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FORMULATION OF FOUR POLES OF THREE-DIMENSIONAL ACOUSTIC SYSTEMS FROM PRESSURE RESPONSE FUNCTIONS WITH SPECIAL ATTENTION TO SOURCE MODELLING

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The procedure proposed by Kim, who is also one of the authors of this paper, and Soedel (*Journal of Sound and Vibration* **129**, 237–254) for formulation of four-pole parameters of three-dimensional cavities is revised. In the procedure, four poles were formulated in terms of the pressure responses of the cavities to a point source. However, it is shown that using the point source model for such a purpose may not always be valid because the pressure response function becomes singular at the source point in such a case. In this work, the procedure is modified by employing the surface source model and a new definition of the input point impedance. It is shown that the modified procedure can be applied to three-dimensional acoustic systems. Also, an interesting concept of using a sub-system composed of the cavity and a pair of short pipes is suggested for more accurate analysis.

1. INTRODUCTION

The four-pole matrix is a very convenient concept to analyze complex acoustic systems. It allows various acoustic elements of a system to be formulated independently, then assembled later to form the system equation. Also, related computational effort is reduced substantially because the system equation always remains a two by two matrix. Many application examples are found on the analyses of one-dimensional systems and lumped parameter systems [2, 3].

It is very useful to have four poles of three-dimensional cavities. If four poles of a three-dimensional cavity in an acoustic system are available, the model of the cavity can be easily integrated with one-dimensional or lumped parameter elements to form the system equation. Kim and Soedel proposed a method to formulate four-pole parameters of three-dimensional cavities [1, 4]. In their

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proposed method, four poles of a continuous system were formulated as functions of the pressure responses of the system at the input and output points. The method was applied to analysis of annular cylindrical cavities [1, 4, 5]. Lai and Soedel [6] applied the procedure to analyze thin three-dimensional cavities by specializing it to a two-dimensional formulation. In these works [1, 4–6], pressure response functions are obtained by solving the wave equation of the cavity when it is subjected to a point source input. The approach used in these references may not be valid due to convergence problems as will be shown in this paper.

Because deriving four-pole matrix implies that the three-dimensional cavity is connected to one-dimensional systems, the size of the mass flow source of concern is generally much smaller compared to other dimensions of the cavity. Therefore, it appears to be logical to model the source as a point source, as was done in references [1, 4–6]. However, in this work it is shown that the point source model cannot be used because of its singularity at the source point, and that the surface source model has to be used for the purpose of deriving four poles of three-dimensional cavities. Analytical solutions based on the modal expansion method are used for the related discussion.

For practical applications, it may be more appropriate to use the concept of an extended model made of the cavity and two short pipes attached to input and output ports of the cavity. The geometric complexity due to the two additional pipes may make it very complicated to find analytical solutions to the four poles of the extended model. However, it is not considered a limitation since numerical approach may have to be used for the analysis of virtually any practical three-dimensional cavity. It is explained how the concept can be used to obtain more accurate system four poles.

2. FORMULATION OF FOUR-POLE PARAMETERS FROM PRESSURE RESPONSE FUNCTIONS

A four-pole matrix defines the relationship between the input and output variables of an acoustic system in the frequency domain. For the acoustic system shown in Figure 1, its four-pole equation is defined as

$$\begin{Bmatrix} Q_1 \\ P_1 \end{Bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{Bmatrix} Q_2 \\ P_2 \end{Bmatrix}, \tag{1}$$

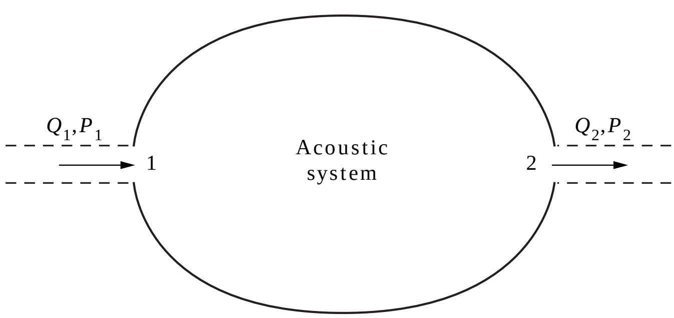


Figure 1. Acoustic system.

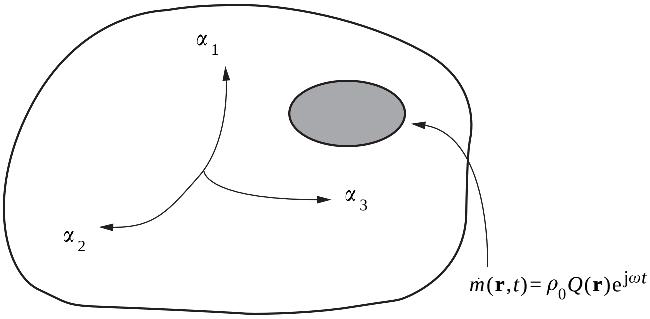


Figure 2. An acoustic cavity with a distributed volume flow source.

where P and Q are the harmonic amplitudes of the acoustic pressure and volume flow rate, subscripts 1 and 2 indicate the input and the output points, respectively, and A, B, C and D are the four-pole parameters.

