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NON-LINEAR VIBRATION OF TIMOSHENKO BEAM DUE TO A MOVING FORCE AND THE WEIGHT OF BEAM

R.-T. WANG AND T.-H. CHOU

Department of Engineering Science, National Cheng Kung University, Tainan, Taiwan, Republic of China

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The large deflection theory is employed to derive the equations of motion of the Timoshenko beam due to the coupling effect of an external force with the weight of the beam. Galerkin's method is employed to obtain the dynamic responses of the beam. A set of two discrete moving forces is taken as an example to investigate the characteristics of these responses. Results show that the effect of weight of the beam decreases the fundamental period of the structure. Further, both the dynamic deflection and the dynamic moment of the beam predicated by the theory including the effect of weight of the beam are less than those of the beam predicated either by the small deflection theory or by the large deflection theory without including the effect of weight of the beam.

1. INTRODUCTION

Various types of beams are widely encountered in structures. The small deflection theory for beams is normally employed to obtain the responses of a beam induced by an external load. However, the responses of the beam predicated by the theory may be unreasonable, particularly while the magnitude of the external load is large. Such erroneous results can be corrected by employing the large deflection theory instead of the small deflection theory to get the responses of the beam.

According to the small deflection theory, the modal frequencies of the beam are independent of both the magnitude and the frequency of an external load. The large deflection theory for beams indicates that the product of longitudinal extension and transverse deflection of the beam stiffens the structure. Consequently, the effects of initial imperfections [1–3], large amplitudes [4, 5] and longitudinal extension [6] of the beam increase the fundamental frequency of the structure. Similarly, both effects of the weight of the beam and the magnitude of the external load may increase the fundamental frequency of the beam.

The responses of a beam due to the moving loads are a function of both time and velocity. Neglecting the effect of the weight of the beam, the small deflection theory implies that both the maximum deflection as well as the maximum moment of the structure caused by a moving load are greater than those induced by the

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load in a static situation [7]. Using the large deflection theory and neglecting the effect of the weight of the beam, Hino *et al*. [8, 9] have demonstrated that the fundamental frequency of the beam increases as the magnitude of the load traversing on the structure increases. Sometimes the weight of the beam is heavier than the magnitude of the external load acting on the structure. Neglecting the weight of the beam while studying the dynamics of the beam may yield erroneous results for the structure. The large deflection theory and the weight of the beam should be, therefore, considered simultaneously for obtaining precise results.

In the present study, the large deflection theory is adopted to study the non-linear vibration of the Timoshenko beam caused by the coupling effect of a moving force with the weight of the beam. The equations of static equilibrium of the beam due to its own weight are derived via the theory to obtain the static responses of the beam. The effects of thickness and length of the beam on the differences in its responses, as obtained by the small deflection theory and the large deflection theory, are investigated. Further, the static responses of the beam are considered while deriving the equations of motion of the beam subjected to an external load. The transverse deflection is known to be larger than the longitudinal displacement for the beam. Further, the inertia of the transverse motion is more important than the longitudinal inertia on dominating the transverse vibration of the beam due to external loads. Therefore, the longitudinal inertia of the beam is neglected in this study.

Due to the coupling effect of longitudinal force with the transverse deflection, the equations of motion of the beam cannot be solved analytically. Therefore, the set of mode shape functions obtained form the small deflection theory for the beam is incorporated in Galerkin's method to solve the non-linear problem. A set of two discrete moving forces traversing on the beam is used to simulate a vehicle moving on the beam. The dynamic responses obtained respectively from the small deflection theory, the large deflection theory including the effect of weight of the beam, and the large deflection theory without including the effect of weight of the beam are discussed. Furthermore, the responses obtained for the Timoshenko beam are compared with those for the Bernoulli–Euler beam.

2. EQUATIONS OF EQUILIBRIUM

A distributed force $F(x, t)$ acting on a simply supported Timoshenko beam is depicted in Figure 1. Both ends of the beam are immovable. The beam is considered to be homogeneous and isotropic with Young's modulus E, Poisson's ratio ν, shear modulus G, mass density ρ, length L, thickness h and width b. The

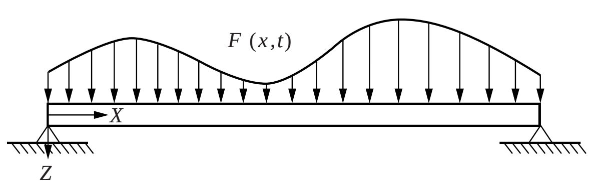

Figure 1. A distributed force on a simply supported Timoshenko beam.

co-ordinate of the neutral axial of the beam is denoted as x. The longitudinal displacement, transverse deflection and bending slope of the beam due to its own weight are denoted as $u_o(x)$, $w_o(x)$ and $\psi_o(x)$, respectively. Further, the dynamic deformations induced by the distributed force are denoted as $u_*(x, t)$, $w_*(x, t)$ and $\psi_*(x, t)$, respectively. The displacement fields u and w of the beam due to the combined action of its own weight and the distributed load can be obtained by modifying the Mindlin plate theory [10] to be of the forms

$$u(x, z, t) = u_o(x) + u_*(x, t) - z\psi_o(x) - z\psi_*(x, t), \tag{1a}$$

$$w(x, z, t) = w_o(x) + w_*(x, t). \tag{1b}$$

The longitudinal strain, shearing strain and the corresponding longitudinal force and shear force caused by the weight of the beam are denoted as ϵ_o, γ_o, n_o and q_o, respectively. Further, the dynamic longitudinal strain, shearing strain and the corresponding longitudinal force and shear force caused by the distributed force are denoted as ϵ_*, γ_*, n_* and q_*, respectively.

