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LETTERS TO THE EDITOR

FREE VIBRATIONS AND MODES OF CHIMNEYS ON AN ELASTIC FOUNDATION

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1. INTRODUCTION

Vertical structures, such as chimneys, elevated water tanks and TV towers can be modelled as cantilever columns with or without a heavy mass or masses along their height. Often such structures have rotationally symmetric base plate foundations. Generally the dynamic behaviour of chimneys is analyzed by using a lumped model approximation where the lumped masses are assumed to be connected to one another through elastic massless elements, and usually, the structure is assumed to be fixed at the base without considering any interaction between the soil and the structure/foundation.

There are several interaction models which have various degrees of complexity and sophistication. In one of the simpler approximations the translation and rotation of the base of the structure is taken into account by considering the structure and its foundation to be supported by elastic springs. SSI (soil-structure interaction) is used in a pre-design analysis phase of the project and is important especially in designing tall chimneys. The interaction has an effect on free vibration periods, as well as on mode shapes of the structure.

The potential beneficial effect of SSI was apparently first noted by Housner [1] who considered the rocking motion of the structure on the soil. A number of researchers have analytically investigated the foundation uplift in detail [2–5]. Novak [6] studied the effect of soil on structural response of structures. By use of the Stodola method, Rumman [7] and Abu-Saba [8] obtained the natural modes of reinforced concrete, steel and masonry chimneys. The importance of SSI effects on the seismic behaviour of tall chimneys is also noted by Navarro [9]. The dynamic response of slender towers under seismic ground motion is studied by Zembaty and Boffi [10].

In the present study a solution for the determination of natural frequencies and mode shapes of some tall chimney structures has been presented by considering distributed mass and rigidities along the column, and numerical results are given for two chimneys depending on various degrees of soil flexibility with no separation of the base plate from the foundation.

2. PROBLEM

The system considered consists of a column and a rigid circular plate supported on a Winkler foundation. The elastic column has concentrated mass of M_1, $M_2, \ldots, M_n$ having rotatory inertia's $J_1, J_2, \ldots, J_n$ located at points $X_1, X_2, \ldots, X_n$ respectively, as shown in Figure 1. Furthermore, it is assumed that

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the column has a variable flexural rigidity of $EI(X)$ and an additional variable distributed mass of $m_c(X)$ per unit length. For modelling the behaviour of the rigid plate–column system on an elastic foundation of Winkler type that reacts in both compression and tension is considered. The governing equations of the system are obtained by considering the plate and column separately. The vibrations of the column are governed by

$$L[U(X,t),\theta(t)] = [EI(X)U''(X,t)]'' + \left[m_c(X) + \sum_{i=1}^{n} M_i\delta(X-X_i)\right]\ddot{V}$$

$$+\sum_{i=1}^{n} J_i\delta'(X-X_i)\ddot{V}' = 0, \tag{1}$$

where prime and dot denotes differentiation with respect to X and t, respectively; $U(X,t)$ denotes the structural displacement of the column δ the Dirac delta function, $V(X,t)$ the total lateral displacement of the column which consists of the displacement due to the rotation of the rigid support $\theta(t)$ and the structural displacement of the column as shown in Figure 1:

$$V(X,t) = \theta(t)X + U(X,t). \tag{2}$$

The variations of the bending rigidity $EI(X)$ and mass per unit length of the column $m_c(X)$ along the column height are assumed to be controlled by

$$EI(X) = EI_o g(X/H), \qquad m_c(X) = m_{co} f(X/H), \tag{3}$$

respectively, where $g(0) = f(0) = 1$ and H is the length of the column. The total mass of the column and the bending moment at the base of the column can be

