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FORCE FREQUENCY SHIFTING FOR STRUCTURAL EXCITATION

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This paper develops concepts and equations for the characterisation of a frequency shifting force shaker which can be employed to excite low frequency structures into resonance. Force which is developed at a high frequency can be employed for excitation at a very low frequency. Frequency shifting is accomplished by moving a shaker to and fro in a reciprocating manner parallel to the structure which it is exciting while the shaker force acts perpendicular to the structure. Simple equations which relate the generalised force ratio to a length ratio are developed along with the positions on a structure where a frequency shifting shaker should be placed. This information allows the engineer some simple choices for the design and operation of a frequency shifting shaker. Also, this investigation develops criteria for which the frequency shifting shaker outperforms an ordinary shaker operating at the natural frequency of a structure.

1. INTRODUCTION

Large structures such as buildings, towers and bridges may have natural frequencies much lower than one Hertz. For the purpose of experimental study of the dynamic behaviour of such structures a shaker is usually required to force them into motion, normally at the lowest natural frequency. At low frequencies the performance of out-of-balance mass shakers is poor due to the low operational frequency. The purpose of this paper is to present a method of force frequency shifting which employs an out-of-balance mass shaker, electrodynamic shaker or other device which operates efficiently at a high frequency and shifts the force to a lower frequency. The starting point for the approach is an analysis given by Timoshenko [1] of a pulsating load, moving at a constant velocity across a bridge. The generalised force acting on any mode of the bridge has a sum and difference frequency under such a loading condition. From the Timoshenko analysis the concept of a frequency shifting vibration shaker was introduced by Koss [2] and Koss *et al*. [3] giving both theoretical and experimental results of investigations which verify the frequency shifting principle. The principle is based upon moving a vibration shaker to and fro at one frequency whilst the vibration shaker acts as a different frequency. A force is generated at a sum and difference frequency from these two operational frequencies; the amplitude of the force at these two

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frequencies only depends upon the amplitude of the force of the shaker frequency. Work presented herein applies this concept to four beam/bridge types resulting in simple formulas for calculating generalised force from geometrical properties of the structure and the throw of the shaker. This paper also gives criteria for the frequency shifting shaker, FSS, to generate a generalised force greater than an out-of-balance mass shaker which operates at a low frequency, i.e., the difference frequency. A second method of analysis is given in Appendix 1 where the results are similar to that given in section 3 in the body of the paper. Experimental verification is given in Appendix 2.

2. ANALYSIS OF SHAKER

2.1. TIMOSHENKO ANALYSIS

The Timoshenko analysis is for a simply supported bridge whose mode shape, $\phi_i(x)$, for vibrational mode i is given by

$$\phi_i(x) = \sin(i\pi x/\ell), \tag{1}$$

where x is distance across the bridge and ℓ is the bridge length; see Figure 1. A generalised force for mode i, Q_i, is given in [1] for the condition of a force P moving at a constant velocity, v, across the bridge. This generalised force is

$$Q_i = P\cos\omega t \sin(i\pi vt/\ell). \tag{2}$$

In equation (1) the term vt replaces x in order to obtain equation (2); this is the Timoshenko method. The force, P, is given by

$$P = m_{ob} e\omega^2, \tag{3}$$

where ω is radian frequency and m_{ob} is out-of-balance mass with eccentricity e. The generalised force in equation (2) can be expanded out to give Q_i as follows

$$Q_i = P(\sin(i\pi v/\ell - \omega)t + \sin(i\pi v/\ell + \omega)t)/2. \tag{4}$$

Thus, each mode i of the simply supported bridge is acted on by a generalised force at a difference and at a sum frequency. Also, the force P is generated at frequency ω and at low frequencies the force P would be small as ω is small, thus operating a shaker at a low frequency would not be of interest for exciting a low frequency structure by this technique. For frequency shifting to be an efficient method of low frequency excitation, the force P should be generated at a relatively high

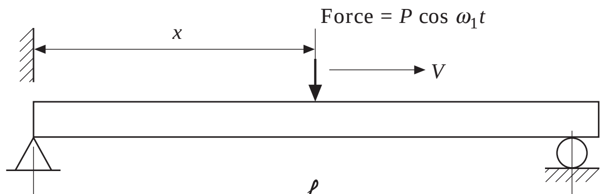

Figure 1. Oscillating force moving at a constant velocity v across a simply supported bridge of length ℓ.

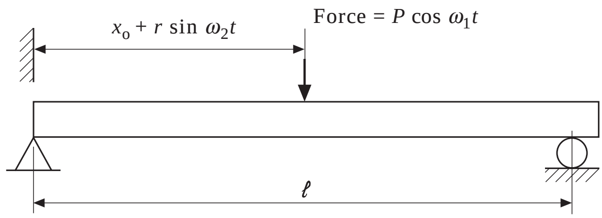

Figure 2. Representation of a reciprocating shaker on a bridge of length ℓ. x_0 is the shaker equilibrium position and r is the amplitude of reciprocating motion (throw).

frequency. The sum frequency term will also excite the ith structural mode at a frequency higher than that of the shaker.