It was shown the the four-pole parameters of any acoustic system could be formulated from the pressure response functions of the system as below [1]:

$$A = \frac{f_{22}(\omega)}{f_{12}(\omega)}, \qquad B = \frac{1}{f_{12}(\omega)}, \qquad C = -f_{21}(\omega) + \frac{f_{11}(\omega)}{f_{12}(\omega)} f_{22}(\omega), \qquad D = \frac{f_{11}(\omega)}{f_{12}(\omega)}, \tag{2}$$

where ω is the circular frequency and $f_{ij}(\omega)$ is defined as the pressure response of the system at location i when the system is subjected to the harmonic volume flow input of a unit strength at location j. From acoustic reciprocity it can be easily shown that

$$f_{12}(\omega) = f_{21}(\omega). \tag{3}$$

Therefore, four-pole parameters of any general acoustic systems can be derived if pressure response functions are available.

A three-dimensional cavity of an arbitrary shape is shown in Figure 2. The input mass flow source is distributed over a space which is small compared to the size of the cavity. The linear wave equation of this three-dimensional cavity becomes [1]

$$\nabla^2 p(\mathbf{r}, t) - \frac{1}{c_0^2} \frac{\partial^2 p(r, t)}{\partial t^2} = -\frac{\partial m(\mathbf{r}, t)}{\partial t}, \tag{4}$$

where p is the acoustic pressure, c_0 is the speed of sound, $\dot{m}(\mathbf{r}, t)$ defines the mass flow source, and ∇^2 is the Laplacian operator given as

$$\nabla^2 = \frac{1}{A_1 A_2 A_3} \left[\frac{\partial}{\partial \alpha_1} \left(\frac{A_2 A_3}{A_1} \frac{\partial}{\partial \alpha_1} \right) + \frac{\partial}{\partial \alpha_2} \left(\frac{A_3 A_1}{A_2} \frac{\partial}{\partial \alpha_2} \right) + \frac{\partial}{\partial \alpha_3} \left(\frac{A_1 A_2}{A_3} \frac{\partial}{\partial \alpha_3} \right) \right], \tag{5}$$

where α_1, α_2 and α_3 are the curvilinear co-ordinates necessary to define the system, and A_1, A_2 and A_3 are the Lamé parameters.

The solution of equation (4) may be obtained by using the modal expansion method if the natural frequencies and the mode shapes of the acoustic cavity are available. If the acoustic pressure is small compared to the mean pressure of the system, the harmonic mass flow source can be represented as

$$\dot{m}(\mathbf{r}, t) = \dot{M}(\mathbf{r}, \omega)\, \mathrm{e}^{\mathrm{j}\omega t} = \rho_0 Q(\mathbf{r}, \omega)\, \mathrm{e}^{\mathrm{j}\omega t}, \tag{6}$$

where $Q(\mathbf{r}, \omega)$ is the volume flow distribution, and ρ_0 is the average density of the gas in the cavity. According to the procedure described in references [1, 7], the pressure response is obtained as

$$p(\mathbf{r}, t) = P(\mathbf{r}, \omega)\, \mathrm{e}^{\mathrm{j}\omega t}$$

$$= \sum_{l=0}^{\infty} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{\mathrm{j} c_0^2 \omega P_{lmn}(\mathbf{r}) \iiint P_{lmn}(\bar{\mathbf{r}}) Q(\bar{\mathbf{r}}, \omega) A_1 A_2 A_3 \, \mathrm{d}\alpha_1 \, \mathrm{d}\alpha_2 \, \mathrm{d}\alpha_3}{N_{lmn}(\omega_{lmn}^2 - \omega^2)} \mathrm{e}^{\mathrm{j}\omega t}, \tag{7}$$

where $\bar{\mathbf{r}} = \bar{\mathbf{r}}(\alpha_1, \alpha_2, \alpha_3)$ is the integration variable, $\mathrm{j} = \sqrt{-1}$, ω_{lmn} and $P_{lmn}(\mathbf{r})$ are the natural frequencies and the natural modes of the system, and l, m, n are mode numbers. N_{lmn} is given as

$$N_{lmn} = \iiint P_{lmn}^2(\bar{\mathbf{r}}) A_1 A_2 A_3 \, \mathrm{d}\alpha_1 \, \mathrm{d}\alpha_2 \, \mathrm{d}\alpha_3. \tag{8}$$

Equation (2) shows that four poles can be easily formulated if the general expression of the pressure in equation (7) has a unique convergent value at the input and output points. In the following, it will be shown that this condition is not satisfied when a point source model is used. A rectangular cavity with rigid wall is used as an example to explain this problem, taking advantage of the fact that the exact solution is available.