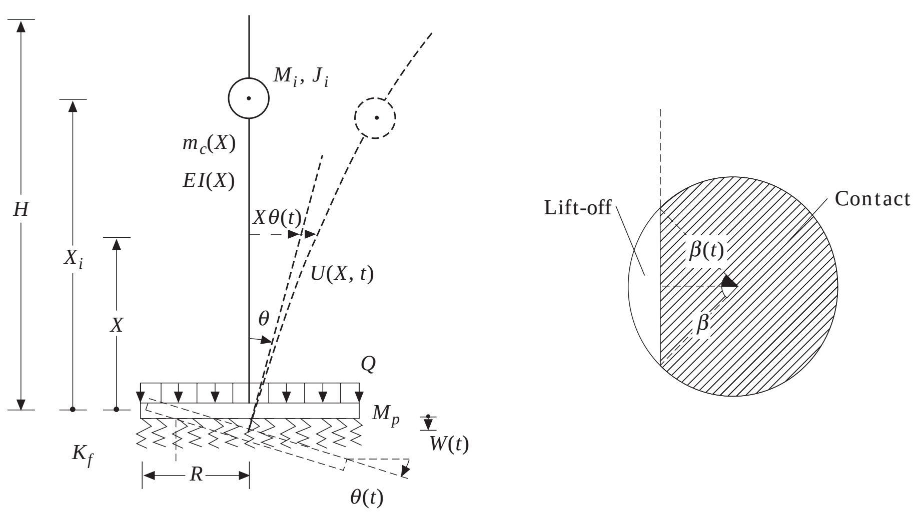

Figure 1. Rigid plate–elastic column on elastic foundation.

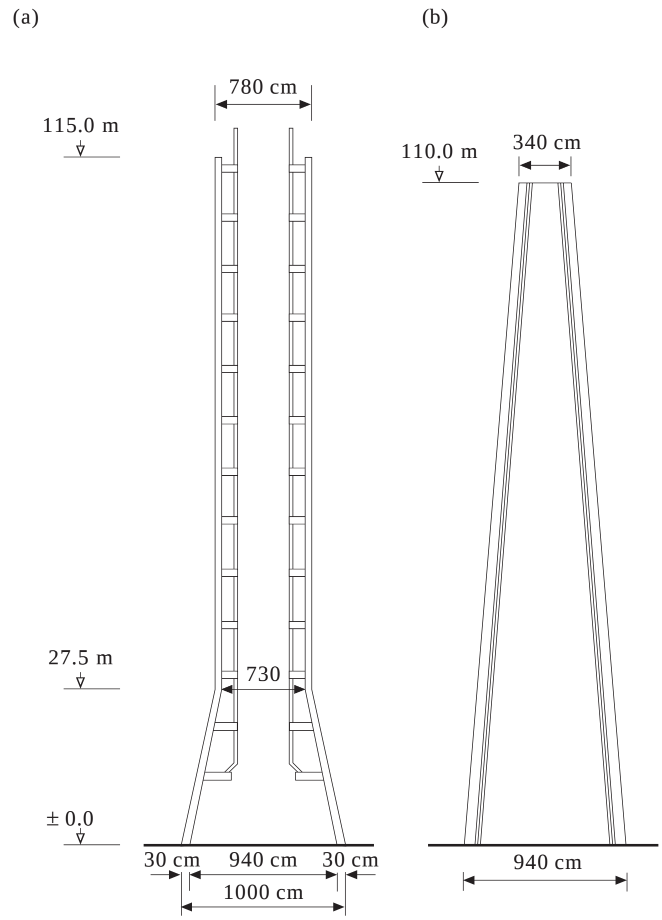

Figure 2. Configurations of chimneys a) first and b) second chimney.

expressed as follows:

$$M_c = m_{co} \int_0^H f(X/H)\,\mathrm{d}X, \qquad M_1 = EI_o U''(X = 0, t). \tag{4}$$

The discretization of the differential equation (1) which is of continuous form is carried out by adopting the Galerkin Method. For this purpose the lateral displacement is approximated as

$$V(X, t) = X\theta(t) + \sum_{m=1}^{\infty} T_m(t) U_m(X), \tag{5}$$

TABLE 1

First three free vibration periods for two chimneys (s)

Free vibration periods	Mode 1	Mode 2	Mode 3
First chimney [11]	3·018 (3·047)	0·470 (0·537)	0·169 (0·332)
second chimney [12]	2·140 (2·080)	0·530 (0·541)	0·152 (0·225)

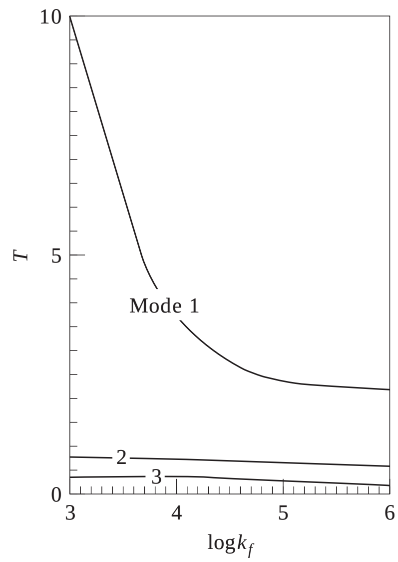

Figure 3. Variation of first three mode periods for various soil stiffnesses for the second chimney.