The difference frequency term of equation (4) presents somewhat of a practical problem as illustrated by the following example. Given a bridge of 20 m length, natural frequency of 0·5 Hz (π rads^{-1}) and a shaker operating frequency of 5 Hz, the velocity of the shaker across the bridge would have to be 180 ms^{-1} to shift the force from 5 Hz to 0·5 Hz. This method of frequency shifting seems to be impractical due to the high velocities required. Another method of frequency shifting is presented in the next section using reciprocating motion to slide an out-of-balance mass shaker to and fro along a structure. The method of analysis is the same as used by Timoshenko [1].

3. GENERALISED FORCE FOR A FREQUENCY SHIFTING SHAKER (FSS)

In this section the case of a reciprocating vibration shaker which acts on several different structures is considered. A general layout of the shaker on a structure is shown in Figure 2. In this case two frequencies exist, a shaker operating frequency ω_1 in rads^{-1} and a reciprocating frequency ω_2 in rads^{-1}. Reciprocating motion amplitude is r in m and the equilibrium position of the shaker on the structure from the origin is x_0. The appropriate mode shape for each structure has to be employed to obtain the correct generalised force for the given structure. Simple relationships between the generalised force, P, r and the length of the structure, ℓ, are developed in a non-dimensional format for a simply supported bridge, a cantilever column, a free–free beam and a fixed–fixed beam (bridge). Also, the force gain of a reciprocating frequency shifting shaker over that of a shaker operating at a low natural frequency is developed herein.

3.1. SIMPLY SUPPORTED BEAM (BRIDGE)

Consider the case of a vibration shaker the position of which along a simply supported beam is given by

$$x = x_0 + r \sin(\omega_2 t), \tag{5}$$

where x_0 is an equilibrium position, r is amplitude of reciprocating motion and ω_2 is the frequency of the reciprocating motion. The virtual work, δ_w, performed by the shaker force, $P \sin \omega_1 t$, operating through a virtual displacement δ_{q_i} for mode i is given by

$$\delta_w = P \sin(\omega_1 t) \sin(i\pi(x_0 + r \sin \omega_2 t)/\ell)\delta_{q_i}, \tag{6}$$

where equation (5) is substituted into equation (1) for x.
The generalised force for mode i, Q_i, is then given by δ_w/δ_{q_i}

$$Q_i = P \sin(\omega_1 t) \sin(i\pi(x_0 + r \sin \omega_2 t)/\ell). \tag{7}$$

If the generalised force, Q_1, for the first mode if a stucture is considered, i.e., the lowest natural frequency, and the force P is brought to the left side of equation (7), equation (7) becomes

$$Q_1/P = \sin(\omega_1 t) \sin(\pi(x_0/\ell + (r/\ell) \sin \omega_2 t)). \tag{8}$$

Equation (8) is in an non-dimensional format, thus Q_1/P can be described by

$$Q_1/P = \text{function}\ ((r/\ell)^a, \quad (x_0/\ell)^b, \quad (\omega_1/\omega_2)^c), \tag{9}$$

where a, b, c and the functional relationship can be determined by analysis of equation (8).

The procedure for the analysis is as follows: (a) calculate Q_1/P time histories for various values of the three ratios in equation (9); (b) Fourier transform the time histories obtained in (a) and take absolute value of the spectral peak at the difference frequency (the same amplitude exists at the sum frequency); (c) Obtain relationships between Q_1/P and the dimensionless terms in equation (9).

An analysis for a simply supported beam is given first. A Q_1/P time history for $r/\ell = 0{\cdot}05$, $x_0/\ell = r/2$ and $\omega_1/\omega_2 = 1{\cdot}25$ is shown in Figure 3. This time history has a low frequency component and several high frequency components and its Fourier spectrum is given in Figure 4 which demonstrates the existence of these

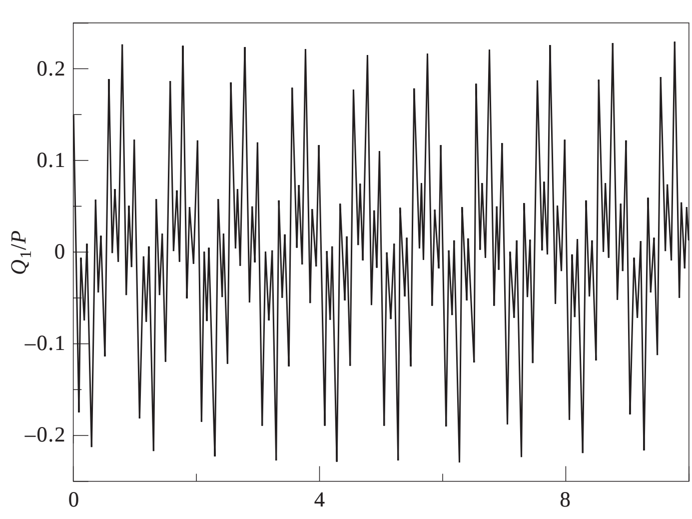

Figure 3. A Q_1/P time history for a simply supported beam for ratio values of $r/\ell = 0{\cdot}05$, $x_0/\ell = r/2$ and $\omega_1/\omega_2 = 1{\cdot}25$. $\omega_1 = 10\pi$ rads^{-1}, (5 Hz) and $\omega_2 = 8\pi$ rads^{-1} or (4 Hz).