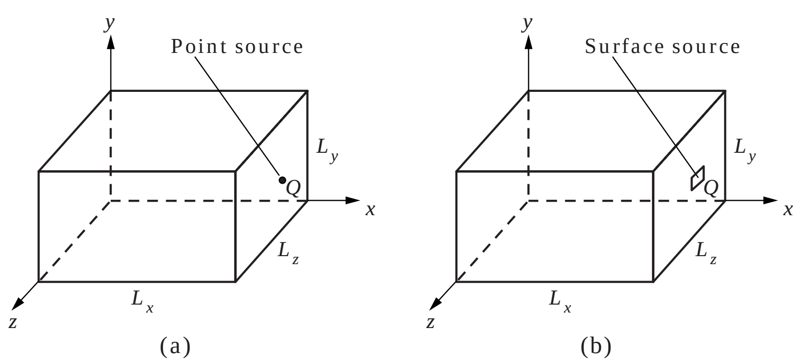


Figure 3. Rectangular cavities with a point source or a surface source: (a) point source, (b) surface source.

3. PRESSURE RESPONSE SOLUTIONS OF A RECTANGULAR CAVITY

For a rectangular cavity shown in Figure 3, $A_1 = 1$, $A_2 = 1$ and $A_3 = 1$ in equations (5), (7) and (8). Also, the natural mode P_{lmn}, and the natural frequency ω_{lmn} become [8]

$$P_{lmn}(x, y, z) = \cos\frac{l\pi x}{L_x}\cos\frac{m\pi y}{L_y}\cos\frac{n\pi z}{L_z}, \qquad l, m, n = 0, 1, 2, \ldots . \tag{9}$$
$$\omega_{lmn} = \pi c_0\left[\left(\frac{l}{L_x}\right)^2 + \left(\frac{m}{L_y}\right)^2 + \left(\frac{n}{L_z}\right)^2\right]^{1/2},$$

Therefore, the pressure response function in equation (7) becomes

$$P(x, y, z, \omega) = \sum_{l,m,n=0}^{\infty}\frac{\mathrm{j}c_0^2\omega P_{lmn}(x, y, z)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\dot{M}(\bar{x}, \bar{y}, \bar{z}, \omega)P_{lmn}(\bar{x}, \bar{y}, \bar{z})\,\mathrm{d}\bar{x}\,\mathrm{d}\bar{y}\,\mathrm{d}\bar{z}}{(\omega_{lmn}^2 - \omega^2)N_{lmn}}, \tag{10}$$

where N_{lmn} becomes

$$N_{lmn} = \begin{cases} L_xL_yL_z/8, & l, m, n = 1, 2, \ldots, \\ L_xL_yL_z/4, & l, m = 1, 2, \ldots, \quad n = 0, \quad \text{and circulations}, \\ L_xL_yL_z/2, & l = 1, 2, \ldots, \quad m = n = 0, \quad \text{and circulations}, \\ L_xL_yL_z, & l = m = n = 0. \end{cases} \tag{11}$$

3.1. RESPONSE TO THE POINT SOURCE

A point source locating at $\mathbf{r}_s = (x_s, y_s, z_s)$ is described as,

$$\dot{M}(x, y, z, \omega) = \rho_0 Q_0\delta(x - x_s)\delta(y - y_s)\delta(z - z_s), \tag{12}$$

where ρ_0 is the mean density of the acoustic medium, $\mathbf{r}_s$ defines the location of the source point, Q_0 is the volume flow harmonic amplitude, and $\delta(\cdot)$ indicates the Dirac delta function.

Substituting equation (12) into equation (10), the pressure response due to the point source becomes

$$P(x, y, z, \omega) = \mathrm{j}\rho_0 c_0^2\omega Q_0\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{P_{lmn}(x_s, y_s, z_s)P_{lmn}(x, y, z)}{(\omega_{lmn}^2 - \omega^2)N_{lmn}}. \tag{13}$$

By letting $(x_1, y_1, z_1) = (x_s, y_s, z_s)$ and $Q_0 = 1$ in equation (13), $f_{11}(\omega)$ and $f_{21}(\omega)$ are considered as

$$f_{11}(\omega) = P(x_1, y_1, z_1, \omega)|_{Q_0=1}, \qquad f_{21}(\omega) = P(x_2, y_2, z_2, \omega)|_{Q_0=1}. \tag{14}$$

$f_{22}(\omega)$ is obtained in a similar manner. It will be shown that $f_{11}(\omega)$ and $f_{22}(\omega)$ obtained from this model are divergent, and therefore cannot be used to formulate four poles.

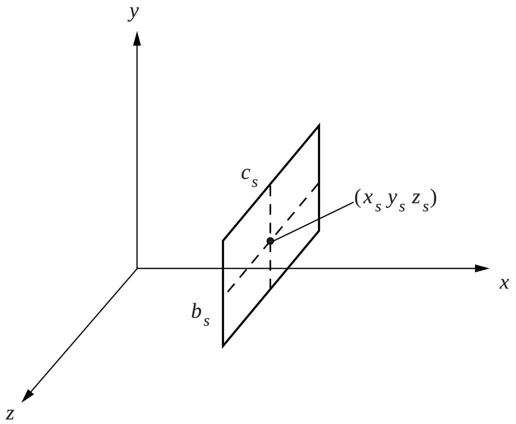


Figure 4. Geometry of the source: b_s and c_s are the dimensions of the source surface; (x_s, y_s, z_s) is the co-ordinate of the center of the surface source or the location of the point source.

