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FUNDAMENTAL FREQUENCY OF TRANSVERSE VIBRATION OF A CLAMPED RECTANGULAR PLATE OF CYLINDRICAL ORTHOTROPY

D. V. BAMBILL AND P. A. A. LAURA

Institute of Applied Mechanics (CONICET) and Department of Engineering, Universidad Nacional del Sur 8000 Bahía Blanca, Argentina

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1. INTRODUCTION

A vast amount of scientific literature is available for vibrating isotropic, orthotropic and anisotropic plates [1]. The present paper deals with a case which, apparently, has not been treated previously: the case of a rectangular plate of polar orthotropy (Figure 1). These constitutive characteristics are common in natural materials like wood and in certain situations of composites. On the other hand they may be caused by metallurgical processes [2]. It is assumed that the plate shown in Figure 1 is clamped at its four edges.

The fundamental frequency coefficient is determined using a variational formulation which yields excellent accuracy when the plate is isotropic [3]. Finally, the same approach is followed in the case of a clamped plate with a central hole with free edge using the polynomial co-ordinate function employed when dealing with the solid plate.†

2. APPROXIMATE ANALYTICAL SOLUTION

The problem of transverse vibrations of the structure element depicted in Figure 1 will be solved by minimization of the governing functional [4]

$$J[W(r,\theta)] = \frac{1}{2}\left\{\iint\left[D_r\left(\frac{\partial^2 W}{\partial r^2}\right)^2 + D_\theta\left(\frac{1}{r}\frac{\partial W}{\partial r} + \frac{1}{r^2}\frac{\partial^2 W}{\partial \theta^2}\right)^2 \right.\right.$$
$$+ 2\mu_\theta D_r \frac{\partial^2 W}{\partial r^2}\left(\frac{1}{r}\frac{\partial W}{\partial r} + \frac{1}{r^2}\frac{\partial^2 W}{\partial \theta^2}\right)$$
$$\left. + 4D_k\left(\frac{1}{r}\frac{\partial^2 W}{\partial r\,\partial \theta} - \frac{1}{r^2}\frac{\partial W}{\partial \theta}\right)^2\right] r\,\mathrm{d}r\,\mathrm{d}\theta$$
$$\left. - \rho h \omega^2 \iint W^2 r\,\mathrm{d}r\,\mathrm{d}\theta\right\}, \qquad (1)$$

† Obviously the natural boundary conditions at the hole edge are not satisfied.

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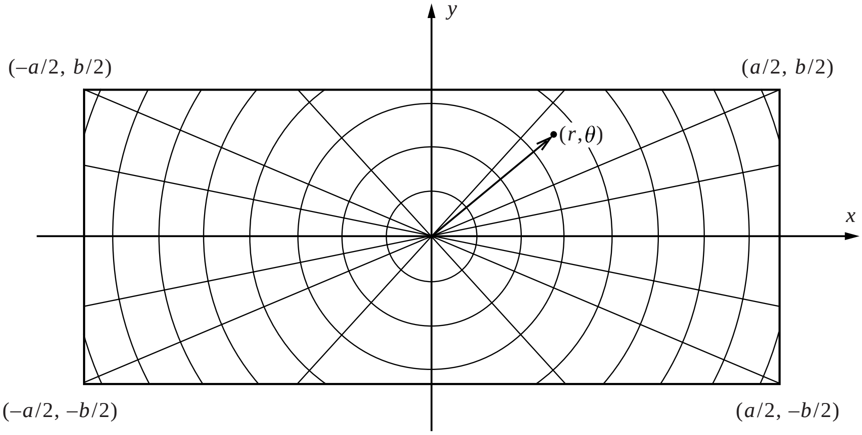

Figure 1. Vibrating clamped rectangular plate.

subject to the governing essential boundary conditions

$$W[L(x, y) = 0], \qquad \frac{\partial W}{\partial n}[L(x, y) = 0], \tag{2a, b}$$

where $L(x, y) = 0$ is the functional relation which defines the boundary of the domain and "n" is its outer normal.

Following previous studies [3] the following polynomial co-ordinate function is used:

$$W \cong W_a(x, y) = A\left[x^2 - \left(\frac{a}{2}\right)^2\right]^2\left[y^2 - \left(\frac{b}{2}\right)^2\right]^2, \tag{3}$$

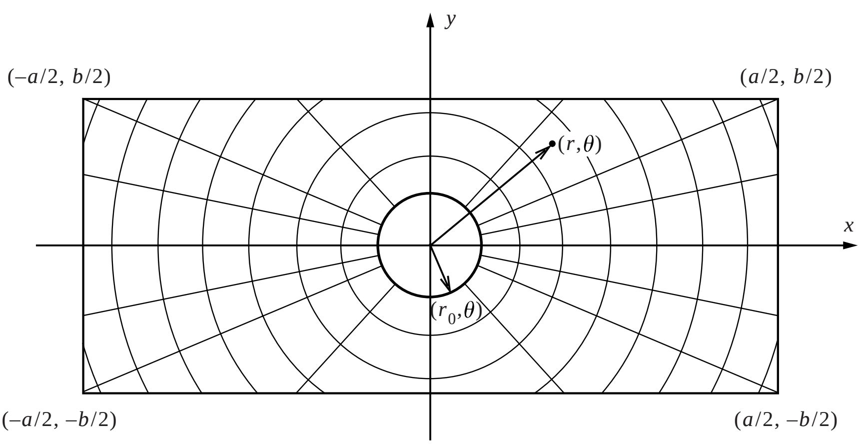

Figure 2. Vibrating clamped rectangular plate with a central, circular perforation of free edge.

