tfrac{1}{16}\sqrt{m_i K_y / \rho h D^*}. \tag{33, 34}$$

For an infinite beam, the velocity at the impact point is related to the contact force by [18]

$$\dot{w}_t(t) = \frac{1}{\sqrt{2\pi}} \frac{1}{\rho h} \left(\frac{\rho h}{D_{11}}\right)^{1/4} \int_0^t \frac{F_c(\xi)}{\sqrt{t-\xi}} \,\mathrm{d}\xi. \tag{35}$$

Substitution of equation (35) into the equations of motion and normalization yield

$$\ddot{\bar{\alpha}} + 2\zeta_b \Omega_n \int_0^\tau \frac{\dot{\bar{\alpha}}(\xi)}{\sqrt{\tau - \xi}} \,\mathrm{d}\xi + \Omega_n^2 \bar{\alpha} = 0, \tag{36}$$

where

$$\Omega_n = 1, \qquad \zeta_b = \frac{1}{2\sqrt{2\pi}} \sqrt{\frac{m_i}{\rho b^2 h}} \left[\frac{m_i K_y}{\rho h D_{11}}\right]^{1/4}. \tag{37, 38}$$

Since equations (32) and (36) are obtained by assuming infinite sizes, for a finite target, they are valid only for the initial impact response until the waves are reflected back from the boundaries. Clearly, in this case, the impact response is governed by a single parameter, termed as "loss factor", ζ, and as the nature of equations (32) and (36) suggests, this parameter can be used to characterize the energy "lost" by the impactor to the structure during the contact duration, and as shown it is evaluated differently for beams and plates. It is interesting to note that in beams, the resulting equation is a somewhat complicated integro-differential equation whereas in plates the resulting equation is the well known second order oscillator with ζ playing the role of the damping ratio. Therefore, the loss factor would affect the initial impact response in the same way the damping ratio affects the response of a second order oscillator.

From the foregoing discussion three non-dimensional parameters can be identified as key parameters in determining the nature of impact response. The characteristic impact number, β, was shown to govern the local contact behavior. The relative stiffness, λ, of the structure is related to the structural contribution and completely governs the response for a quasi-static impact while the loss factor, ζ, governs the initial response. Therefore, it is expected that in general, the type of impact response is related to the values of these three key parameters. For the purpose of this paper, the dependence on β is eliminated by the use of a linearized contact law and by normalizing the impact response with respect to the half-space solution. This should not be a problem since only low velocity impacts without damage effects are under consideration here.

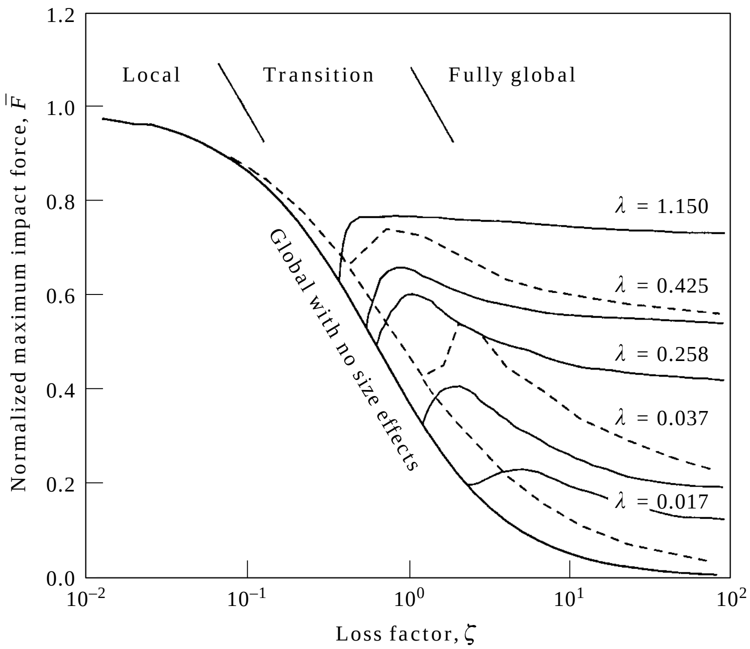

Figure 2. Normalized maximum impact force for structures as a function of the loss factor and relative stiffness: ——, plates; ----, beams.