By modifying the large deflection theory for the Mindlin plate [10], the total strain fields of the beam are expressed as

$$\epsilon = \epsilon_o + \epsilon_* - z\frac{\mathrm{d}\psi_o}{\mathrm{d}x} - z\frac{\partial\psi_*}{\partial x}, \qquad \gamma = \gamma_o + \gamma_*, \tag{2a, b}$$

in which

$$\epsilon_o = \frac{\mathrm{d}u_o}{\mathrm{d}x} + 0{\cdot}5\left(\frac{\mathrm{d}w_o}{\mathrm{d}x}\right)^2, \qquad \gamma_o = \frac{\mathrm{d}w_o}{\mathrm{d}x} - \psi_o, \tag{3a, b}$$

$$\epsilon_* = \frac{\partial u_*}{\partial x} + \frac{\partial w_*}{\partial x}\frac{\mathrm{d}w_o}{\mathrm{d}x} + 0{\cdot}5\left(\frac{\partial w_*}{\partial x}\right)^2, \qquad \gamma_* = \frac{\partial w_*}{\partial x} - \psi_*. \tag{4a, b}$$

The total longitudinal force n, shear force q and bending moment m of the beam are

$$n = n_o + n_*, \qquad q = q_o + q_*, \qquad m = m_o + m_*, \tag{5}$$

where

$$(n_o, n_*) = EA(\epsilon_o, \epsilon_*), \qquad (q_o, q_*) = \kappa GA(\gamma_o, \gamma_*), \tag{6a}$$

$$(m_o, m_*) = -EI\left(\frac{\mathrm{d}\psi_o}{\mathrm{d}x}, \frac{\partial\psi_*}{\partial x}\right), \tag{6b}$$

in which κ is the shear coefficient, $A(=bh)$ is the cross-sectional area and $I(=bh^3/12)$ is the second moment of area about the y-axis of the beam.

The strain energy V and kinetic energy K of the beam are

$$V = \frac{1}{2}\int_0^L \left(\frac{n^2}{EA} + \frac{m^2}{EI} + \frac{q^2}{\kappa GA}\right) \mathrm{d}x, \tag{7a}$$

$$K = \frac{1}{2}\int_0^L \rho\left(A\left(\frac{\partial u}{\partial t}\right)^2 + A\left(\frac{\partial w}{\partial t}\right)^2 + I\left(\frac{\partial \psi}{\partial t}\right)^2\right) \mathrm{d}x. \tag{7b}$$

The work done on the beam by the combination of its own weight and the distributed force is

$$P = \int_0^L (\rho g A w + F w_*)\, \mathrm{d}x. \tag{7c}$$

Substituting V, K and P into Hamilton's principle and neglecting the longitudinal inertia of the beam yields the equations of static equilibrium

$$\frac{\mathrm{d}n_o}{\mathrm{d}x} = 0, \qquad -\frac{\mathrm{d}m_o}{\mathrm{d}x} + q_o = 0, \qquad n_o\frac{\mathrm{d}^2 w_o}{\mathrm{d}x^2} + \frac{\mathrm{d}q_o}{\mathrm{d}x} + \rho g A = 0, \tag{8a–c}$$

and the equations of motion

$$\frac{\partial n_*}{\partial x} = 0, \qquad \frac{\partial m_*}{\partial x} - q_* + \rho I \frac{\partial^2 \psi_*}{\partial t^2} = 0, \tag{9a, b}$$

$$-n_o\frac{\partial^2 w_*}{\partial x^2} - n_*\left(\frac{\mathrm{d}^2 w_o}{\mathrm{d}x^2} + \frac{\partial^2 W_*}{\partial x^2}\right) - \frac{\partial q_*}{\partial x} + \rho A\frac{\partial^2 w_*}{\partial t^2} = F(x, t). \tag{9c}$$

The boundary conditions at both simply supported and immovable ends of the beam are

$$w_o = w_* = 0, \qquad u_o = u_* = 0, \qquad m_o = m_* = 0. \tag{10}$$

3. STATIC RESPONSES

The solution of equation (8a) that satisfies the boundary conditions at immovable ends is

$$n_o = \frac{EA}{2}\int_0^L \left(\frac{\mathrm{d}w_o}{\mathrm{d}x}\right)^2 \mathrm{d}x. \tag{11}$$