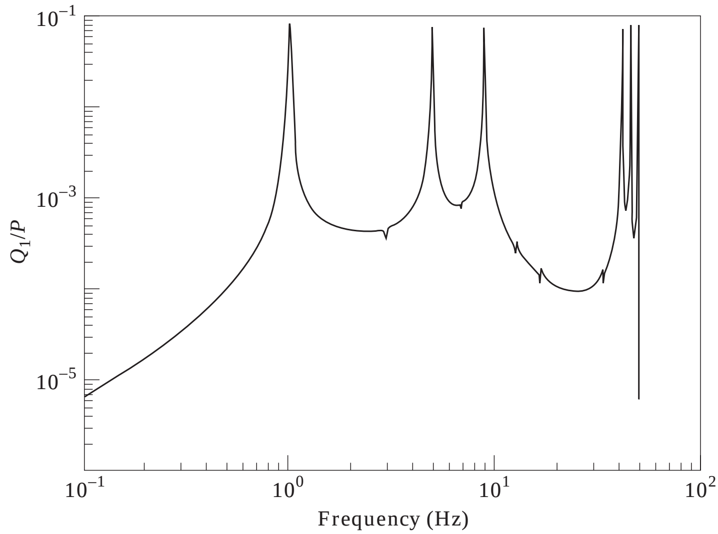

Figure 4. Frequency spectrum of Q_1/P for the time history given in Figure 3. The Matlab routine used to generate the spectrum gives a mirror image for frequencies above 26 Hz.

components. In this example ω_1 is 10π rads^{-1} (5 Hz) and ω_2 is 8π rad^{-1} (or 4 Hz), which gives a difference frequency of 2π rads^{-1} (1 Hz) and a sum frequency of 18π rads^{-1} (9 Hz). In Figure 4 the difference frequency, the forcing frequency and the sum frequency are present and components at these frequencies may excite the first mode of vibration. A spectrum of Q_1/P for $r/\ell = 0{\cdot}005$ and $x_0 = r/2$ is shown in Figure 5. Further analysis shows that the amplitudes of Q_1/P at the difference and

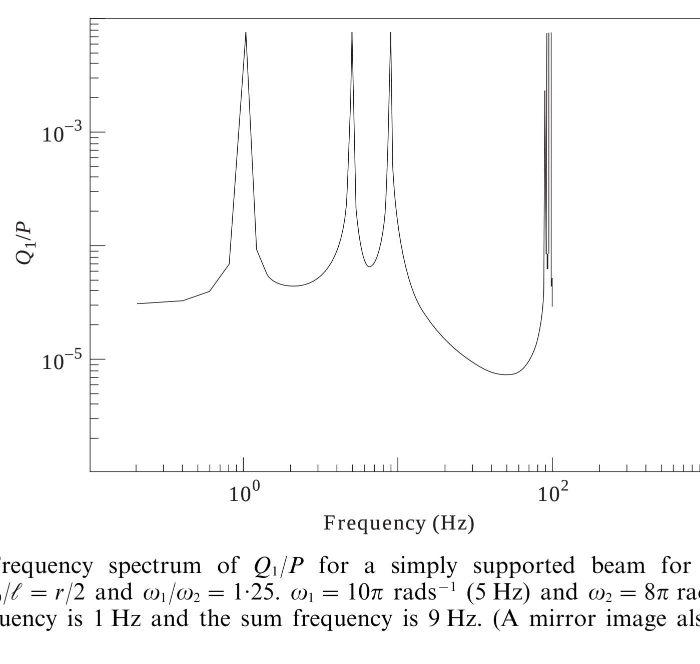

Figure 5. Frequency spectrum of Q_1/P for a simply supported beam for ratio values of $r/\ell = 0{\cdot}005$, $x_0/\ell = r/2$ and $\omega_1/\omega_2 = 1{\cdot}25$. $\omega_1 = 10\pi$ rads^{-1} (5 Hz) and $\omega_2 = 8\pi$ rads^{-1} (4 Hz). The difference frequency is 1 Hz and the sum frequency is 9 Hz. (A mirror image also exists for this spectrum.)