5. NUMERICAL EXPERIMENTS

From the preceding analysis and discussion, it is clear that the effect of two non-dimensional parameters, namely, ζ and λ, on the impact response should be investigated. This has been done by solving the full dynamic equations of beams and plates with sufficient number of modes for various (ζ, λ) pairs. The maximum normalized impact force, $\bar{F}$, obtained from each simulation is plotted as a function of these two parameters in Figure 2. The solid lines represent plate responses whereas the dashed lines are for beams. For clarity, only two beam cases are shown. Although the curves for beams and plates with the same relative stiffness follow a similar trend, they only coincide when the response is either local or fully global (quasi-static). This is because, in these extreme cases both responses are governed by the same equations (equations 27, 28). However, in the transition zone, the dynamics of the two type of structures are inherently different and can not be matched. This can also be seen by comparing infinite structure solutions for plates and beams (see equations 32 and 36).

A normalized force close to unity signifies a locally dominated response (e.g., single impacts). For a flexible structure, in the range labelled as "local", the impact durations are relatively short. During this time the structural displacements do not change significantly. Therefore, the effect of local contact behavior can be included through the use of an appropriate coefficient of restitution and the momentum balance method can be used with a very good accuracy to predict post impact behavior [10]. Furthermore, the impact is completed before the waves are reflected back from the boundaries. Therefore, the size effects do not play a major role and the maximum normalized impact force does not depend on λ.

A small $\bar{F}$ signifies a more complicated globally dominated response. As ζ increases the fraction of the energy transferred to the target increases. For large values of ζ the impactor essentially remains in contact with the target throughout the impact event. Moreover, for globally dominated cases, not only ζ but also the relative structure stiffness affect the maximum force. As the relative stiffness of the structure increases, the maximum impact force approaches that of the half-space. Also, as ζ increases, each curve in Figure 2 approaches an asymptote which can be obtained from the quasi-static approximation. In this region, the maximum normalized impact force is a function of the relative structure stiffness alone. A quasi-static impact model which is a single-degree-of-freedom lumped model can be used to predict the impact response.

For any case on the infinite structure solution, the maximum normalized impact force is a function of ζ alone and the response is global in nature, (i.e., the energy transferred to the structure is not negligible), except for small values of ζ where the response can be considered as local. For large values of ζ the local contact behavior is not dominant and the response is not sensitive to the details of the contact law used.

In general, for a given relative stiffness, as ζ increases the maximum normalized impact force decreases. However, the functional relationship between $\bar{F}$ and ζ changes at a certain value of ζ, exhibiting a local minimum and a local maximum. This is an indication of a transition from locally dominated to globally dominated behavior. This transitory behavior can be explained as follows. For locally dominated cases the maximum impact force is the maximum force due to a single impact. As ζ increases, structural effects become more significant and multiple impacts occur. In these cases the maximum impact force may be due to subsequent impacts rather than the initial impact. As the relative stiffness of the structure increases, the response is governed by the global stiffness.