3.2. RESPONSE TO THE SURFACE SOURCE

As shown in Figures 3 and 4, the same cavity is subjected to the input flow distributed uniformly over a small rectangular surface. This model approximates a three-dimensional cavity with a small attached rectangular pipe. The mass flow source of unit strength is expressed as

$$\dot{M}(x, y, z, \omega) = \rho_0 \frac{Q_0}{b_s c_s} \delta(x - x_s)\left[H\left(y - y_s + \frac{b_s}{2}\right) - H\left(y - y_s - \frac{b_s}{2}\right)\right]$$

$$\times\left[H\left(z - z_s + \frac{c_s}{2}\right) - H\left(z - z_s - \frac{c_s}{2}\right)\right], \tag{15}$$

where $\mathbf{r}_s = (x_s, y_s, z_s)$ is the location of the center of the source surface, b_s and c_s indicate the size of the rectangular source surface, and $H(\cdot)$ is the unit step function.

Substituting equation (15) into equation (10), the pressure response becomes

$$P(x, y, z, \omega) = \mathrm{j}\rho_0 c_0^2 \omega \frac{Q_0}{b_s c_s} \sum_{l=0}^{\infty} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{P_{lmn}(x, y, z) \cos \dfrac{l\pi x_s}{L_x} C_m C_n}{(\omega_{lmn}^2 - \omega^2) N_{lmn}}, \tag{16}$$

where

$$C_m = \begin{cases} b_s, & \text{if } m = 0, \\ \dfrac{2L_y}{m\pi} \cos \dfrac{m\pi y_s}{L_y} \sin \dfrac{m\pi b_s}{2L_y}, & \text{if } m \neq 0, \end{cases} \tag{17}$$

and

$$C_n = \begin{cases} c_s, & \text{if } n = 0, \\ \dfrac{2L_z}{n\pi} \cos \dfrac{n\pi z_s}{L_z} \sin \dfrac{n\pi c_s}{2L_z}, & \text{if } n \neq 0. \end{cases} \tag{18}$$

Pressure response functions $f_{ij}(\omega)$ $(i, j = 1, 2)$ may also be obtained by substituting equation (16) into equation (14).

4. COMPARISON OF THE TWO PRESSURE RESPONSE SOLUTIONS TO SHOW THE CONVERGENCE PROBLEM OF THE POINT SOURCE SOLUTION

The pressure responses defined in equation (13) and equation (16) are considered. The following conditions are used for numerical calculations. The cavity has a cubic geometry, each side of which is 0·2 m long ($L_x = L_y = L_z = 0{\cdot}2$). $\rho_0 = 1{\cdot}21$ kg/m^3 and $c_0 = 343$ m/s. Flow sources are on the boundary surface defined by $x = L_x$, as shown in Figure 3. Both the point source and the center of the rectangular surface source are located at the point of (L_x, $5L_y/9$, $5L_z/9$). The size of the surface source is taken as $b_s = L_y/14$ and $c_s = L_z/14$.

4.1. PRESSURE RESPONSES AT POINTS NOT ON THE SOURCE SURFACE

At first, two pressure response solutions in equations (13) and (16) are compared at two points, $\mathbf{r} = (0{\cdot}01L_x, 5L_y/9, 5L_z/9)$ and $\mathbf{r} = (0{\cdot}99L_x, 5L_y/9, 5L_z/9)$. The first point is relatively far away from the source, while the second point is very close to the source.

Figure 5 compares the pressure responses at the first point (far from the source) at three frequencies (100, 500 and 1000 Hz) as functions of the number of natural modes used in the expansion in equations (13) and (16). It is seen that responses from either model converge to approximately the same value at each frequency.

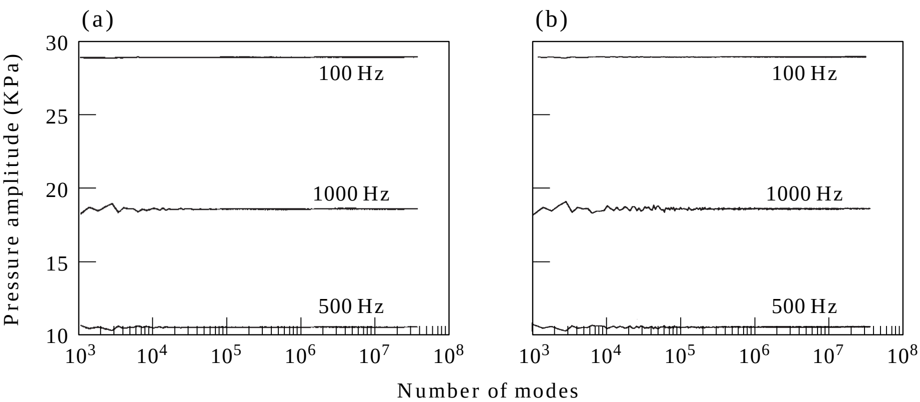


Figure 5. Pressure response at a far field point: (a) response to a surface source, (b) response to a point source.