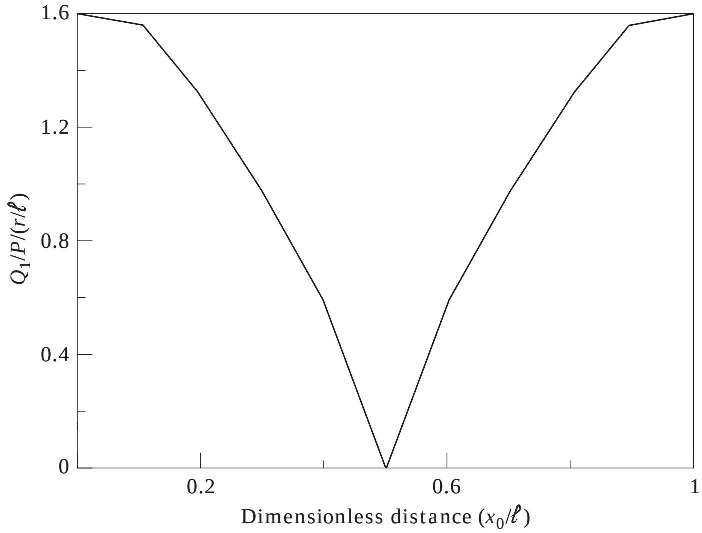

Figure 6. $Q_1/P/(r/\ell)$ versus x_0/ℓ for a simply supported beam.

sum frequencies are independent of force or reciprocating frequencies, and the ratio of Q_1/P is a trigonometric quantity given by Equation (8).

The dependence of Q_1/P on r/ℓ is linear as can be seen by comparing the amplitude of the difference frequency component in Figures 4 and 5 for different values of r/ℓ. The relationship between $Q_1/P/(r/\ell)$ and x_0/ℓ is shown in Figure 6 and is symmetrical about the centre line of the bridge. If a FSS is placed at the bridge centre, neither a difference or sum frequency would be developed; Maximum Q_1/P is obtained when the shaker is placed at the beginning or at the end of the bridge.

For a FSS placed at the beginning or at the end of the bridge, the relationship for Q_1/P at the difference or sum frequency and r/ℓ is independent of frequency ratio and is given by

$$Q_1/P = 1{\cdot}65r/\ell. \tag{10}$$

To understand how this shaker, FSS, operates, it is noted that the slope of a simply supported beam is maximum at the supports and is zero of the bridge centre and the FSS operates most efficiently near the supports. Thomson [4] gives a relationship between generalised force, moment and slope for a beam structure as

$$Q_i = M(x, t)\phi_i'(x), \tag{11}$$

where $M(x, t)$ is a moment and $\phi_i'(x)$ is the mode shape slope. The relationship shows that Q_i would be greatest for maximum slope for a given moment or torque acting on the bridge, thus suggesting that the FSS is a moment shaker. The force P operates over a distance r, thus giving a moment. The analysis given in Appendix 1 is based upon equation (11).

Table 1

Data for different beam types

Beam type	$\beta_1\ell$	σ_1
Free–free	4·73	0·98
Cantilever	1·87	0·73
Fixed–fixed/free–free	4·73	0·98

3.2. CANTILEVER, FIXED–FIXED AND FREE–FREE BEAMS

Using an analysis similar to that given in section 3.1, the dimensionless generalised force for mode one, Q_1/P, of cantilever, fixed–fixed and free–free beams can be obtained. Inman [5] gives the form of the mode shape of the above beams as

$$\phi_1(x) = \cosh(\beta_1 x) + \cos(\beta_1 x) - \sigma_1(\sinh(\beta_1 x) - \sin((\beta_1 x)), \tag{12}$$

where $\beta_1\ell$ and σ_1 are given in Table 1 (note that for a free–free beam the signs have a different value.) For the evaluation of the generalised force for these three beam types x is replaced by $x_0 + r\sin(\omega_2 t)$ from equation (5) and the virtual work procedure is used to obtain for the first mode of vibration

$$Q_1 = P\sin(\omega_1 t)\phi_1(\beta_1(x_0 + \sin\omega_2 t)). \tag{13}$$

Equation (13) is then evaluated for the three beam types.

A spectral result for Q_1/P for a cantilever beam for r/ℓ 0·05, $x_0/\ell = 0{\cdot}995$ and $\omega_1/\omega_2 = 1{\cdot}04$ is given in Figure 7. For this beam the spectral amplitudes at the sum and difference frequencies do not have the same amplitude as that at the force

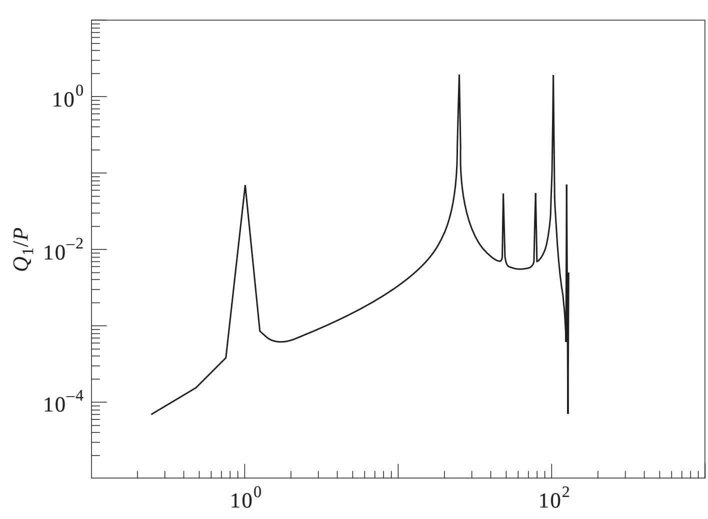

Figure 7. Frequency spectrum of Q_1/P for a cantilever beam for ratio values of $r/\ell = 0{\cdot}05$, $x_0/\ell = 0{\cdot}995$ and $\omega_1/\omega_2 = 1{\cdot}04$. $\omega_1 = 50\pi$ rads^{-1} (25 Hz) and $\omega_2 = 48\pi$ rads^{-1} (24 Hz). Difference frequency is 2π rads^{-1} (1 Hz). Mirror image of spectrum above 56 Hz.