A well known similar transitory behavior is the transition from laminar to turbulent flow in fluid mechanics where in most cases the Reynolds number is used to classify the type of flow. In the case of flow inside rough pipes, for example, the plots of friction factor versus Reynolds number for different roughness ratios (Moody diagram, see e.g., Figures 6.12 and 6.13 in reference [21]) closely resemble Figure 2. Although there is no evidence at this point to suggest that the underlying physics of the two phenomena are similar, an analogy may improve the understanding of the transitory behavior observed in impact dynamics. The normalized maximum impact force, loss factor and relative stiffness may be considered analogous to friction factor, Reynolds number and relative roughness, respectively. Local response with no size effects is similar to laminar flow (small Reynolds number with no roughness effects); global response with no size effects (only ζ governing the response) is similar to turbulent flow in smooth pipes, where the friction factor is a function of Reynolds number alone; fully global response (only λ governing the response) is similar to complete turbulence in rough pipes, where the friction factor is a function of the relative roughness alone. In the transition zone, as in the case of flow in rough pipes, the response is a function of both parameters. Furthermore, the impact force history in locally dominated and globally dominated cases closely resemble the velocity profiles observed in laminar and turbulent flows, respectively.

Therefore, as in the case of fluid mechanics, the information provided in Figure 2 could be very useful to analysts and designers in parametric type studies, as well as in designing experiments to study the impact response of structures. This figure can be used to estimate the maximum impact force and the type of response for a given ζ and λ without running any simulation or experiment. Also, determining the type of response is important since the damage mechanisms involved are sometimes closely related to the type of response. In addition, knowing the type of response prior to a simulation, will help in choosing an adequate impact model.

6. SCALING OF IMPACT RESPONSE

At this point, it is important to reiterate the power and usefulness of characterizing impact response by the two non-dimensional key parameters proposed. Although the curves in Figure 2 were obtained for simply supported beams and square plates of certain material systems, each point on these curves may represent many different impact cases with different material and support conditions. Seemingly different impact situations may be similar in non-dimensional form and need not be duplicated. The difficulty of covering all possible impact situations in an experimental program is well known. Thus, the reduction of the various impact parameters into two non-dimensional parameters not only has provided physical insight into the impact problem but also has provided a valuable tool for generalizing and correlating experimental results through the use of minimum data and model tests.

From the foregoing discussion, it is clear that plates or beams having the same relative stiffness and loss factor will result in the same normalized response. Therefore, by using the information provided here, scaling impact response among the same type of structures is straightforward. In order to demonstrate how Figure 2 can be used for scaling impact response among different structures, full simulations for a plate and a beam with the same relative stiffness ($\lambda = 0{\cdot}425$), and loss factor ($\zeta = 11{\cdot}3$) were carried out. As would be predicted from Figure 2 both responses of the plate and the beam shown in Figures 3 and 4 approach the quasi static or fully global type response. As can be seen, both impact force and deflection scale well between the two structures. Though the overall behavior is similar, naturally the frequency content is different. As ζ increases, the accuracy of the quasi-static approximation will improve and the results asymptotically approach each other. Practically, however, in most real tests, the high frequency components will be attenuated, and the results will be more similar for scaled problems. It should be noted that a quasi-static approximation (equation (28)) for both the plate and the beam will be identical and can be used instead of the results of the full simulation.

Figures 5 and 6 show a locally dominated case with $\zeta = 0{\cdot}038$. Again, both the beam and the plate result in very similar impact force responses. Note however