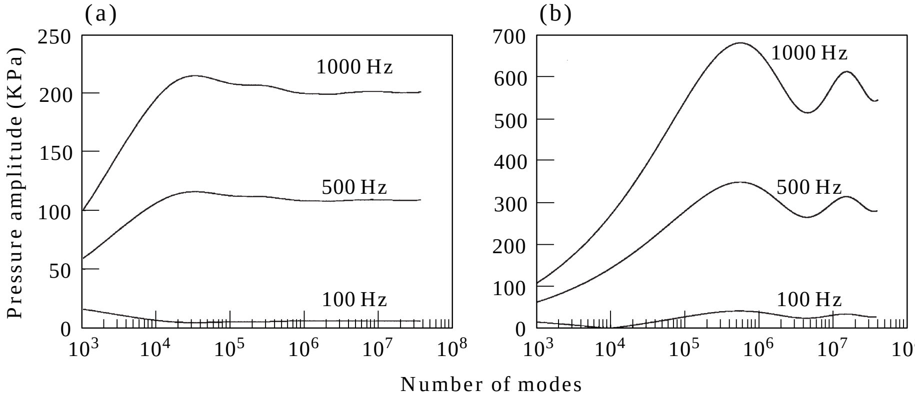


Figure 6. Pressure response at a near field point: (a) response to a surface source, (b) response to a point source.

This means that the source model influences very little at far field, as should be expected.

Figure 6 compares the responses at the second point (very close to the source) at the same three frequencies as in Figure 5. The figure shows that the results from the two models, while both are bounded, converge to completely different pressure response values. For example, while the pressure at 1000 Hz is calculated to be approximately 210 Pa from the surface source model, it is calculated to be approximately 590 Pa from the point source model. This indicates that the pressures induced by the point source and by the surface source (or any other realistic sources) which have the same source strength are completely different in the near field. Figure 7 is a conceptual explanation of this near field effect. Imagining that one moves toward the source point or source surface, as he approaches very close to the source, the surface source (Figure 7(a)) and the point source (Figure 7(b)) would look completely different.

Furthermore, Figure 6 shows that a huge number of modes have to be included for the expansion solution to obtain a converged pressure response solution in the near field, particularly if the point source model is used. For example, the pressure response shown in Figure 6(b) still fluctuates even after ten million modes are added in equation (13).

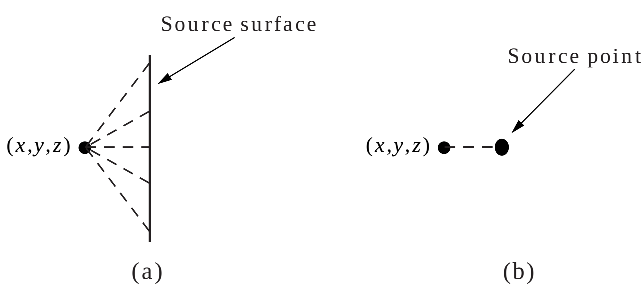


Figure 7. Illustration of the difference in the responses at a near field point due to source geometry: (a) surface source, (b) point source.

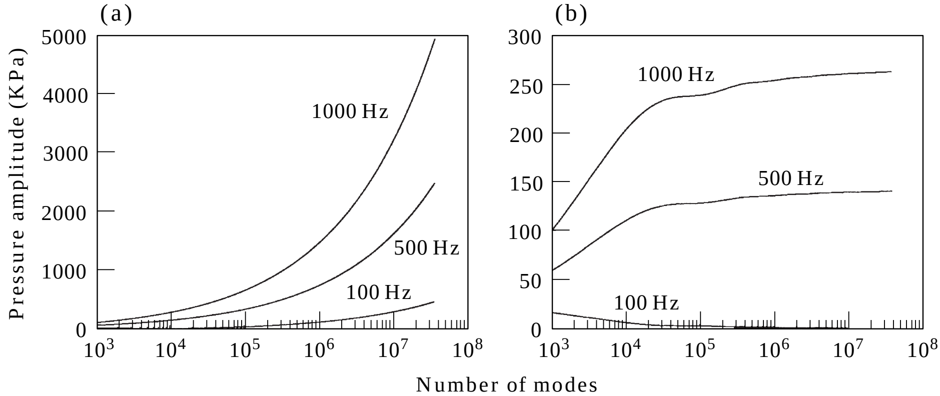


Figure 8. Pressure responses at source point: (a) point source, (b) surface source (at the center of source surface).

4.2. PRESSURE RESPONSES AT SOURCE POINTS CALCULATED FROM THE TWO SOURCE MODELS

Figure 8(a) shows the pressure responses at the source point (x_s, y_s, z_s) obtained from the point source model as functions of the number of modes used in the calculation. The figure clearly shows divergence of the solution. In comparison, Figure 8(b) shows the pressure response obtained from the surface source model at the center of the source surface, also as a function of the number of terms used in the modal expansion. It shows a converging trend of the solution although more than ten million terms have to used due to a very slow convergence rate.