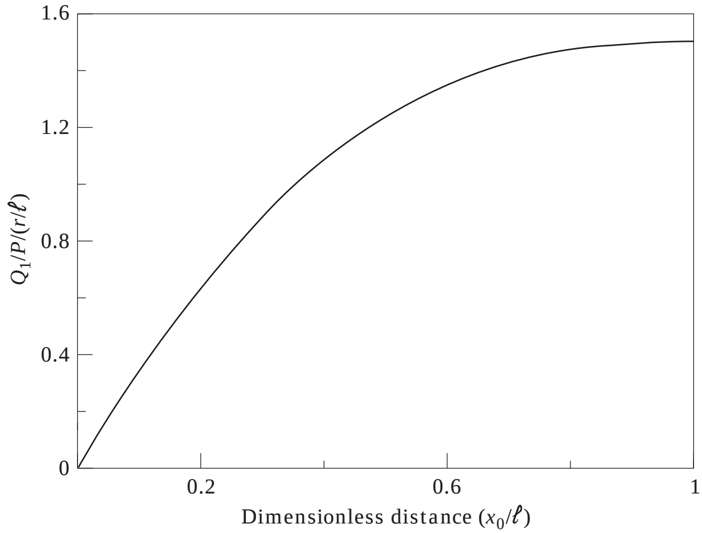

Figure 8. $Q_1/P/(r/\ell)$ versus x_0/ℓ for a cantilever beam.

frequency. This is also true for the free–free and fixed–fixed beams. The dependence of $Q_1/P/(r/\ell)$ with x_0/ℓ for a cantilever beam is given in Figure 8 and follows the concept that at a small mode shape slope amplitude Q_1/P will be small, i.e., near the fixed end, and at large mode shape slope values Q_1/P will be large, i.e., at the free end.

Thus, the maximum value of Q_1/P for a cantilever beam is

$$Q_1/P = 1{\cdot}50r/\ell. \tag{14}$$

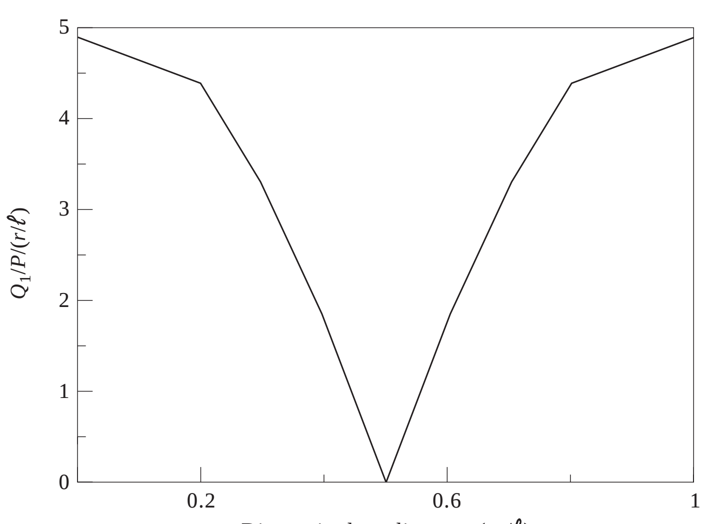

Figure 9. $Q_1/P/(r/\ell)$ versus x_0/ℓ for a free–free beam.

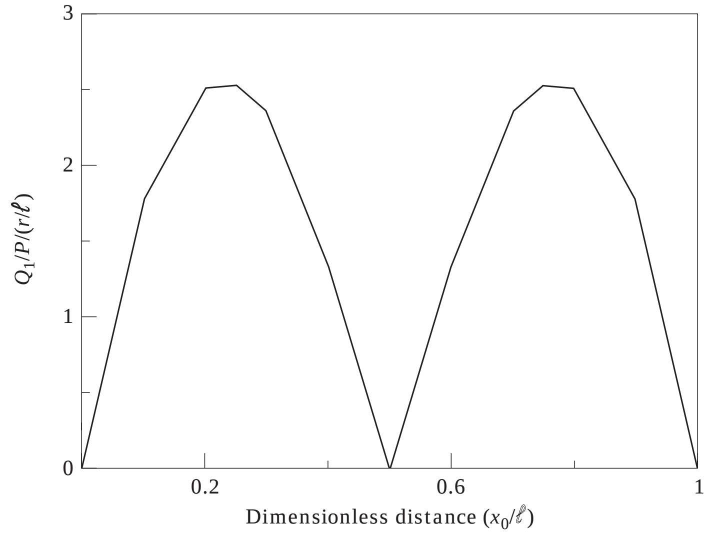

Figure 10. $Q_1/P/(r/\ell)$ versus x_0/ℓ for a fixed–fixed beam.