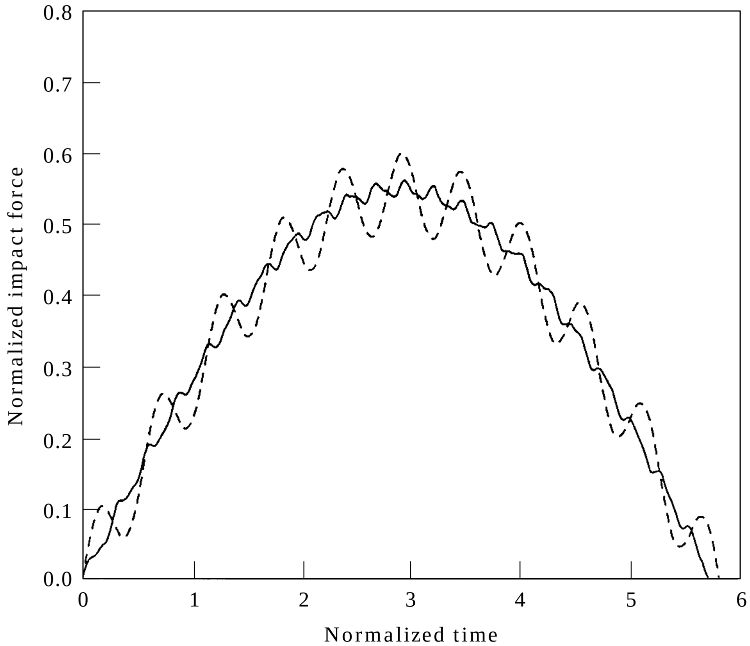

Figure 3. Scaling of impact force for a fully global impact ($\lambda = 0{\cdot}425$, $\zeta = 11{\cdot}3$); key as Figure 2.

that the deflections are only similar during the initial response. Therefore, while the impact force can be scaled between the two structures for these type of impacts, the deflections can not be scaled. This is to be expected, since the similitude is only

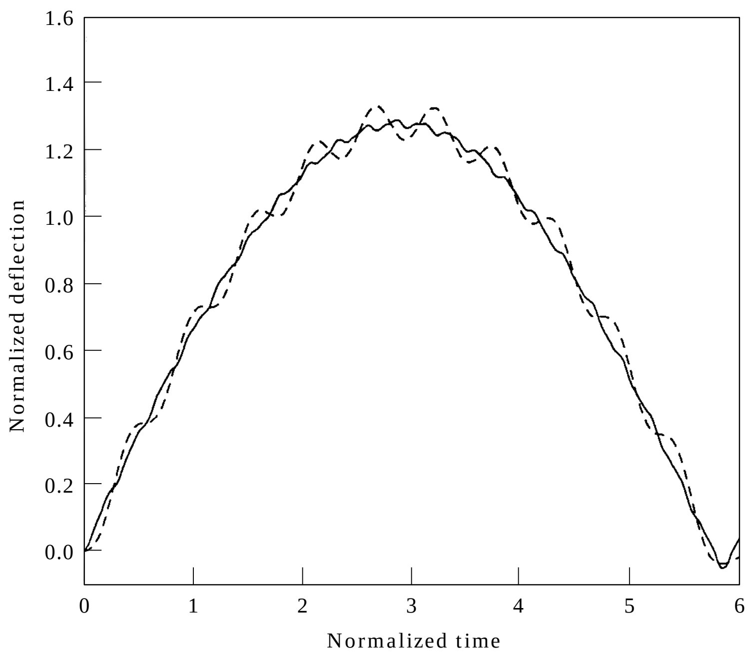

Figure 4. Scaling of central deflection for a fully global impact ($\lambda = 0{\cdot}425$, $\zeta = 11{\cdot}3$); key as Figure 2.