4.3. DIVERGENCE OF THE POINT SOURCE SOLUTION

The pressure response at the source point obtained from the point source model is, from equation (13):

$$P(x_s, y_s, z_s, \omega)$$

$$=\mathrm{j}\rho_0 c_0^2 \omega Q_0 \sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{P_{lmn}^2(x_s, y_s, z_s)}{\left\{\pi^2 c_0^2\left[\left(\frac{l}{L_x}\right)^2+\left(\frac{m}{L_y}\right)^2+\left(\frac{n}{L_z}\right)^2\right]-\omega^2\right\}N_{lmn}}. \tag{19}$$

For a given frequency, ω, the convergence of the above series is equivalent to the convergence of the series

$$\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{c_2}{l^2+m^2+n^2-c_1^2}, \tag{20}$$

where c_1 and c_2 are constants. Performing the summation of the series for n, with l and m fixed, results in a finite value. The second summation for m, with l fixed, is a sum of infinite terms with finite values, and therefore its value will become infinite. Therefore, the series in equation (20), or its double summation form (which is the pressure response of a two-dimensional cavity to a point source at the source point) becomes divergent. Conditions to have a finite pressure response at the source point are then summarized as follow. (1) The dimension of the source geometry should be less only by one than that of the cavity. Therefore, the line source model must be used for two-dimensional cavities and the surface source model must be used for three-dimensional cavities. (2) A point model can be used only for one-dimensional systems.

Convergence of the series in expression (20) may also be checked from the convergence at infinity of the following integration, which is an equivalent integral form of the series in terms of convergence:

$$\int_0^\infty \int_0^\infty \int_0^\infty \frac{c_2\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z}{x^2 + y^2 + z^2 - c_1^2}. \tag{21a}$$

This integration may be conducted in the spherical co-ordinate system, and it becomes

$$\frac{\pi}{2} \int_0^\infty \frac{c_2}{r^2 - c_1^2}\, r^2\,\mathrm{d}r, \tag{21b}$$

where $r = \sqrt{x^2 + y^2 + z^2}$. It becomes obvious that the integration in expression (21) does not have a finite value. It indicates that the series in expression (20) or equation (19) is not bounded as well.

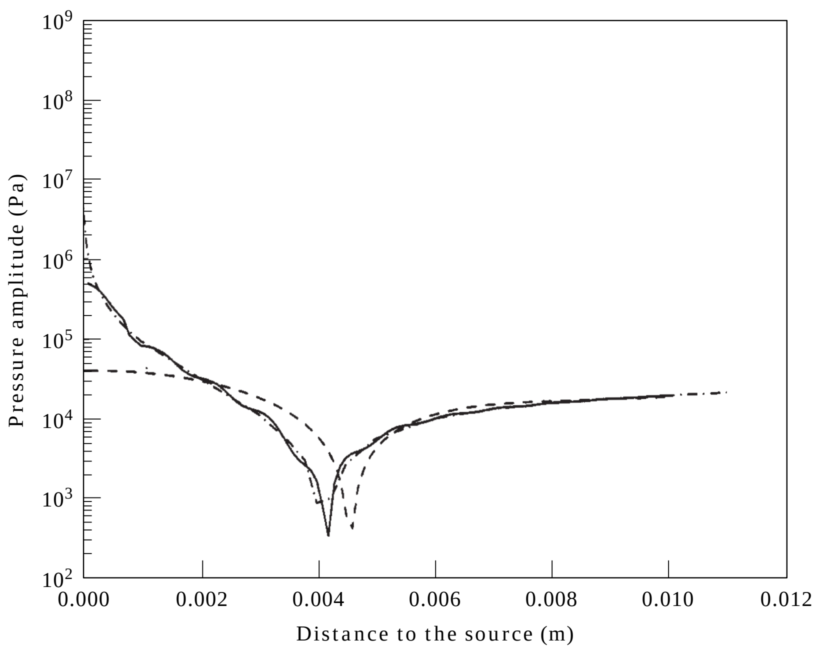


Figure 9. Pressure responses to a point source calculated by different methods ($f = 100$ Hz): ——, solution using modal expansion method with 47×10^6 terms; —–, solution using modal expansion method with $0{\cdot}1 \times 10^6$ terms; – · –, solution using the BEM method.

Figure 9 shows the pressure responses to the point source at the frequency of 100 Hz plotted as a function of the distance from the source point. The solid line is obtained from equation (13) by adding up almost 47 million terms. The dashed line is obtained using 100 000 modes. The dash-dot line is calculated using the Boundary Element Method (BEM). The BEM solution and the summation of 47 million term indicate that the pressure at zero distance, which corresponds to $f_{11}(\omega)$ or $f_{22}(\omega)$, becomes infinite.

Because the series in equation (19) converges extremely slowly, the authors used a special scheme to prevent higher order terms from being ignored due to limited digits available in a computer. In the scheme, higher order terms were added into an intermediate series until their sum reached a value sufficiently large for the computer to recognize relative to the current total sum. In the works reported in references [1] and [6], the method formulating four poles by equation (2) and using point source models was applied to an annular cavity, which required use of Bessel's functions. This made accurate numerical calculation of higher modes even more difficult.