Eliminating ψ_o between equations (8b) and (8c) and simplifying the result yields

$$\frac{\mathrm{d}^4 w_o}{\mathrm{d}x^4} - c^2\frac{\mathrm{d}^2 w_o}{\mathrm{d}x^2} = f, \tag{12}$$

where

$$\alpha = \frac{n_o}{\kappa GA}, \qquad c^2 = \frac{n_o}{EI(\alpha + 1)}, \qquad f = \frac{\rho g A}{EI(\alpha+1)}.$$

The solution w_o of equation (12) and the corresponding moment that satisfy the boundary conditions at both simply supported ends are

$$w_o = \frac{f}{c^2}\left\{a_1\left[\cosh(cx) + \frac{x}{L} - \frac{x\cosh(cL)}{L} - 1\right] + a_2\left[\sinh(cx) - \frac{x\sinh(cL)}{L}\right] - \frac{x(x-L)}{2}\right\}, \quad (13)$$

$$m_o = -EIf\left[(a_2 + \alpha c a_1)\sinh(cx) + (a_1 + \alpha c a_2)\cosh(cx) - \frac{1}{c^2}\right], \quad (14)$$

where

$$a_1 = \frac{1}{1-\alpha^2 c^2}\left\{\frac{1}{c^2} - \alpha c\left[\frac{1-\cosh(cL)}{\sinh(cL)}\right]\right\},$$

$$a_2 = \frac{1}{1-\alpha^2 c^2}\left\{-\frac{\alpha}{c} + \frac{1-\cosh(cL)}{\sinh(cL)}\right\}.$$

Further, the shear force obtained from equation (8b) is

$$q_o = -EIcf[(a_2 + \alpha c a_1)\cosh(cx) + (a_1 + \alpha c a_2)\sinh(cx)]. \quad (15)$$

Substituting equation (13) into equation (11) yields the non-linear equation in terms of n_o as the symbolic form

$$n_o = N(n_o, \rho g A, L) \quad (16)$$

The solution n_o of equation (16) can be obtained by the Bisection method [11].

4. DYNAMIC RESPONSES

The dynamic longitudinal force obtained from equation (9a), satisfying the condition of zero displacements at both the immovable ends, is

$$n_*(t) = EA\int_0^L\left[\frac{\mathrm{d}w_o}{\mathrm{d}x}\frac{\partial w_*}{\partial x} + \tfrac{1}{2}\left(\frac{\partial w_*}{\partial x}\right)^2\right]\mathrm{d}x. \quad (17)$$

Solutions of the set of non-linear partial differential equations (9b) and (9c) cannot be obtained exactly. Therefore, Galerkin's method is adopted here to find the approximate solutions of equations (9b) and (9c). Any two distinct sets of mode shape functions of the Timoshenko beam, obtained from the small deformation theory, have been shown to be orthogonal [7], i.e.,

$$\int_0^L \left[\Psi_i \left(\frac{\mathrm{d}M_j}{\mathrm{d}x} - Q_j \right) + W_i \left(-\frac{\mathrm{d}Q_j}{\mathrm{d}x} \right) \right] \mathrm{d}x = 0, \qquad i \neq j, \tag{18a}$$

$$\int_0^L \rho (A W_i W_j + I \Psi_i \Psi_j)\, \mathrm{d}x = 0, \qquad i \neq j, \tag{18b}$$

where W_i, Ψ_i, Q_i and M_i are the ith mode shape functions of transverse deflection, bending slope, shear force and bending moment of the beam, respectively. Further, the following relation has also been obtained [7]

$$\int_0^L \left[\Psi_i \left(\frac{\mathrm{d}M_i}{\mathrm{d}x} - Q_i \right) + W_i \left(-\frac{\mathrm{d}Q_i}{\mathrm{d}x} \right) \right] \mathrm{d}x = \omega_i^2 \int_0^L (A W_i^2 + I \Psi_i^2)\, \mathrm{d}x, \tag{18c}$$

where ω_i is the ith modal frequency. Therefore, the dynamic responses of transverse deflection, bending slope, shear force and bending moment of the beam can be expressed in the following form for the Galerkin's method

$$\{w_*, \psi_*, q_*, m_*\}(x, t) = \sum_{i=1} B_i(t) \{W_i, \Psi_i, Q_i, M_i\}(x). \tag{19}$$

In equation (19), the ith modal amplitude B_i is required to be determined.