The relationship between $Q_1/P/(r/\ell)$ and x_0/ℓ for a free–free beam is given in Figure 9 and is somewhat similar to that for the simply supported beam given in Figure 6. For a fixed–fixed beam the relationship between $Q_1/P/(r/\ell)$ and x_0/ℓ is given in Figure 10 where maximum values are at 22% and 78% along the beam length. The maximum generalised force ratio for a free–free beam is

$$Q_1/P = 5r/\ell; \tag{15}$$

and for a fixed–fixed beam is

$$Q_1/P = 2{\cdot}58r/\ell. \tag{16}$$

Results for generalised forced ratios are summarised in Table 2.

Table 2

Maximum non-dimensional first mode generalised force ratios

Beam type	Q_1/P	x_0
simply supported	$1{\cdot}65\ r/\ell$	$r/2$ or $l - r/2$
fixed–fixed	$2{\cdot}58\ r/\ell$	$0{\cdot}22\ \ell$ or $0{\cdot}78\ \ell$
cantilever	$1{\cdot}50\ r/\ell$	$\ell - r/2$
free–free	$5{\cdot}0\ \ r/\ell$	$r/2$ or $\ell - r/2$

TABLE 3

Maximum generalised force gain ratios

Beam type	Gain ratio $(r/\ell)(\omega_1\omega_n)^2$
simply supported	1·65
fixed–fixed	2·58
cantilever	1·50
free–free	5·0

4. GENERALISED FORCE GAIN RATIO

A unidirectional force generated, P_{SH}, by an out-of-balance mass shaker at the lowest natural frequency of a structure, ω_n, is given by

$$P_{SH} = m_{ob}e\omega_n^2 \sin(\omega_n t) \tag{17}$$

and the generalised force of P_{SH} acting on a simply supported beam would be

$$Q_{1SH} = m_{ob}e\omega_n^2\, \phi(\ell/2) \sin(\omega_n t) \tag{18}$$

Usually $\phi(\ell/2)$ is 1 or $\sqrt{2}$ depending upon the mode shape scaling and a value of 1 will be employed here. The amplitude of Q_{1SH} is $m_{ob}e\omega_n^2$. If a FSS shaker is employed for the same task then Q_1 would be, see Table 2,

$$Q_1 = 1{\cdot}65(r/\ell)m_{ob}e\omega_1^2, \tag{19}$$

noting that ω_1 is much greater than ω_n.

If it assumed that the ordinary shaker and the FSS have the same out of balance, $m_{ob}e$, then a ratio of Q_1 over Q_{1SH} gives a gain ratio

$$\text{gain ratio} = 1{\cdot}65(r/\ell)\omega_1^2/\omega_n^2. \tag{20}$$

The gain ratio is greater than one if

$$1{\cdot}65(r/\ell)\omega_1^2 > \omega_n^2, \tag{21}$$

and then a frequency shifting shaker will deliver a generalised force greater than an ordinary shaker. In general, however, a factor greater than one may be applied to the right side of equation (20) to account for the fact that a motive drive for the reciprocating mechanism is required and has mass also. In Table 3 is listed the maximum gain ratios for all four beam types considered in this paper.

4.1. PRACTICAL CONSIDERATIONS

Several practical problems have to be examined prior to the design of a full scale FSS. The first item is the choice of r, ω_1 and ω_2 noting that ℓ is a fixed quantity as it is determined by the structure to be tested. Also, the constant in Table 2 column 1, (which multiplies r/ℓ) is also determined by structure type and if the FSS is to generate a force greater than an ordinary shaker, then equation (20) has to be satisfied for a simply supported beam and similar equations for other beam types. In general then a FSS shaker would generate a force greater than an

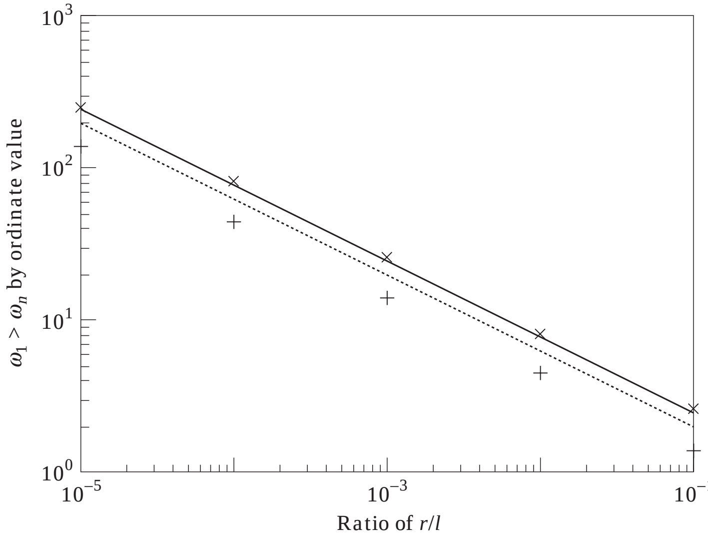

Figure 11. Plot of $\omega_1 = \text{SQRT}\ (1/(r/\ell)\omega_n$ for frequency shifting shaker to have some force as ordinary shaker operating at ω_n. Notation is ——, simply supported beam; $\times$, cantilever beam; ····, fixed–fixed beam and $+$, free–free beam.