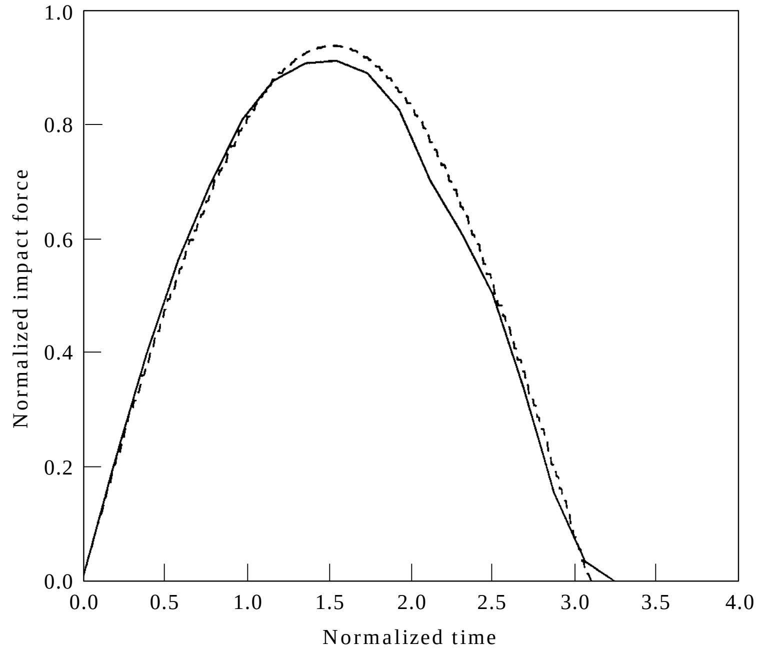

Figure 5. Scaling of impact force for a locally dominated impact ($\zeta = 0{\cdot}038$); key as Figure 2.

valid during the initial response, before the waves are reflected back from the boundaries. Clearly, after the impact, the dynamics of the beam and the plate are different.

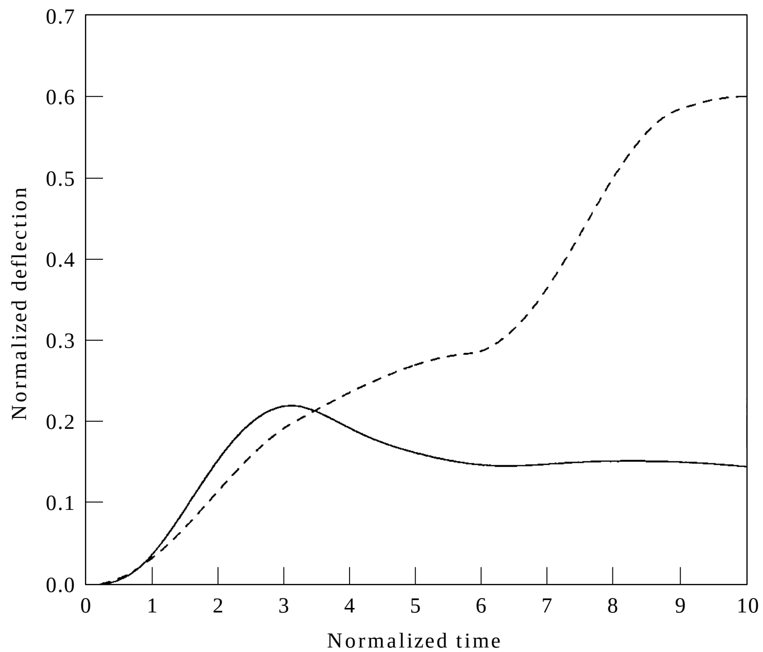

Figure 6. Scaling of central deflection for a locally dominated impact ($\zeta = 0{\cdot}038$); key as Figure 2.