Substituting equation (19) into equations (9b) and (9c), respectively, yields

$$\sum_{i=1} \left\{ B_i \left(-Q_i + \frac{\mathrm{d}M_i}{\mathrm{d}x} \right) + \rho I \Psi_i \frac{\mathrm{d}^2 B_i}{\mathrm{d}t^2} \right\} = 0, \tag{20a}$$

$$\sum_{i=1} \left\{ -B_i \left(\frac{\mathrm{d}Q_i}{\mathrm{d}x} \right) + \rho A W_i \frac{\mathrm{d}^2 B_i}{\mathrm{d}t^2} \right\} = F(x, t) + \sum_{i=1} B_i (n_* + n_o) \left(\frac{\mathrm{d}^2 W_i}{\mathrm{d}x^2} \right) + n_* \left(\frac{\mathrm{d}^2 W_o}{\mathrm{d}x^2} \right). \tag{20b}$$

Multiplying equation (20a) by Ψ_j, equation (20b) by W_j and integrating their sum from $x = 0$ to $x = L$ yields

$$\frac{\mathrm{d}^2 B_j}{\mathrm{d}t^2} + \omega_j^2 B_j = \frac{F_j(t)}{s_j} + \frac{n_o + n_*}{s_j} \sum_{i=1} B_i \int_0^L W_j \frac{\mathrm{d}^2 W_i}{\mathrm{d}x^2}\, \mathrm{d}x + \frac{n_*}{s_j} \int_0^L W_j \frac{\mathrm{d}^2 W_o}{\mathrm{d}x^2}\, \mathrm{d}x,$$

$$j = 1, 2, \ldots, \tag{21}$$

where the jth modal mass s_j and its corresponding modal excitation $F_j(t)$ are

$$s_j = \int_0^L \rho (A W_j^2 + I \Psi_j^2)\, \mathrm{d}x, \qquad F_j(t) = \int_0^L F(x, t) W_j\, \mathrm{d}x. \tag{22a, b}$$

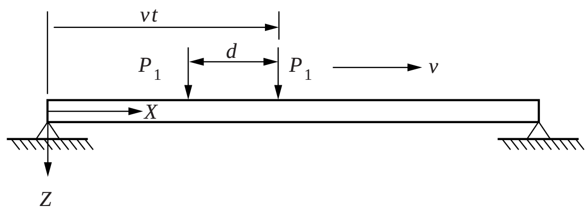

Figure 2. A set of two discrete forces traversing on the beam at constant velocity v.

The value of n_* in equation (21) is

$$n_*(t) = EA\left[\sum_{k=1} B_k \int_0^L \frac{\mathrm{d}w_o}{\mathrm{d}x}\frac{\mathrm{d}W_k}{\mathrm{d}x}\,\mathrm{d}x + \frac{1}{2}\sum_{l=1}\sum_{k=1} B_k B_l \int_0^L \frac{\mathrm{d}W_l}{\mathrm{d}x}\frac{\mathrm{d}W_k}{\mathrm{d}x}\,\mathrm{d}x\right]. \quad (23)$$

Substituting n_* from equation (23) into equation (21) yields a set of non-linear differential equations with coupling of modal amplitudes, which can be solved by the Runge–Kutta method.

A set of two discrete forces moving on the Timoshenko beam at a constant velocity v is considered here as an example to investigate the dynamic responses of the beam. These moving forces with the same magnitude P_1 are depicted in Figure 2. The form of this kind of moving forces is

$$F(x, t) = P_1\delta(x - vt) + P_1\delta(x - vt + d), \quad (24)$$

where $d(\leqslant L)$ is the distance between two forces and δ is the impulse function. The history of the ith modal excitation is

1. $$0 \leqslant t \leqslant t_1(=d/v), \qquad F_j(t) = P_1 W_j(vt); \quad (25a)$$
2. $$t_1 \leqslant t \leqslant T(=L/v), \qquad F_j(t) = P_1 W_j(vt) + P_1 W_j(vt - d); \quad (25b)$$
3. $$T \leqslant t \leqslant T + t_1, \qquad F_j(t) = P_1 W_j(vt - d); \quad (25c)$$
4. $$T + t_1 \leqslant t, \qquad F_j(t) = 0. \quad (25d)$$

5. EXAMPLES

The material properties $\nu = 0{\cdot}2$, $E = 30$ Gpa, $\rho = 2400$ kg/m^3 and shear coefficient $\kappa = 0{\cdot}85$ of the beam are considered in the numerical computation.

5.1. STATIC RESPONSES

The responses of a Timoshenko beam due to its own weight are normally different from those of Bernoulli–Euler beam. The shearing effect is negligible for the ratio of thickness to length being small. Therefore, the results for these two beams will be consistent when the ratio of thickness to length is small. The large deflection theory indicates that the coupling of longitudinal force with transverse deflection stiffens the beam. Consequently, both the deflection as well as the moment of a beam as given by the large deflection theory are less than those by the small deflection theory.

The effects of length on w_o and m_o at the mid-span of the beam ($h = 0{\cdot}5$ m, $b = 0{\cdot}5$ m) are depicted in Figures 3(a) and (b), respectively. According to the small deflection theory for beams, the deflection of the beam is proportional to the fourth order of its own length, while the moment of the beam is proportional to the second order of its length. In the large deflection theory for beams, a large deflection of the beam produces a strong coupling of the longitudinal force with deflection of the beam. Consequently, both the deflection deviation as well as the moment deviation between the small deflection theory and the large deflection theory increase as the length of the beam increases.

