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UNBALANCE IDENTIFICATION AND FIELD BALANCING OF DUAL ROTORS SYSTEM WITH SLIGHTLY DIFFERENT ROTATING SPEEDS

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The identification of unbalance is the crux of field balancing of dual rotors system with slightly different rotating speeds. On the basis of correlation theory, this paper explains a method called "Single Point Discrete Fourier Transformation (DFT)" to identify the unbalance. By theoretical analysis, the correlation integral time and its maximum possible error are determined. The field balancing experiment on WLZY-350 horizontal spiral centrifuge verifies its precision, reliability and applicability in practice.

1. INTRODUCTION

Nowadays, the horizontal spiral centrifuge, a special kind of solid–liquid separating machine with powerful productivity, high mechanization and good separation property, is applied in many industries. However, in its field operation the excess vibration is still an unsolved problem. As there are two rotors with slightly different rotating speeds in this centrifuge, when the unbalances emerge on the two rotors separately, two vibration vectors with slightly different frequencies will be produced simultaneously, and it is difficult to identify them effectively [1, 2]. Hereafter, this paper explains a method called "Single Point Discrete Fourier Transformation (DFT)" to separate and identify the unbalances. The field balancing experiment on WLZY-300 horizontal spiral centrifuge proves that this method can work efficiently.

2. STRUCTURE OF HORIZONTAL SPIRAL CENTRIFUGE

As shown in Figure 1, the WLZY-350 horizontal spiral centrifuge is a machine which consists mainly of an outer drum rotor (5) and an inner spiral rotor (6). The inner rotor is set in the outer rotor by two ball bearings (3, 9), while the outer rotor is supported on the machine basis (11) by another two ball bearings (2, 10). These rotors are all driven by a motor through a gear reducer (1).

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In order to identify the unbalances and achieve field balancing, the balancing planes for adding correction masses have to be determined first. For the outer drum rotor, the balancing planes are the two ends of the drum, on which there are many fixing bolts (8). The correction mass may be produced by adding the bolt washers to the fixing bolts. For the inner spiral rotor, the left balancing plane is the exposed part of the spiral in solid outlet (4). When necessary, the bolts serving as correction mass may be added to it. The right balancing plane of the inner rotor is the right end (13) of the spiral, and the suitable correction mass may be appended to it through two liquid outlets (7).

As for the key phases, they are easy for the outer rotor to achieve. A piece of metal attached to anywhere on the outer rotor, matching with the induce sensor, may work. But, for the inner rotor, it is comparatively inconvenient. As the inner rotor is enclosed wholly in the outer drum rotor, a short shaft (12), which is regarded as an extension of the inner rotor, is mounted on the solid–liquid inlet. The combination of another piece of metal attached to the shaft (12) and another induce sensor may cause unbalance in the key phase signal of the inner rotor.

For the WLZY-300 horizontal spiral centrifuge, the rotating speed of the inner rotor is 1572 rpm, while that of the outer rotor is 1560 rpm. Between them there is a speed difference of 12 rpm, or a rotating frequency difference of 0·2 Hz.

3. IDENTIFICATION PRINCIPLE OF VIBRATION AND UNBALANCE

The vibration caused by the unbalances on the rotors is picked up by the sensor mounted on the bearing seat. Suppose the rotating angle speeds of the inner rotor and the outer rotor are ω_1 and ω_2, respectively, then the vibration signal picked up by the sensor mounted on either bearing seat may be written as follows:

$$X(t) = X_1(t) + X_2(t) = A_1 \times \cos(\omega_1 t - \phi_1) + A_2 \times \cos(\omega_2 t - \phi_2), \qquad (1)$$

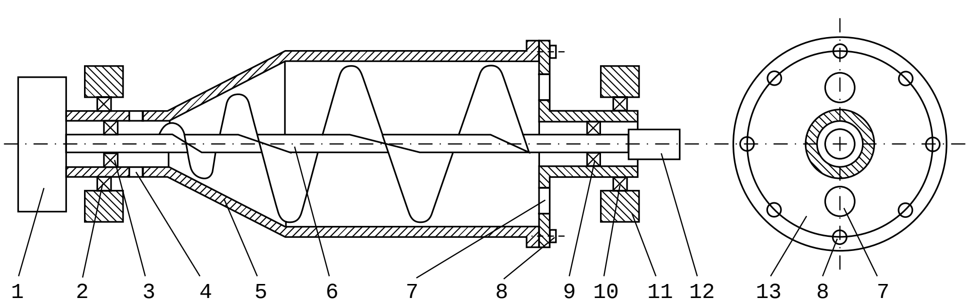

Fig. 1. Sketch showing the structure of the horizontal spiral centrifuge. 1. Gear reducer, 2. Ball bearing, 3. Ball bearing, 4. Solid outlet, 5. Outer drum rotor, 6. Inner spiral rotor, 7. Liquid outlet, 8. Fixed bolt, 9. Ball bearing, 10. Ball bearing, 11. Machine basis (bearing seat), 12. Extension part of inner rotor, 13. Right end of drum.