It should be noted that this divergence problem at the source point is not caused by the modal expansion method, but by the inherent limitation of the point source model. Another way to look at this divergence problem is as follows.

The total pressure due to a point source at an point in a three-dimensional cavity with closed boundary can be considered as the result of the combination of the direct response to the point source and the response to the waves reflected from the boundary surface [9] as

$$P(\mathbf{r},\omega)=\frac{\mathrm{j}\rho_0\omega Q_0}{4\pi|\mathbf{r}-\mathbf{r}_s|}\,\mathrm{e}^{-\mathrm{j}k|\mathbf{r}-\mathbf{r}_s|}+\text{(reflected waves from the boundaries)},\qquad(22)$$

where $|\mathbf{r}-\mathbf{r}_s|$ is the distance from the source to the response point. The response due to the boundary reflections, the second term in equation (22), is always finite because it is the response to surface sources. However, the first term becomes infinite as the response point approaches the source point ($|\mathbf{r}-\mathbf{r}_s|\to 0$).

This divergence problem associated with the point source model in three- or two-dimensional cavities does not become an issue unless the response at the source point itself or at a point very near to the source has to be found. Unfortunately, the pressure responses at the source points ($f_{11}(\omega)$ or $f_{22}(\omega)$) are used in the formulation of four-pole parameters based on equation (2). Therefore, a modification of the procedure is proposed in the next section to overcome this problem.

5. REVISED METHOD TO FORMULATE FOUR-POLE PARAMETRS OF THREE-DIMENSIONAL CAVITIES

A straightforward revision of the method would be to calculate pressure response functions using the surface source model as follows:

$$f_{ii}(\omega)=\int_{\Gamma_s}\frac{P(\mathbf{r},\omega)}{u(\mathbf{r},\omega)}\,\mathrm{d}\Gamma_s/A_s^2,\qquad i=1,2,\qquad(23)$$

$$\int_{\Gamma_s}u(\mathbf{r},\omega)\,\mathrm{d}\Gamma_s=1,$$

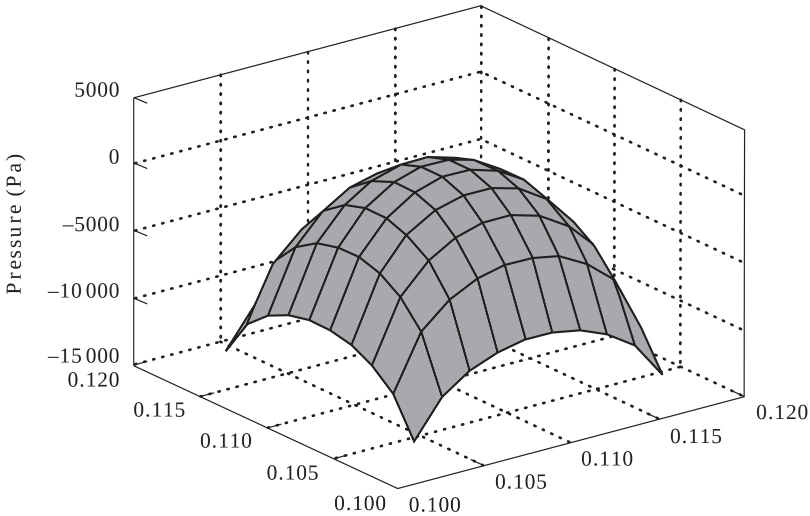


Figure 10. Pressure distribution on the source surface ($f = 100$ Hz).

where $P(\mathbf{r}, \omega)$ is the pressure response calculated based on the surface source model, Γ_s indicates the source surface, A_s is the area of the source surface, and $u(\mathbf{r}, \omega)$ is the velocity distributed on the source surface. If a uniformly distributed source of unit strength is employed, equation (23) can be simplified as

$$f_{ii}(\omega) = \int_{\Gamma_s} P(\mathbf{r}, \omega)\, \mathrm{d}\Gamma_s / A_s, \qquad i = 1, 2. \tag{24}$$

Figure 10 show the pressure induced on the source surface in response to the surface source of a uniform strength. The calculation is made at 100 Hz, while all other parameters are kept the same as in the calculation for Figure 8(b). One problem immediately observed from the figure is that the pressure distribution on the source surface varies in a very wide range, which makes the validity of the averaging process in equations (23) and (24) questionable.