The effects of thickness on w_o and m_o at the mid-point of the beam ($L = 20$ m, $b = 0{\cdot}5$ m) are displayed in Figures 4(a) and (b), respectively. The total mass of the beam is linearly proportional to its own thickness, while the bending rigidity of the beam is proportional to the third order of its thickness. According to the small deflection theory for beams, the deflection is inversely proportional to the second order of thickness and the moment is linearly proportional to the thickness. Therefore, a thin beam has a large deflection, which implies that the thin beam has a strong coupling of the longitudinal force with transverse deflection for the large deflection theory. Consequently, both the deflection deviation and the moment deviation between the small deflection theory and the large deflection theory increase as the thickness decreases.

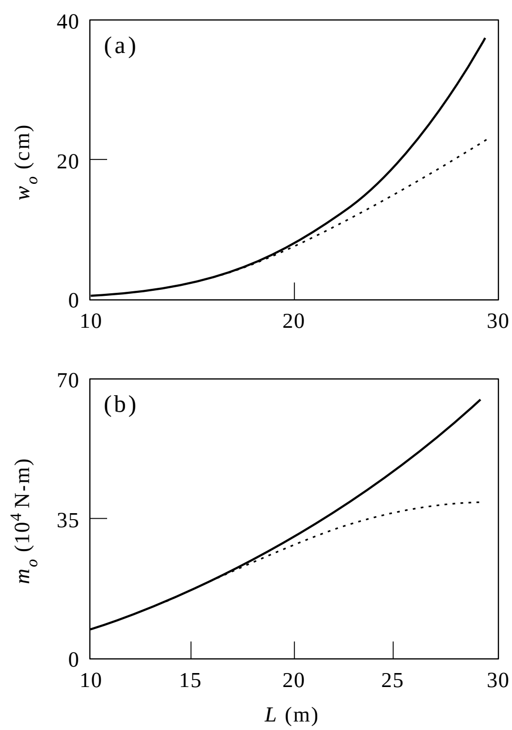

Figure 3. Effect of length of the beam on (a) w_o and (b) m_o at the mid-point of the beam ($h = 0{\cdot}5$ m, $b = 0{\cdot}5$ m). ——, small deflection theory; - - - - , large deflection theory.

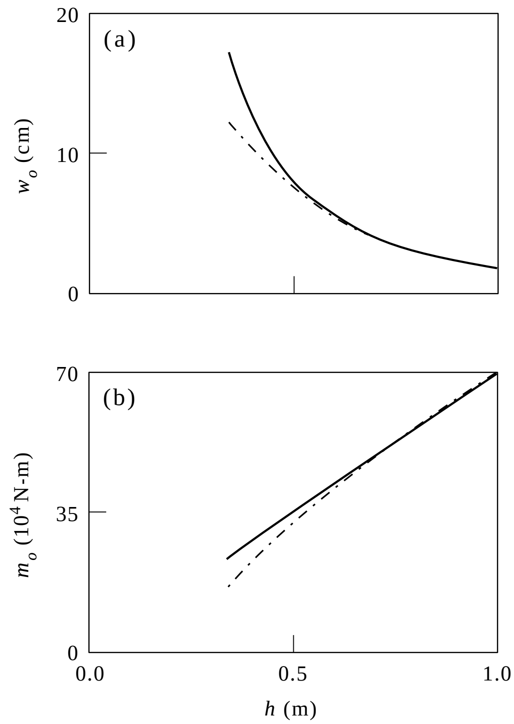

Figure 4. Effect of thickness of the beam on (a) w_o and (b) m_o at the mid-point of the beam ($h = 0{\cdot}5$ m, $b = 0{\cdot}5$ m). ——, small deflection theory; - - - - , large deflection theory.

5.2. DYNAMIC RESPONSES

Based on the small deflection theory, the dynamic responses of the beam obtained by the method of modal analysis converge rapidly. Therefore, it is sufficient to employ the first ten modal frequencies and their corresponding sets of mode shape functions in the numerical computations. The velocity range considered here is from 0 to 200 km/h. The following nomenclature and parameters are defined to illustrate the numerical results: small deflection theory,

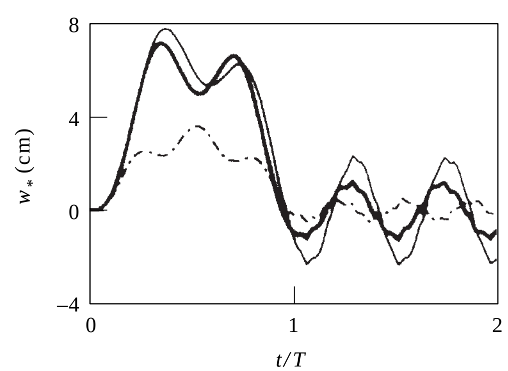

Figure 5. Comparison of three deflection theories on the history of deflection at the mid-point of the Timoshenko beam ($L = 20$ m, $b = 0{\cdot}5$ m, $h = 0{\cdot}5$ m) due to a moving concentrated force ($2P_1 = 3000$ kg). ——, SDT; ▬▬, LDTN; – · – · –, LDTW.