where X_1, X_2 represent the vibration, A_1, A_2 the amplitude, ω_1, ω_2 the rotating speed, and ϕ_1, ϕ_2 the phase. If the initial phases in equation (1) are neglected, then

$$X(t) = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\omega_1 - \omega_2)t} \times \cos\left\{\operatorname{arctg}\left[\frac{A_1\sin(\omega_1 t) + A_2\sin(\omega_2 t)}{A_1\cos(\omega_1 t) + A_2\cos(\omega_2 t)}\right]\right\}. \tag{2}$$

From equation (2), it is found that the amplitude of the vibration is $\sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\omega_1 - \omega_2)t}$ with the maximum value $A_1 + A_2$, the minimum value $|A_1 - A_2|$, and the change cycle of the amplitude $T = 2\pi/|\omega_1 - \omega_2|$.

In order to separate the two vibration vectors in equation (1), correlation theory may be used [3]. The correlation function $R_{xy}(\tau)$ is defined as:

$$R_{xy}(\tau) = \lim_{T\to\infty}\frac{2}{T}\int_0^T X(t) \times Y(t+\tau)\,\mathrm{d}t.$$

In the equation above, let $X(t)$ be the vibration signal expressed by equation (1), $Y(t)$ be $\cos(\omega_1 t)$ and $\tau = 0$. Then

$$A_{1R} = R_{xy}(0) = \lim_{T\to\infty}\frac{2}{T}\int_0^T [A_1\cos(\omega_1 t + \phi_1) + A_2\cos(\omega_2 t + \phi_2)]\cos(\omega_1 t)\,\mathrm{d}t$$

$$= \lim_{T\to\infty}\frac{1}{T}\left\{\frac{A_1}{2\omega_1}\sin(2\omega_1 t + \phi_1) + A_1 t\cos\phi_1 + \frac{A_2}{(\omega_1 + \omega_2)}\sin[(\omega_1 + \omega_2)t + \phi_2]\right.$$

$$\left.+\frac{A_2}{(\omega_1 - \omega_2)}\sin[(\omega_1 - \omega_2)t - \phi_2]\right\}\Bigg|_0^T$$

$$= A_1\cos\phi_1. \tag{3}$$

The integral low limit 0 in equation (3) is the moment when the key phase pulse of the inner rotor comes and $\cos(\omega_1 t)$ begins.

If $Y(t)$ is supposed to be $\sin(\omega_1 t)$, then

$$A_{1I} = A_1\sin(\phi_1), \tag{4}$$

so

$$A_1 = \sqrt{A_{1I}^2 + A_{1R}^2} \qquad \phi_1 = \operatorname{arctg}(A_{1I}/A_{1R}). \tag{5}$$

In this way the vibration vector according to the rotating frequency of the inner rotor is separated. If $Y(t)$ is suppposed to be $\sin(\omega_2 t)$ and $\cos(\omega_2 t)$ separately, the vibration vector that conforms to the outer rotor may be identified in the same way.

It is found that, with the application of the correlation theory and on condition that $T\to\infty$ is met, the expected vibration vector according to a certain frequency may be separated accurately from the vibration signal even if there are many vibration vectors corresponding to various frequencies in the vibration signal. But in practice, the ideal condition $T \to \infty$ is seldom met, and as a result the frequency leak happens. So, in order to separate the expected vibration vector effectively, it is necessary to determine the shortest integral time T.