ordinary shaker if the operational frequency of a FSS satisfies the following inequality

$$\omega_1 > (1/\text{factor} * r/\ell)^{1/2}\omega_n \tag{22}$$

where the factor term is given in column 1 of Table 2 for the different beam types e.g., 1·50 for a cantilever beam. A plot of equation (22) for the different beam types is given in Figure 11 where ω_1 should be greater than the ordinate value to satisfy equation (22).

The second item to consider is balancing of inertia forces generated by the movement of the shaker parallel to the structure due to the reciprocating motion to and fro. Inertia forces would be larger than the shaker forces due to motors, bearings and shafts all of which are moving parallel to the structure. Thus, balancing of forces in the parallel direction may be required.

A force frequency shifting shaker could also be employed with an active vibration controller to reduce motions for tall structures.

5. CONCLUSIONS

Concepts for the development of a frequency shifting shaker for use on several different beam structures have been developed in this paper. This shaker is based upon moving an ordinary shaker, and an out-of-balance mass or electrodynamic with inertial mass, to and fro along a structure. If the ordinary shaker operates at frequency ω_1 and the reciprocative motion occurs at ω_2, then forces are developed at a difference frequency, $\omega_1 - \omega_2$ and at a sum frequency, $\omega_1 + \omega_2$. Analysis of equation (7) gives a relationship between the generalised force ratio, Q_1/P and r/ℓ which is a ratio of shaker throw to structural length. These relations are given in Table 2 and further analysis demonstrates that a frequency shifting shaker can develop a greater generalised force at a lower frequency than an ordinary shaker acting at the low frequency for the same amount of out of balance. The structure type dictates where a frequency shifting shaker should be placed on the structure to achieve maximum force shifting; these positions are given in Table 2. Analysis of the relationships between Q_1/P and x_0/ℓ indicates that the shaker behaves as a moment shaker in its action as demonstrated in Appendix 1.

Benefits include the generalised force ratio to be independent of shaker operating frequency, ω_1, and thus a constant force sine sweep can be developed. A very wide operating frequency range can be achieved and the requirement of having only one speed controller if the shaker is operated at constant speed.

This shaker is considered to be an important development for testing of structures at low frequencies. Some practical aspects were also considered for the use and design of a frequency shifting shaker. Experimental verification is given in Koss *et al*. [3], Koss [2] and in Appendix 2.

ACKNOWLEDGMENTS

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APPENDIX 1. ALTERNATE METHOD OF CALCULATING FREQUENCY SHIFTED GENERALISED FORCES

The generalised force for a moment acting on a beam at position x_0 is given in Thomson [4] as

$$Q_i = M(x, t)\phi_i'(x_0), \tag{A1}$$

which is equation (11) in the main body of this paper. The fluctuating moment $M(x_0, t)$ is given by

$$M(x_0, t) = P \sin(\omega_1 t) r \sin \omega_2 t. \tag{A2}$$

TABLE A1

Maximum mode shape slope values

Beam type	Root	Normalised mode shape slope	Slope
Fixed–free	$1{\cdot}87/\ell$	1·468	$2{\cdot}75/\ell$
Fixed–fixed	$4{\cdot}73/\ell$	1·03	$4{\cdot}87/\ell$
Free–free	$4{\cdot}73/\ell$	1·96	$9{\cdot}27/\ell$

For a simply supported beam, the most efficient position to place this moment is at either support position i.e., $x_0 \cong 0$ or $x_0 \approx \ell$ where the moment slope $\phi_i'(x_0)$ is maximum. The mode shape for a simply supported beam is

$$\phi_i(x) = \sin(i\pi x/\ell) \tag{A3}$$

and the slope of the mode shape is

$$\phi_i'(x) = (i\pi/\ell)\cos(i\pi x/\ell). \tag{A4}$$

Thus, the generalised force becomes

$$Q_i = P\sin(\omega_1 t) r \sin(\omega_2 t)((i\pi/\ell)\cos(i\pi x/\ell)). \tag{A5}$$

Letting $i = 1$ and employing sum and difference trigonometric formulas; the generalised force becomes

$$Q_1 = -(P\pi r/2\ell)(\cos(\omega_1 - \omega_2)t + \cos(\omega_1 - \omega_2)t), \tag{A6}$$

where the amplitude of Q_1 in equation (A6) for the simply supported beam has a value of $1{\cdot}57\ Pr/\ell$ which is compared to $1{\cdot}65\ Pr/\ell$ given by equation (10) in the body of the paper.