TABLE 1

Both effects of the value of h and the deflection theory on the fundamental period of the Timoshenko beam ($L = 20$ m, $b = 0{\cdot}5$ m) *due to a moving concentrated force* ($2P_1 = 3000$ kg, $v = 50$ km/h)

Theories	$h = 0{\cdot}34$ m	$h = 0{\cdot}5$ m
SDT	0·756 s	0·518 s
LDTN	0·751 s	0·515 s
LDTW	0·504 s	0·373 s

TABLE 2

Both effects of the velocity of the moving concentrated force ($2P_1 = 3000$ kg) *and the deflection theory on the fundamental period of the Timoshenko beam* ($L = 20$ m, $b = 0{\cdot}5$ m, $h = 0{\cdot}34$ m)

Theories	$v = 50$ km/h	$v = 100$ km/h
SDT	0·756 s	0·756 s
LDTN	0·751 s	0·721 s
LDTW	0·504 s	0·345 s

TABLE 3

Both effects of the magnitude of the moving concentrated force ($v = 50$ km/h) *and the deflection theory on the fundamental period of the Timoshenko beam* ($L = 20$ m, $b = 0{\cdot}5$ m, $h = 0{\cdot}34$ m)

Theories	$2P_1 = 1500$ kg	$2P_1 = 3000$ kg
SDT	0·756 s	0·756 s
LDTN	0·753 s	0·751 s
LDTW	0·511 s	0·504 s

SDT; large deflection theory without including the effect of weight of the beam, LDTN; large deflection theory including the effect of weight of the beam, LDTW; maximum dynamic deflection of the beam during motion of the force, w_{*max}; maximum dynamic moment of the beam during motion of the force, m_{*max}.

The reaction moment of the beam at both the simply supported ends is zero, while both the maximum deflection as well as the maximum moment of the beam always occur near the mid-span of the structure.

A comparison of the results obtained by three deflection theories for the history of w_* at the mid-span of the Timoshenko beam due to a moving concentrated force ($v = 50$ km/h, $2P_1 = 3000$ kg, $d = 0$ m) is presented in Figure 5. The coupling effect of n_o with w_* and that of w_o with n_* stiffen the beam. Because of this, LDTW yields minimum w_* among these deflection theories. The period of w_* of the beam during free vibration after the force has left the beam is called the fundamental period of the beam. In Table 1 it is indicated that a thick beam ($L = 20$ m, $b = 0{\cdot}5$ m) has a small value of the fundamental period due to the moving force ($2P_1 = 3000$ kg, $v = 50$ km/h, $d = 0$ m). In Table 1 it is also shown that LDTW yields a minimum value for the fundamental period among these deflection theories. The effects of two different velocities of the moving concentrated load ($2P_1 = 3000$ kg, $d = 0$ m) and three deflection theories for the beam on the fundamental period of the beam ($L = 20$ m, $b = 0{\cdot}5$ m, $h = 0{\cdot}34$ m) are listed in Table 2. The results in Table 2 show that the fundamental period obtained by SDT is independent of the velocity of the moving force, and also that LDTW yields a minimum value for the fundamental period among these deflection theories. A rapidly moving force excites a larger number of modes of the beam than a slow moving force does. The coupling between high frequency modes and low frequency modes causes the magnitude of the fundamental period of the beam to be small for a rapidly moving force. A moving force with a larger magnitude will cause a stronger coupling of low frequency modes with high frequency modes of the beam. Consequently, as can be seen from Table 3, the beam ($L = 20$ m, $b = 0{\cdot}5$ m, $h = 0{\cdot}34$ m) exhibits a smaller value of the fundamental period due to a larger moving force ($v = 50$ km/h, $d = 0$ m).

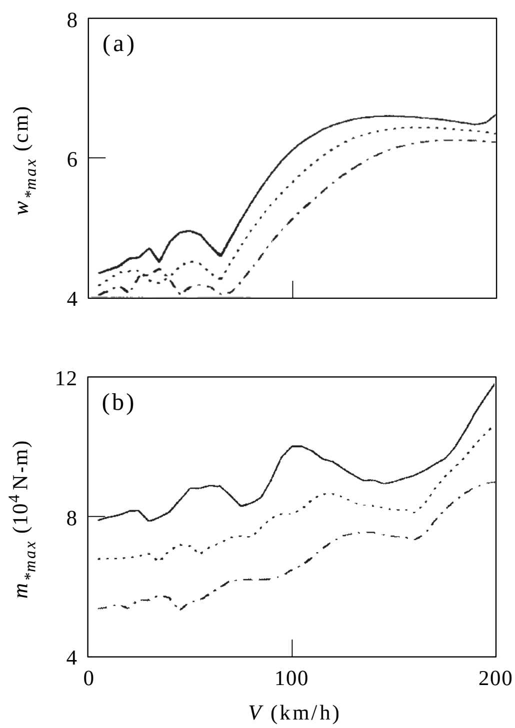

Figure 6. Comparison of three d values of the force ($2P_1 = 3000$ kg) on (a) w_{*max}–v and (b) m_{*max}–v distributions of the Timoshenko beam ($L = 20$ m, $b = 0{\cdot}5$ m, $h = 0{\cdot}34$ m) based on LDTW. ——, $d = 0$ m; - - - - , $d = 2{\cdot}5$ m; – · – · – , $d = 4{\cdot}0$ m.