In equation (3) if the integral time T is finite, then

$$A_{1R} = \frac{A_1}{2\omega_1 T}[\sin(2\omega_1 T + \phi_1) - \sin\phi_1] + \frac{A_2}{(\omega_1 + \omega_2)T}\{\sin[(\omega_1 + \omega_2)T + \phi_2] - \sin\phi_2\} + \frac{A_2}{(\omega_1 - \omega_2)T}\{\sin[(\omega_1 - \omega_2)T - \phi_2] + \sin\phi_2\} + A_1\cos\phi_1. \tag{6}$$

In equation (6) if $T \to \infty$,

$$A_{1R}^{ideal} = A_1 \cos(\phi_1) \tag{7}$$

will be obtained, which is defined as the ideal condition. But, if $T = 2\pi/|\omega_1 - \omega_2|$, then

$$A_{1R} = A_1\cos\phi_1 + \frac{A_1|\omega_1 - \omega_2|}{4\pi\omega_1}\left[\sin\left(\frac{4\pi\omega_1}{|\omega_1 - \omega_2|} + \phi_1\right) - \sin\phi_1\right] + \frac{A_2|\omega_1 - \omega_2|}{2\pi(\omega_1 + \omega_2)}\left\{\sin\left[\frac{2\pi(\omega_1 + \omega_2)}{|\omega_1 - \omega_2|} + \phi_2\right] - \sin\phi_2\right\}. \tag{8}$$

The error between A_{1R} and the ideal condition A_{1R}^{ideal} is written as:

$$\varepsilon \leqslant \max|A_{1R} - A_{1R}^{ideal}| = \frac{A_1|\omega_1 - \omega_2|}{2\pi\omega_1} + \frac{A_2|\omega_1 - \omega_2|}{\pi(\omega_1 + \omega_2)}.$$

In view of the fact $\omega_1 \approx \omega_2$, then

$$\varepsilon \leqslant \max|A_{1R} - A_{1R}^{ideal}| = \frac{A_1|\omega_1 - \omega_2|}{2\pi\omega_1} + \frac{A_2|\omega_1 - \omega_2|}{2\pi\omega_1}. \tag{9}$$

The relative error is as follows:

$$\delta \leqslant \frac{\varepsilon}{A_1} = \left[1 + \frac{A_2}{A_1}\right]\frac{A_1(\omega_1 - \omega_2)}{2\pi\omega_1}. \tag{10}$$

For the centrifuge in this paper, $(\omega_1 - \omega_2)/\omega_1 = 12/1572 = 0{\cdot}0076$. Suppose that $A_2/A_1 = 10$ (when $A_2/A_1 = 10$ is not met, the machine can be treated as a single rotor system), the relative error is $\delta \leqslant 1{\cdot}34\%$. For the dual rotors type system concerned, the rotating speed difference is very small, say $(\omega_1 - \omega_2)/\omega_1 \leqslant 0{\cdot}015$. Still supposing that $A_2/A_1 = 10$, then the error is $\delta \leqslant 2{\cdot}63\%$, which may be allowed in the application. Hence, the integral time $T = 2\pi/|\omega_1 - \omega_2|$ is acceptable.

TABLE 1

$(Z_2 - Z_1)$	1	2	3	4	5
$Z_2 \geqslant$	66	113	200	266	333

It must be pointed out that, for the centrifuge driven by the motor through the gear reducer, equation (8) may be raised to the ideal status of equation (7). This means that the error in equation (9) or (10) is zero. The reason for this is as follows: since the centrifuge is driven by the gear reducer, it is supposed that the tooth numbers of the gears which drive the inner and the outer rotors are Z_1, Z_2 separately. Then the following equation may be obtained:

$$\frac{\omega_1}{\omega_2} = \frac{Z_2}{Z_1},$$

or

$$\frac{\omega_1 + \omega_2}{\omega_1 - \omega_2} = \frac{Z_1 + Z_2}{Z_2 - Z_1}, \tag{11}$$

for $(\omega_1 - \omega_2)/\omega_1 \leqslant 0{\cdot}015$

$$\frac{(Z_2 - Z_1)}{Z_2} \leqslant 0{\cdot}015. \tag{12}$$

Let the tooth number difference $(Z_2 - Z_1)$ be a series of integers, say $1, 2, 3, 4, \ldots$; the values of Z_2 are listed in Table 1. Owing to the limitation of the tooth number, Z_2 cannot exceed 200. Hence $(Z_2 - Z_1)$ must be 1 or 2. As a result, $(Z_1 + Z_2)/(Z_2 - Z_1)$ must be an integer regardless of $(Z_2 - Z_1)$ being 1 or 2. Subsequently the following equation is obtained:

$$\frac{\omega_1 + \omega_2}{\omega_1 - \omega_2} = N = \text{an integer}. \tag{13}$$

Substituting equation (13) into equation (8), then $A_{1R} = A_1 \cos(\phi_1)$, which is the same as equation (7).