Generalised forces for a fixed–fixed beam, cantilever beam and free–free beam are obtained from data for mode shape slopes given in tables of Appendix C of Thomson [4]. The mode shape slope values obtained from a normalised mode shape slope and the product of the root of the first mode are given in Table A1. Combining equations (A1), (A2) and Table A1 values and using trigonometric identities gives the following maximum amplitudes for Q_1 at the sum and difference frequencies:

$$\text{Fixed–free}\quad 1{\cdot}38 Pr/\ell,\quad \text{Fixed–fixed}\quad 2{\cdot}43 Pr/\ell,\quad \text{Free–free}\quad 4{\cdot}63 Pr/\ell. \tag{A7}$$

Comparison of the results of (A7) to that in Table 2 indicates that the maximum amplitudes calculated by the two methods are within 10% of each other. Note that the data given in Table 2 were calculated digitally and amplitudes were obtained by cursor picking.

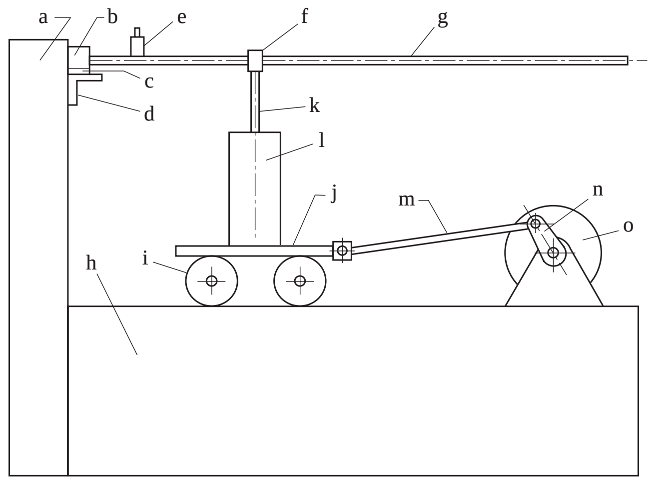

Figure A1. Schematic of experimental testrig: support *a*; brackets *b*, *c* and *d*; accelerometer *e*; roller *f*; beam *g*, steel table *h*; *i* roller; *j* roller table; shaker quill *k*; shaker 1; connecting rod *m*; crank *n*; and flywheel *o*.

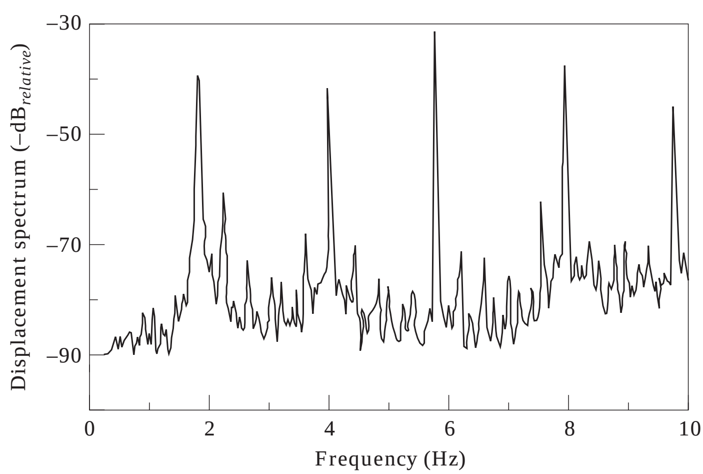

Figure A2. Frequency spectrum displacement of a 2 m long cantilever beam with a natural frequency of 1·8 Hz with accelerometer attached. Shaker frequency is 5·8 Hz and slider crank frequency is 4 Hz with a throw of 2 cm. The difference frequency of 1·8 Hz, the sum frequency of 9·8 Hz, the shaker frequency of 5·8 Hz and the crank frequency of 4 Hz and it harmonic at 8 Hz are all clearly evident.

APPENDIX 2. EXPERIMENTAL VERIFICATION

The concept of force frequency shifting was verified by testing a cantilever beam using an experimental rig shown schematically in Figure A1; reference will now be made to this figure. The base of an electrodynamic shaker *l*, is attached to a slider crank mechanism, *j*, *m* and *n*, which rolls on wheels *i*. The quill of the shaker, *k*, is attached to the beam, *g*, by a roller device, *f*. A Kistler accelerometer, *e*, sits on top of the beam and the beam is attached to the support, *a*, by brackets *b*, *c* and *d*. The slider crank rolls on the steel table *h*.

The beam with accelerometer attached has a natural frequency of 1·8 Hz and the throw of the crank, n, is 2 cm. The flywheel, o, has a rotational frequency of 4 Hz and the shaker has an operational frequency of 5·8 Hz. The acceleration spectrum was measured with a 2 channel AND spectrum analyser; to obtain a displacement spectrum the acceleration spectrum was divided by frequency (rads^{-1}) squared. Peaks on the frequency spectrum should appear at the difference frequency of 1·8 Hz, the shaker frequency of 5·8 Hz and the sum frequency of 9·8 Hz. An examination of the displacement spectrum given in Figure A2, shows the presence of these frequencies plus the slider crank frequency of 4 Hz and its harmonic at 8 Hz. This experimental result verifies the frequency shifting concept.