where the co-ordinate functions $U_m(X)$ are assumed to satisfy the essential boundary conditions of the column. The unsatisfied boundary conditions are compensated to increase the accuracy by updating the application of the method as follows:

$$\int_0^H L(U,\theta)\delta U_m \,\mathrm{d}X - (EIU'')'\delta U_m\Big|_0^H + EIU''\delta U'_m\Big|_0^H = 0. \qquad (6)$$

The remaining part of the system is a circular rigid foundation plate of radius R on a Winkler foundation of stiffness K_f. In addition a uniformly distributed load

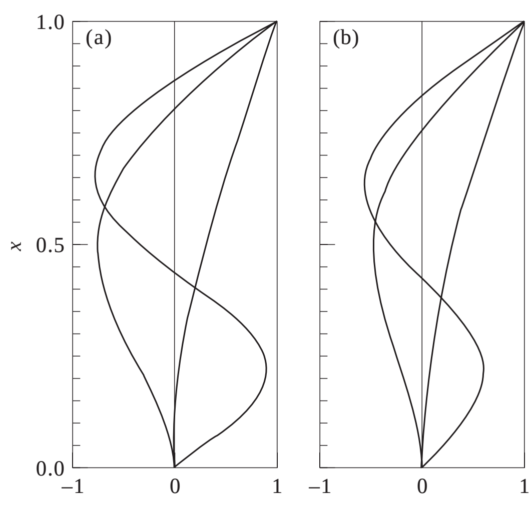

Figure 4. First three mode shapes for a) first and b) second chimney.

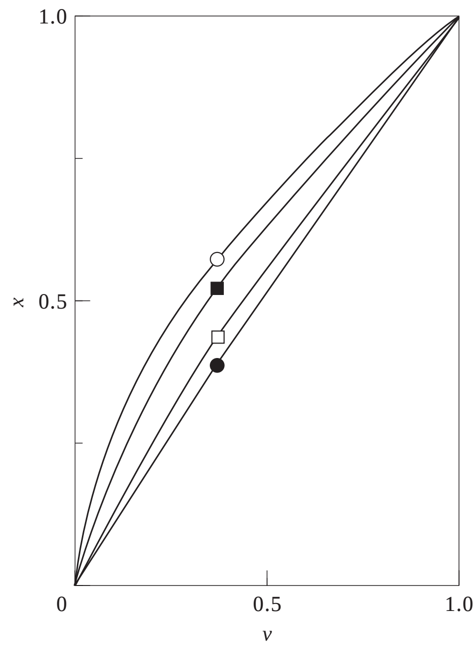

Figure 5. Variation of lateral displacement of second chimney for various soil stiffnesses. k_f values: —●—, 1000; —□—, 5000; —■—, 10 000; —○—, 25 000.

Q is considered on the plate. The governing equation of the rigid plate can be written as:

$$Q\pi R^2 + \left(M_c + M_p + \sum_{i=1}^{\infty} M_i\right)(g - W^{\cdot\cdot}) - F_f = 0, \qquad M_1 - M_p R^2 \theta^{\cdot\cdot}/4 - M_f = 0, \tag{7, 8}$$

with M_p the total mass of the plate, $W(t)$ the vertical displacement of the center of the foundation plate and $\theta(t)$ its rotation and g the gravitational acceleration. F_f and M_f are the force and moment, respectively, exerted by the foundation in the displaced configuration of the plate which can be given as follows in the case of full contact:

$$F_f = K_f R^2 W\pi, \qquad M_f = K_f R^4 \theta\pi/4. \tag{9}$$

The equations (6–8) are coupled through the continuity of the deformation at the joint between the plate and the column. Thus the governing equation of the rigid plate column system can be combined and written in the following matrix form:

$$\mathbf{MV}^{\cdot\cdot} + \mathbf{KV} = \mathbf{0} \tag{10}$$

where $\mathbf{V}^{\mathrm{T}} = (w, \theta, T_1, T_2, T_3, \ldots)$ and the matrix equation (10) represents the small amplitude motion of the plate–column system. The coefficients of the matrix involve integrations of the co-ordinate functions, rigidity and distributed mass of the column. The numerical integration of the coefficients of the matrices and vectors in equation (10) are carried out by using the Gauss–Legendre quadrature procedure. It was found that an acceptable accuracy can be obtained for the graphical representation given here by considering 48 points. The present study deals with the evaluation of natural periods and modes of the chimneys, where an eigenvector problem arises. In this case equation (10) can be expressed as:

$$\mathbf{M}^{-1}\mathbf{K}\mathbf{V} = \omega^2\mathbf{V} \tag{11}$$

where the solution of equation (11) leads to natural vibration periods, mode shapes and eigenvalues ω corresponding to the natural circular frequencies of vibration of the system considered. The mode shapes can be obtained by evaluating $\mathbf{V}$ for each ω in

$$(\mathbf{M}^{-1}\mathbf{K} - \omega^2\mathbf{I})\mathbf{V} = \mathbf{0}. \tag{12}$$