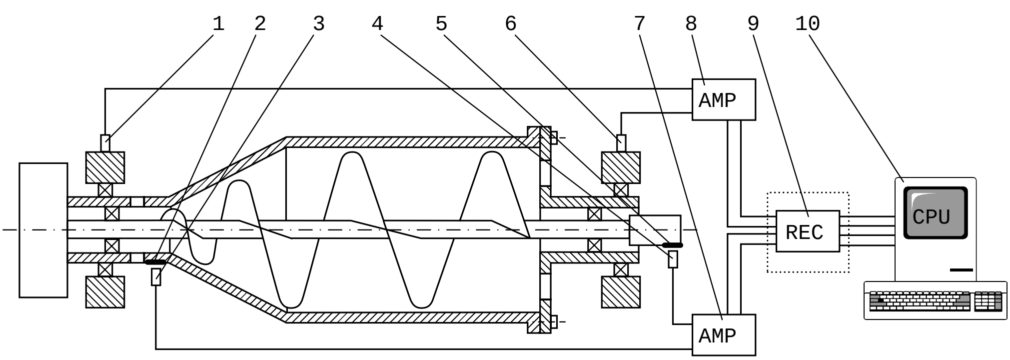

Fig. 2. Experimental system. 1. Velocity sensor, 2. Key phase label of the outer rotor, 3. Induce sensor, 4. Induce sensor, 5. Keyphase label of inner rotor, 6. Velocity sensor, 7. Amplifier, 8. Amplifier, 9. Cassette recorder, 10. Personal computers.

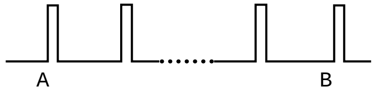

Fig. 3. Key phase signal

In summary, with the application of correlation theory, and the integral time being the change cycle of the vibration amplitude, the vibration vectors caused by the unbalances on two rotors respectively may be separated effectively. In addition, the unbalances on both rotors may be calculated conveniently with the conventional influence coefficient balancing method.

4. EXPERIMENT

The experiment system is shown in Figure 2: two inducer sensors (3, 4) matching with two metal flakes (2, 5) act as the key phases of the outer and the inner rotors separately. Two velocity sensors (1, 6) are mounted on the left and right ball bearing seats to pick up the vibrations. After being amplified by the amplifiers (7, 8), all the signals may be recorded on a cassette recorder, or passed to a personal computer in the field, in which the signals are sampled, saved and processed. From the analysis above, it is noted that in order to apply the correlation theory to separate the vibration vectors of the inner and outer rotors, the rotating speeds corresponding to the two rotors should be very accurate. Here the speeds are measured by the method of timing and counting the pulse number of the key phase signal simultaneously. In a personal computer, the I/O ports numbered by 0×40 and 0×43 are the ones relating to the time. They form a 16-byte counter with a counting frequency of 2·38 MHz, that is to say, the time spent in a counting circulation is $(2^{16}/2{\cdot}38) \times 10^{-6}\,\mathrm{s} = 27{\cdot}54$ ms. It is suggested that Figure 3 is the key phase signal obtained. In Figure 3, point A is the moment when the first pulse comes, point B is the moment when the last pulse comes, the pulse number between the point A and point B is Q, the counter value difference between A and B is P. Then the rotating speed of the rotor corresponding to the above key phase signal may be written as:

$$\mathrm{rpm} = \frac{60Q}{P \times \dfrac{1}{2{\cdot}38 \times 10^6}} = 1{\cdot}428 \times 10^6 \times \frac{Q}{P}. \tag{14}$$

Since the signals sampled by the computer are discrete, it is necessary for the correlation integral equation to be discreted. For example, equation (3) may be changed to:

$$A_{11} = \frac{1}{T}\int_0^T X(t) \times \sin(\omega_1 t)\,\mathrm{d}t = \frac{1}{N} \times \sum_{n=0}^{N-1} X(n) \sin(\omega_1 \times \Delta T \times n), \tag{15}$$

where ΔT is the sampling time interval, N is the sampling points number and it is found that $T = N \times \Delta T$ is met.