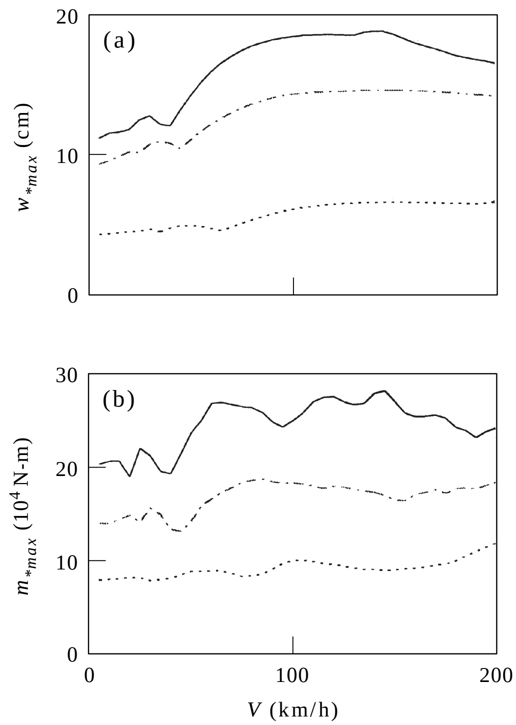

Figure 7. Comparison of three deflection theories on (a) w_{*max}–v and (b) m_{*max}–v distributions of the Timoshenko beam ($L = 20$ m, $b = 0{\cdot}5$ m, $h = 0{\cdot}34$ m) due to a moving concentrated force ($2P_1 = 3000$ kg). ——, SDT; – · – · –, LDTN; - - - - , LDTW.

The effects of three values of d of the moving force ($v = 50$ km/h, $2P_1 = 3000$ kg) on w_{*max}–v and m_{*max}–v distributions of the Timoshenko beam ($L = 20$ m, $b = 0{\cdot}5$ m, $h = 0{\cdot}5$ m) based on LDTW are presented in Figures 6(a) and (b), respectively. Both these figures show that the moving concentrated force induces maximum dynamic deflection and dynamic moment among these forces. Therefore, the dynamic responses induced by the moving concentrated force will only be discussed in the following.

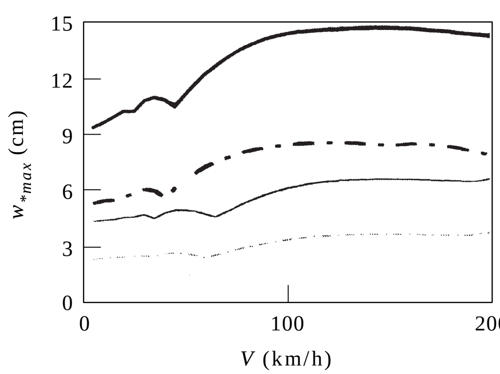

Figure 8. Comparison of two h values on the w_{*max}–v distribution of the Timoshenko beam ($L = 20$ m, $b = 0{\cdot}5$ m) due to a moving concentrated force ($2P_1 = 3000$ kg). ——, LDTW ($h = 0{\cdot}34$ m); - - - - - , LDTW ($h = 0{\cdot}5$ m); —— , LDTN ($h = 0{\cdot}34$ m); – · – · –, LDTN ($h = 0{\cdot}5$ m).

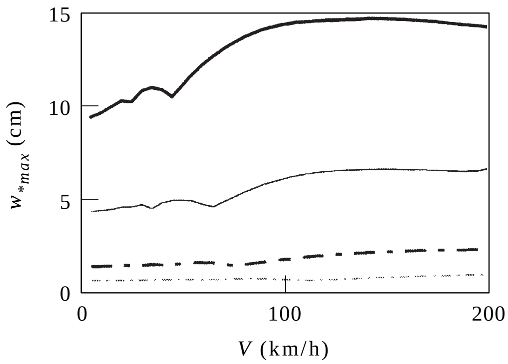

Figure 9. Comparison of two L values on the w_{*max}–v distribution of the Timoshenko beam ($b = 0{\cdot}5$ m, $h = 0{\cdot}34$ m) due to a moving concentrated force ($2P_1 = 3000$ kg). ——, LDTN ($L = 20$ m); -----, LDTN ($L = 10$ m); ——, LDTW ($L = 20$ m); – · – · –, LDTW ($L = 10$ m).

The effects of three different deflection theories on the w_{*max}–v distribution and the m_{*max}–v distribution of the Timoshenko beam ($L = 20$ m, $b = 0{\cdot}5$ m, $h = 0{\cdot}34$ m) due to a moving concentrated force ($2P_1 = 3000$ kg) are plotted in Figures 7(a) and (b), respectively. Both of these figures reveal that the SDT overestimates the maximum deflection as well as maximum moment of the beam.