3. NUMERICAL RESULTS AND DISCUSSION

In the numerical solutions, non-dimensional parameters are used as follows:

$$u = U/H, \quad v = V/H, \quad w = W/H, \quad x = X/H, \quad x_i = X_i/H, \quad r = R/H,$$
$$k_f = K_f H^5/(EI_o), \quad m_p = M_p/(m_{co}H), \quad m_i = M_i/(m_{co}H), \quad j_i = J_i/(m_{co}H^3),$$
$$\tau = t/T_c, \quad T_c^2 = m_{co}H^4/(EI_o) \tag{13}$$

where τ denotes non-dimensional time.

Two different types of chimneys are considered and their free vibration periods and mode shapes are obtained. The geometries considered are shown in Figure 2(a) and Figure 2(b). In the numerical process the function of mass variation of the chimney is evaluated to include the mass of inner brick and insulation material as well as the mass of the reinforced concrete shaft.

The first chimney considered is an industrial chimney 115 m tall [11] that has a linear variation of radius along its height up to a certain level, with a constant radius from that level up to the top of the chimney. Ring plate masses of the chimney are each considered as a concentrated mass at the corresponding height of the chimney. For the variation of distributed mass and rigidity the following functions are used:

$$f(x) = 8{\cdot}590(1 - 0{\cdot}920x)^2 - 7{\cdot}590(1 - 0{\cdot}934x)^2 \quad \text{for } 0 \leqslant x \leqslant 0{\cdot}24;$$
$$g(x) = 4{\cdot}561(1 - 0{\cdot}920x)^2 - 3{\cdot}561(1 - 0{\cdot}934x)^2 \quad \text{for } 0 \leqslant x \leqslant 0{\cdot}24;$$
$$f(x) = 1, \qquad g(x) = 1 \quad \text{for } 0{\cdot}24 \leqslant x \leqslant 1{\cdot}00. \tag{14}$$

The second chimney is 110 m tall [12] and the variation of mass and rigidity along its height can be given as follows:

$$f(x) = 2{\cdot}541(1 - 0{\cdot}638x)^2 - 1{\cdot}541(1 - 0{\cdot}710x)^2 \quad \text{for } 0 \leqslant x \leqslant 1{\cdot}00$$
$$g(x) = 1{\cdot}976(1 - 0{\cdot}638x)^2 - 0{\cdot}976(1 - 0{\cdot}660x)^2 \quad \text{for } 0 \leqslant x \leqslant 1{\cdot}00. \tag{15}$$

Through the solution of equations (11) and (12), the free vibration parameters for the two chimneys are obtained and the first three vibration periods are given in Table 1 for the case of infinite foundation stiffness. The fundamental mode periods obtained here show good agreement with those given by previous studies. The vibration periods (T) are given in Figure 3 for the second chimney for variable soil stiffness. Although the second and third free vibration periods depend on soil stiffness only slightly, the first mode period is effected remarkably. Variation of the first three mode shapes of first and second chimneys are given in Figure 4a and 4b, respectively. In Figure 5, the shape of the lateral displacement of the second chimney for variable soil stiffness is shown. For the case of low soil stiffness, variation of displacement displays approximately linear variations and it corresponds almost entirely to rigid rotation of the foundation plate. However, in the case of a stiff soil, as would be expected, elastic displacement of the column becomes pronounced.

4. CONCLUSIONS

By comparing the results of the available studies, it is found that the present model yields quite accurate results for free vibration modes of chimneys of cantilever shape. As is noted, the method is also applicable for a lumped mass model as well. Numerical results reveal that as the soil softens, free vibration periods get longer and mode shapes change and the displacement due to the foundation rotation becomes pronounced compared to the structural displacement. In other words, rocking vibrations become dominant and flexural vibrations get less pronounced. A very practical application of the present study can be found in evaluation of the earthquake behaviour of industrial chimneys and water towers.

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