Assume that the sampling point number of the inner rotor in a rotating cycle is M_1(M_1 is obtained using the method of timing and counting the pulse number of the key phase signal simultaneously, and it does not have to be or may not be an integer). Then

$$\omega_1 = 2\pi/(M_1 \times \Delta T). \tag{16}$$

By substituting equation (16) into equation (15), it will be found that

$$A_{1I} = \frac{1}{N} \times \sum_{n=0}^{N-1} X(n) \sin(2\pi \times n/M_1). \tag{17}$$

in the same way,

$$A_{1R} = \frac{1}{N} \times \sum_{n=0}^{N-1} X(n) \cos(2\pi \times n/M_1), \tag{18}$$

$$A_{2I} = \frac{1}{N} \times \sum_{n=0}^{N-1} X(n) \sin(2\pi \times n/M_2), \tag{19}$$

$$A_{2R} = \frac{1}{N} \times \sum_{n=0}^{N-1} X(n) \cos(2\pi \times n/M_2). \tag{20}$$

In the equations above, the sampling point number of the inner rotor in a rotating cycle is M_2. It is noted that the essence of equations (17)–(20) is similar to that of DFT (Discrete Fourier Transformation), while the difference between these two is that the M_1, M_2 of the former do not have to be or may not be integers. So this identification method is named "Single Point Discrete Fourier Transformation".

The balancing experiment adopting the method of "Single Point DFT" is introduced as follows.

First, in Figure 2, after being recorded, four signals are sampled by the computer. The sampling frequency is controlled artificially by a delay time variable in the sampling program. The vibration of the right bearing seat is measured six times, and the experimental data are listed in Table 2. From Table 2, it is noted that the vibration data calculated by this method are of good stability.

Second, in order to verify the "Single Point DFT" method further, a balancing experiment with only one balancing plane is carried out to eliminate the vibration of the right bearing seat. The balancing method used here is the conventional influence coefficient method. This time the recorder is put away, the signals are passed directly to the computer for sampling. The experimental results are listed in Table 3. It must be pointed out that when the signals pass through the cassette recorder, they will all be amplified, and so the initial vibrations in Tables 2 and 3 are not the same, but their proportions are identical.

TABLE 2

Sampling frequency (Hz)	5708	5708	5708	4044	4044	4044	5708	5708	5708	4044	4044	4044
Rotating speed (rpm)	1560	1560	1560	1560	1560	1560	1572	1572	1572	1572	1572	1572
Amplitude	247	247	247	250	251	249	328	328	328	326	326	326
Phase (degree)	5	4	2	6	3	7	−88	−85	−87	−85	−85	−85
Max. difference of amplitude				4					3			
Max. difference of phase				5					3			
Relative amplitude difference				1.3%					0.9%			
Relative difference of phase				1.4%					0.8%			

Note: all the amplitudes are relative ones.

TABLE 3

	Outer rotor	Inner rotor
Initial vibration	$93\ e^{i0^\circ}$	$128\ e^{-i75^\circ}$
Add a 13-g trial weight to the inner rotor at the position of 216°	$92\ e^{i2^\circ}$	$139\ e^{-i41^\circ}$
Add a 13·2-g trial weight to the outer rotor at the position of 347·5°	$33\ e^{i7^\circ}$	$130\ e^{-i77^\circ}$
Influence coefficient	$4{\cdot}5\ e^{i152{\cdot}5^\circ}$	$5{\cdot}98\ e^{i167{\cdot}6^\circ}$
Unbalance	$20{\cdot}7\ e^{i152{\cdot}8^\circ}$	$21{\cdot}4\ e^{-i242{\cdot}6^\circ}$
Vibration after balancing	$5\ e^{i58^\circ}$	$17\ e^{-i62^\circ}$
Vibration decreasement percentage	94·6%	87%

TABLE 4

	Right bearing seat		Left bearing seat	
	Inner rotor	Outer rotor	Inner rotor	Outer rotor
Before balancing	$130\ e^{-i75^\circ}$	$93\ e^{i0^\circ}$	$57{\cdot}5\ e^{-i49^\circ}$	$288\ e^{i187^\circ}$
After balancing	$14\ e^{-i68^\circ}$	$13\ e^{i67{\cdot}9^\circ}$	$43\ e^{-i55^\circ}$	$12\ e^{-i87^\circ}$
Vibration decreasement percentage	89·3%	86%	25·2%	95·8%

Finally, a balancing experiment to eliminate the vibration of the whole machinery is carried out. In this experiment there are two balancing planes for each rotor. The balancing results are listed in Table 4.

Because the initial vibration in the left bearing seat caused exclusively by the inner rotor is much smaller than that of the others, the calculated correction weight to be added to the left balancing plane of the inner rotor is neglected. Its decrease is a result of the correction weight mounted on the right balancing plane of the inner rotor.

5. CONCLUSION

The Single Point DFT method, which is based on the correlation theory and the assumption that the integral time T is the change cycle of the amplitude $2\pi/|\omega_1 - \omega_2|$, is successful in identifying the unbalance of the dual rotors system with slightly different rotating speeds.

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