TABLE 1

Fundamental frequency coefficient Ω_1 of (A) isotropic and (B) cylindrically orthotropic clamped rectangular plates

Case	$\frac{D_\theta}{D_r}$	$\frac{D_k}{D_r}$	μ_θ	a/b: 2/5	2/3	1	3/2	5/2
(A)				(23·648)[a]	(27·010)	(35·992)	(60·772)	(147·80)
	1	0·35	$\frac{3}{10}$	23·72	27·04	36·00	60·85	148·29
(B)	$\frac{6}{5}$	$\frac{1}{2}$	$\frac{3}{10}$	25·34	28·29	37·29	63·66	158·38
	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{3}{10}$	26·48	29·46	38·90	66·29	165·51
	2	1	$\frac{3}{10}$	30·57	32·57	41·99	73·29	190·86
	$\frac{5}{2}$	1	$\frac{3}{10}$	32·11	34·26	44·34	77·08	200·69

[a] Value obtained in reference [5].

which satisfies identically the governing boundary conditions

$$W\left(\frac{a}{2}, y\right) = W\left(-\frac{a}{2}, y\right) = W\left(x, \frac{b}{2}\right) = W\left(x, -\frac{b}{2}\right) = 0, \tag{4a}$$

$$\left.\frac{\partial W}{\partial x}\right|_{x=\frac{a}{2}} = \left.\frac{\partial W}{\partial x}\right|_{x=-\frac{a}{2}} = \left.\frac{\partial W}{\partial y}\right|_{y=\frac{b}{2}} = \left.\frac{\partial W}{\partial y}\right|_{y=-\frac{b}{2}} = 0. \tag{4b}$$

TABLE 2

Fundamental frequency coefficient Ω_1 of clamped square (A) isotropic and (B) cylindrically orthotropic plates with a concentric circular perforation

Case	$\frac{D_\theta}{D_r}$	$\frac{D_k}{D_r}$	μ_θ	r_0/a: 0·10	0·20	0·30	0·40	0·50
(A)				(35·67)[a]	(36·30)	(39·14)	–	–
	1	0·35	$\frac{3}{10}$	36·42	37·85	40·79	46·34	56·77
(B)	$\frac{6}{5}$	$\frac{1}{2}$	$\frac{3}{10}$	37·74	39·21	42·21	47·85	58·39
	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{3}{10}$	39·35	40·85	43·89	49·56	60·12
	2	1	$\frac{3}{10}$	42·47	44·08	47·31	53·26	64·21
	$\frac{5}{2}$	1	$\frac{3}{10}$	44·85	46·50	49·80	55·82	66·82

[a] Finite element results obtained in reference [6].

In order to substitute in equation (1), equation (3) is conveniently expressed in polar co-ordinates by means of the well known relations

$$x = r\cos\theta, \qquad y = r\sin\theta. \tag{5}$$

Accordingly, expression (3) becomes

$$W_a(r,\theta) = A\left[r^2\cos^2\theta - \left(\frac{a}{2}\right)^2\right]^2\left[r^2\sin^2\theta - \left(\frac{b}{2}\right)^2\right]^2. \tag{6}$$

Substituting equation (6) into equation (1) and minimizing the functional with respect to A one obtains an approximate expression for the fundamental frequency coefficient $\Omega_1 = \sqrt{\rho h/D_r}\,\omega_1 a^2$ as a function of D_θ/D_r, D_k/D_r and μ_θ. For $D_\theta/D_r = 1$ and $D_k/D_r = 0{\cdot}35$, one has the isotropic case with Poisson's ratio equal to 0·3.†

In the case of the plate with a concentric circular perforation with a free edge, expression (6) is also used as the approximate fundamental mode but the energy functional is evaluated between r_0 and the clamped outer boundary, Figure 2.

3. NUMERICAL RESULTS

Table 1 depicts fundamental frequency coefficients Ω_1 of clamped, isotropic and cylindrically orthotropic rectangular plates. In the case of isotropic plates the results are in excellent agreement with the values obtained by Leissa [5]. When dealing with cylindrically orthotropic plates the results are obtained as a function of D_θ/D_r and D_k/D_r for $\mu_\theta = 0{\cdot}30$.

Table 2 shows values of Ω_1 for the case of a clamped square plate with a central circular perforation with free edge. In the case of an isotropic plate the results are in good engineering agreement with those obtained in reference [6] by means of a finite element code for $r_0/a = 0{\cdot}10$, 0·20 and 0·30 [6].

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† Obviously in the case of a clamped isotropic plate the eigenvalues are independent of Poisson's ratio. On the other hand, when the plate possesses a perforation, Poisson's ratio comes into play.

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