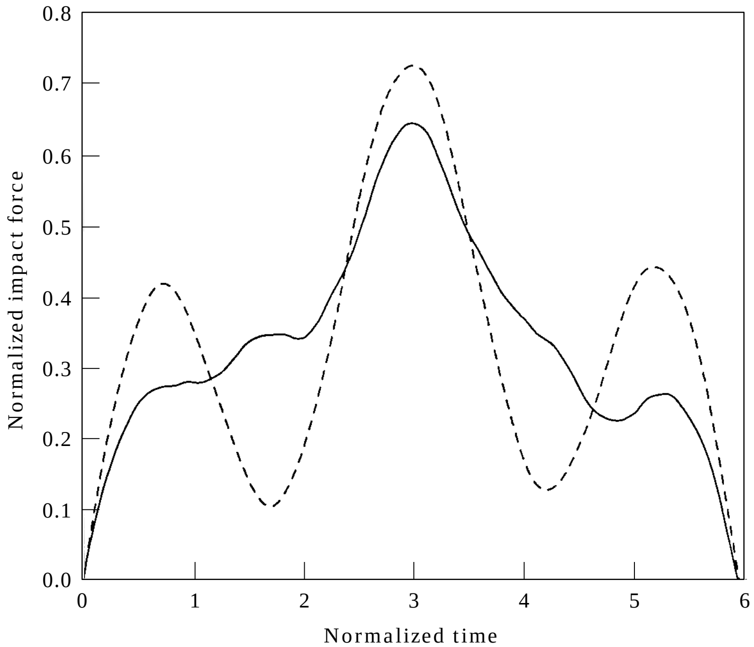

Figure 7. Scaling of impact force for an impact in the transition zone ($\lambda = 0{\cdot}425$, $\zeta = 1{\cdot}198$); key as Figure 2.

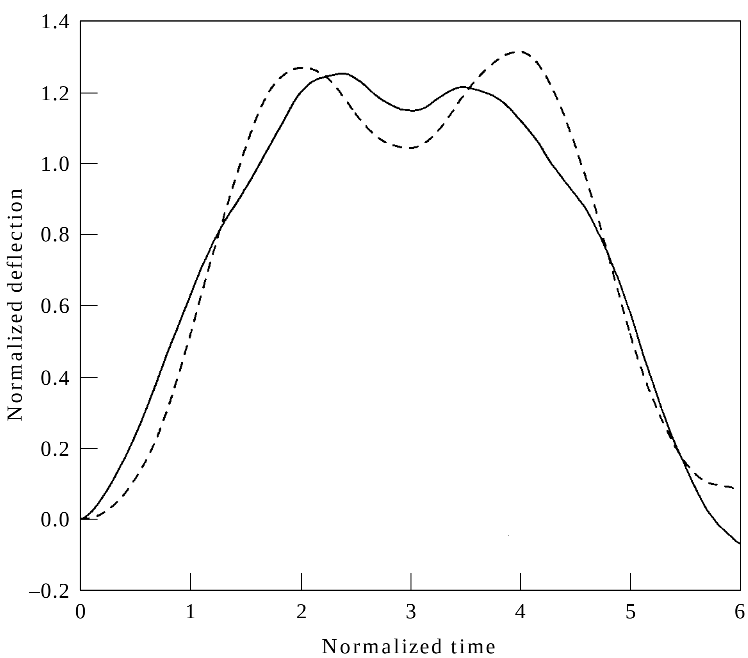

Figure 8. Scaling of central deflection for an impact in the transition zone ($\lambda = 0{\cdot}425$, $\zeta = 1{\cdot}198$); key as Figure 2.

In the transition region, though the type of impact response can be predicted well, the actual impact force and deflection may not be scaled with the same accuracy as can be seen in Figures 7 and 8 ($\lambda = 0{\cdot}425$, $\zeta = 1{\cdot}198$). The maximum impact force for a specific structure in this region can still be predicted from Figure 2 to a reasonable accuracy. However, the actual impact responses can not be scaled among different structures. Therefore, if the objective is to obtain impact response for a structure in the transition region one has to carry out a full simulation with sufficient number of modes.

7. CONCLUSIONS

The dynamic behavior of structures subject to low velocity transverse impact has been investigated. The effects of impact parameters as well as the flexibility of the target have been examined through numerical simulations. It has been shown that the normalized impact force and the type of impact response can be predicted through the functional relationship between the normalized maximum impact force and two non-dimensional parameters termed as "loss factor" and "relative stiffness". It is expected that the results of this study will be of great value in choosing adequate impact and computational models for the dynamic analysis of structures subject to transverse impacts as well as in scaling experimental results through the use of minimum data and model tests. Specifically, simple models can be suggested for cases where the response is governed by either the loss factor alone, or the relative stiffness alone. For the cases in the transition region, however, a full simulation or test may be needed. It is also anticipated that, in this region, the structural damage may be more severe since both local and global structural effects contribute to the energy transfer to the structure. Thus, the ideas proposed here may also be used to predict the type of dominant impact damage mode.

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