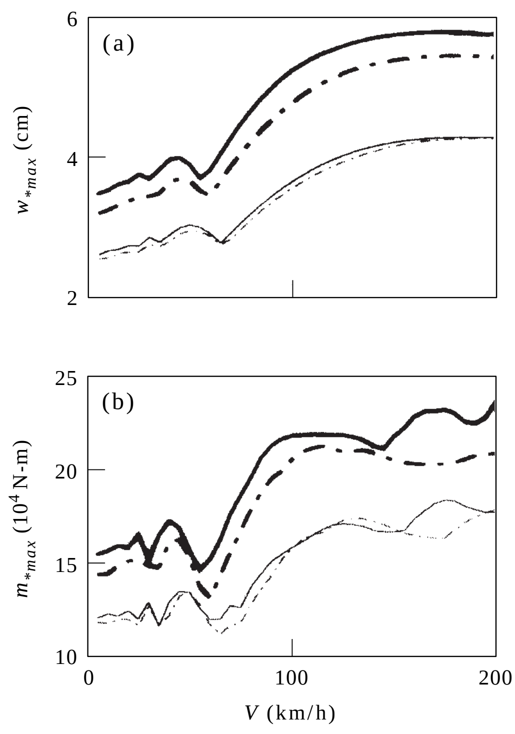

Figure 10. Comparison of two beam models on (a) w_{*max}–v and (b) m_{*max}–v distributions of the beam ($L = 20$ m, $h = 0{\cdot}5$ m, $b = 0{\cdot}5$ m) due to a moving concentrated force ($2P_1 = 3000$ kg). ——, LDTN (Timoshenko); -----, LDTN (Bernoulli–Euler); ——, LDTW (Timoshenko); – · – · –, LDTW (Bernoulli–Euler).

Further, these results also show that the coupling of static longitudinal force with dynamic transverse deflection and that of static deflection with dynamic longitudinal force cause minimum deflection and moment of the beam. Therefore, both static deflection and static longitudinal force of the beam due to its own weight are two very important factors dominating the vibration of the beam.

The effects of two different magnitudes of thickness and two different deflection theories (LDTN, LDTW) on the w_{*max}–v distribution of the Timoshenko beam ($L = 20$ m, $b = 0{\cdot}5$ m) due to a moving concentrated force ($2P_1 = 3000$ kg) are displayed in Figure 8. A thicker beam exhibits a larger bending rigidity and a smaller deflection. In Figure 8 it is shown that the difference between the magnitude of w_{*max} as obtained by LDTN and LDTW decreases as the thickness increases. This indicates the LDTN can be adopted to approximately obtain w_{*max} of a thick beam due to the moving concentrated force without including the effect of weight of the beam.

The effects of two different magnitudes of length and two different theories (LDTN, LDTW) on the w_{*max}–v distribution of the Timoshenko beam ($h = 0{\cdot}34$ m, $b = 0{\cdot}5$ m) due to a moving concentrated force ($2P_1 = 3000$ kg) are presented in Figure 9 to show that a longer beam has large w_{*max}, while the shorter beam has small w_o as well as n_o. Therefore, the effects of coupling of n_o with w_* and w_o with n_* are small for a shorter beam. Consequently, the difference between the magnitude of w_{*max} as obtained by LDTN and LDTW decreases as the length of the beam decreases.

The effects of two different beam models and two different theories (LDTN, LDTW) on w_{*max}–v and m_{*max}–v distributions of the beam ($h = 0{\cdot}5$ m, $L = 20$ m, $b = 0{\cdot}5$ m) due to a moving concentrated force ($2P_1 = 3000$ kg) are displayed in Figures 10(a) and (b), respectively. The Bernoulli–Euler beam is stiffer than the Timoshenko beam. Therefore, in Figure 10(a) it is indicated that w_{*max} of the Timoshenko beam is greater than that of the Bernoulli–Euler beam. The effects of shear deformation and rotatory inertia cause the Timoshenko beam to exhibit greater m_{*max} than that of the Bernoulli–Euler beam, as indicated in Figure 10(b).

6. CONCLUSIONS

For a beam with a small ratio of thickness to length, the deflections as well as moment of the Timoshenko beam caused by its own weight are the same as those of the Bernoulli–Euler beam. The coupling of longitudinal force with transverse deflection stiffens the beam. The effect of this coupling decreases the fundamental period of the Timoshenko beam. A force with a large magnitude traversing on the structure at a high velocity will cause the Timoshenko beam to exhibit a small fundamental period. A long and thin beam exhibits strong coupling of longitudinal force with transverse deflection. The difference in the magnitudes of dynamic deflection and dynamic moment of the Timoshenko beam, as obtained by the large deflection theory including the effect of weight of the beam and without considering this effect is small for a short and thick beam. The Timoshenko beam exhibits both larger deflection and larger moment than the Bernoulli–Euler beam.

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