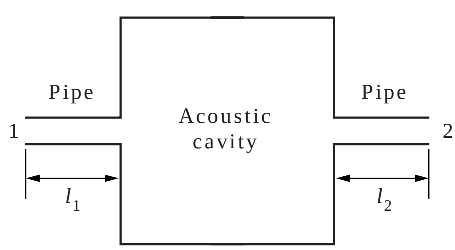
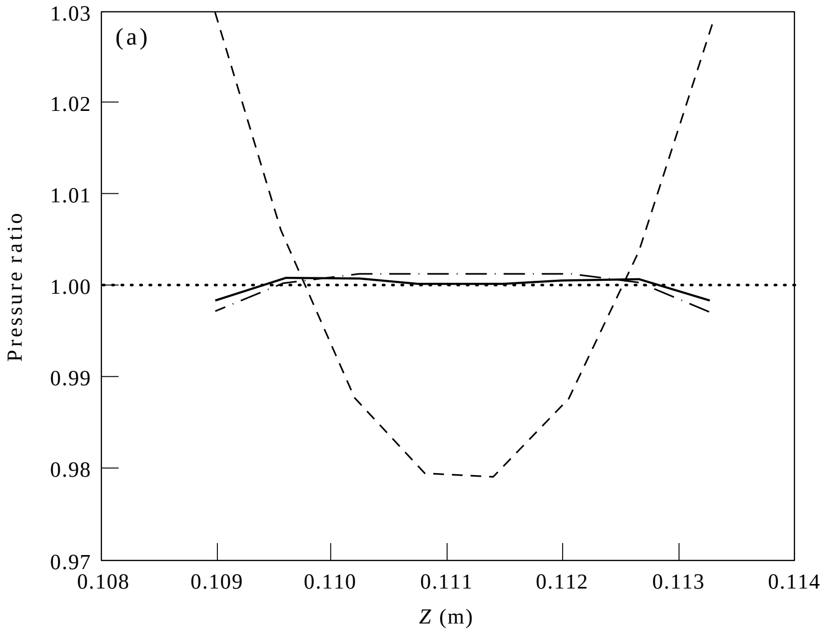


Figure 11. Extended acoustic system with the cavity and two pipes.



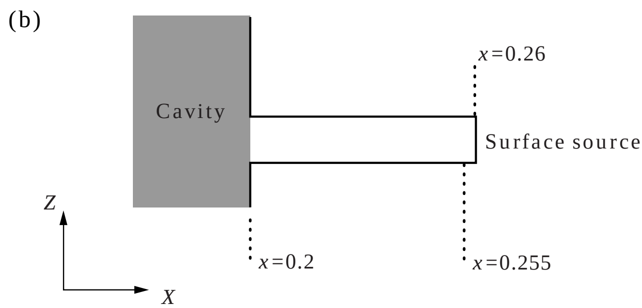


Figure 12. Pressure responses in the attached pipe. (a) Pressure distributions along the lines ($y = y_s$). Pressure ratio is defined as the ratio of the actual pressure and the mean pressure along the line. ——, At location $\mathbf{x} = 0{\cdot}255$ m; – – –, at location $\mathbf{x} = 0{\cdot}2$ m; – · –, at location $\mathbf{x} = 0{\cdot}2275$ m. (b) Illustration of the locations of the lines where pressures are calculated; $\mathbf{x} = 0{\cdot}255$ is very close to the surface source, and $\mathbf{x} = 0{\cdot}2$ is at the intersection of the pipe and cavity.

To avoid this problem, two very short pipes may be added to the cavity, as shown in Figure 11. The length of the pipes may be taken as short as possible, as long as the plane wave condition develops on the source plane to make the pressure across the section almost uniform, which will minimize the error involved in the averaging process in equation (24). In actual application, the cross-section areas of the pipes should be taken as the same as those of the pipes to be connected to the cavity.

Figure 12(b) shows the pressure distribution on the input source surface, calculated by using the Boundary Element Method (BEM), when two 6 cm long pipes are attached to the same cavity used previously. Figure 12(a) shows the locations where pressures are calculated. After obtaining its four poles, the system shown in Figure 11 may be combined with other one-dimensional or lumped parameter elements as needed. It was somewhat surprising to see that the pressure became relatively uniform not only on the source plane but also at the junction to the cavity.

Obviously, a numerical method such as the BEM may have to be used for acoustic analysis when pipes are attached even with a rectangular cavity. However, this is not believed to be a serious limitation of this new method because most three-dimensional cavities in practice have to be analyzed by a numerical method. The proposed method will derive four poles of a sub-system, composed of the cavity and two very short pipes, but not the cavity itself. Again, this is not a serious limitation because four poles of a cavity become useful when the cavity is connected to one-dimensional acoustic elements.

6. CONCLUSIONS

A numerical problem encountered when four-pole parameters of three-dimensional cavities are derived following the method proposed by Kim, who is the second author of this paper, and Soedel [1] is discussed. In their procedure, the pressure responses at the input point and the output point are used to formulate four poles. It has been shown that the point source model may be invalid for deriving four poles of general two- or three-dimensional cavities due to the singularity at the point source. This problem is studied in detail by investigating the exact solutions obtained by using the modal expansion method and numerical solutions obtained by using the Boundary Element Method. Theoretical and practical implications of this singularity at the source point are discussed. Necessary modifications of the original procedure are proposed to overcome this difficulty. The new procedure uses the surface source model and the concept of an extended system model by including two short pipes to the cavity at the input and output sides. It is explained that the revised procedure can be used to formulate four poles of the extended system, which serves virtually the same purpose as the four poles of the three-dimensional cavity itself.

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